PREDICTIVE FORMULAS OF COMPLEX MODULUS FOR HIGH AIR VOID CONTENT MIXES

Giovanni Giuliana, Ph.D.,
Contract Professor
Department of Civil Engineering
University of Rome “Tor Vergata”
Via del Politecnico 1
00133 Rome, Italy
Phone: (+39) 081 7683615 / (+39) 06 72597015
Fax: (+39) 081 7683740
e-mail: giogiuli@unina.it

Vittorio Nicolosi
Associate Professor
Department of Civil Engineering
University of Rome “Tor Vergata”
Via del Politecnico 1
00133 Rome, Italy
Phone: (+39) 06 72597075
e-mail: nicolosi@uniroma2.it

Bruna Festa
Associate Professor
Department of Transportation Engineering
University of Naples Federico II
Via Claudio 21
80125 Naples, Italy
Phone: (+39) 081 7683373
Fax: (+39) 081 7683740
e-mail: festa@unina.it

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ABSTRACT

Over the last 10 years, there has been a number of trials and developments of the porous asphalt (according to definition of European standard EN 13108-7) or open-graded friction course (according to definition used in USA), which has led to the use of high air void content mixes with better acoustic performances. Mixes with air void content greater than 20% are used at present in Europe and their use is spreading in other countries (e.g. USA, New Zealand, etc.).

The dynamic modulus ($|E^*|)$ is one of the most important performance parameters for characterizing bituminous mixes, and it is used as input in the mechanistic and mechanistic-empirical design methods (MEPDG). Since the measurement of stiffness in the laboratory is not straightforward, a commonly used approach is to estimate dynamic modulus by using predictive models. Several models are available in the literature for estimating $|E^*|$, but they were developed and calibrated on hot mixes with air void content less than 15%.

This paper describes a research carried out to evaluate if some predictive dynamic modulus equations work well for porous asphalt with high air void contents. Two predictive models were analyzed: the Witczak-Andrei model, and the Witczak-Bari model. The experimental values of $|E^*|$ on some high air void content mixes are compared with values obtained by previously mentioned predictive formulas. The results showed that proposed formulas “under predict” (Witczak-Andrei) or over predict (Witczak-Bari) the $|E^*|$, but they should work well if they are recalibrated. Basing on experimental results, methodology of calibration of the predictive models was proposed in order to be used for high air void content mixes.
INTRODUCTION

Porous Asphalt (according to definition of European standard EN 13108-7) or open-graded friction course (according to definition used in USA) is used worldwide since 1950 to improve wet weather driving conditions as it reduces hydroplaning, splash and spray behind vehicles, and improve wet pavement friction, furthermore it reduces traffic noise and increases surface reflectivity.

In the last decade bituminous binder modified with polymers and fibers have been extensively used in open-graded friction courses to improve the durability of mixtures and voids content (drainability). As matter of fact in some countries (e.g. European country, New Zealand, etc.) porous asphalt with an air void contents greater than 20% has been successfully used; while in many states of USA, air void content in OGFC tends to be higher, compared to the value (15%) suggested in the Federal Highway Administration mix design procedure (FHWA-RD-74-2) (1).

Effective characterization of pavement materials is a key requirement for a successful design. To be effective and useful for pavement design, characterization should be based on material properties that accurately capture the material response to external stimuli of traffic loading.

The complex modulus has been identified as a suitable parameter for characterizing stiffness of asphalt concrete, because of its ability to capture the visco-elastic response of the material, and it is recommended as input parameter for the design of asphalt pavement in the new mechanistic-empirical pavement design (MEPDG).

The guide provides 3 levels of input depending on the criticality of the project, the sensitivity of the pavement performance to a given input, the resources available to the designer, and the availability of input information at the time of the design. Level 1: Site and/or material specific inputs for the project are to be obtained through direct testing or measurements. Level 2: Correlations are established in the model to determine the required inputs. For example, the dynamic modulus could be estimated on results of tests performed on binders, aggregate gradation and mix properties. Level 3: This level produces the lowest accuracy. Inputs are typically user selected from national or regional default values, such as characterizing the HMA using its physical properties (gradation) and type of binder used.

As measurement of complex modulus in the laboratory is a complicated and time consuming operation, the MEPDG suggests estimating it by using empirical models. Several models are available in the literature for estimating the complex modulus but their performances may vary with the volumetric properties of the mix, as observed by some researchers. Particularly, the models were typically calibrated on dense and open graded asphalt but no high void content mix was usually considered.

OBJECTIVES

This paper presents the results of a study carried on for evaluating and calibrating the Witczak-Andrei and Witczak-Bari predictive dynamic modulus equations for porous asphalt with high void content (greater than 18%).

DYNAMIC MODULUS E* PREDICTIVE MODELS

Various models have been developed over the past several decades to predict the dynamic modulus $E'$ of hot-mix asphalt (HMA) based on regression analysis of laboratory measurements, the most well-known are: Witczak-Andrei model (2), Witczak-Bari model (3),
Hirsch model (4) and Al-Khateeb model (5). In this study, the accuracy of first two models in predicting modulus of high void content HMA was tested, as Witczak-Andrei model (1999) is currently used in the Level 2 and Level 3 designs of the newly developed Mechanistic-Empirical Pavement Design Guide (MEDPG) (6) and the Witczak-Bari model can be considered its evolution.

**Witczak-Andrei Model (1999)**

Witczak dynamic modulus predictive equation seems to be one of the most comprehensive mixture stiffness models used today. This model is capable to predict mixture stiffness over a wide range of temperature and loading frequency from information that is readily available from material specifications or volumetric design of mixture.

Andrei et al. (2) revised the Witczak-Fonseca model, using 2750 test data points from 205 HMA mixes. Witczak-Andrei model is shown in equation (1):

\[
\begin{align*}
\text{Log}|E^*| &= -1.249937 + 0.029232 \cdot P_{200} - 0.001767 \cdot (P_{200})^2 - 0.002841 \cdot P_4 - 0.058097 \cdot V_a + \\
&-0.802208 \cdot \frac{V_{\text{beff}}}{(V_{\text{beff}} + V_a)} + \frac{3.871977 - 0.0021 \cdot P_4 + 0.003958 \cdot P_{38} - 0.000017 \cdot (P_{38})^2 + 0.005470 \cdot P_{34}}{1 + e^{(-0.603313 - 0.313351 \log f - 0.393532 \log \eta)}}
\end{align*}
\]

where:

- \(|E^*| = \) Asphalt mix dynamic modulus (10^5 psi),
- \(\eta = \) Viscosity of binder (10^6 poise),
- \(f = \) Loading frequency (Hz),
- \(V_a = \) Air voids in the mix (% by volume),
- \(V_{\text{beff}} = \) Effective binder content (% by volume),
- \(P_{200} = \) % Passing # 200 (0.075 mm) sieve,
- \(P_{38} = \) Cumulative % retained on # 4 (4.75 mm) sieve,
- \(P_{34} = \) Cumulative % retained on 3/8 inch (9.5 mm) sieve,
- \(P_{34} = \) Cumulative % retained on 3/4 inch (19 mm) sieve.

**Witczak-Bari Model (2006)**

In 2006, Witczak and Bari (3) presented a new revised version of the widely used Witczak-Andrei predictive model, in which complex shear modulus \(G_b^*\) and phase angle \(\delta_b\) of binder replaced the viscosity, from the current A-VTS relationship, as direct input, because \(G_b^*\) can more effectively describes binder stiffness with changing in temperature and loading time.

The new model has the same mathematical structure (sigmoidal function) of the previous one, but it has better goodness of fit, least bias and highest accuracy over previous predictive model, as authors highlighted. Witczak-Bari predictive model is shown in equation (2):

\[
\begin{align*}
\text{Log}|E^*| &= -0.349 + 0.754 \cdot \left( \frac{G_b^*}{0.0052} \right) \cdot \left[ 6.65 - 0.032 \cdot P_{200} + 0.0027 \cdot P_{200}^2 + 0.011 \cdot P_4 - 0.0001 \cdot P_4^2 + \\
&+ 0.006 \cdot P_{38} - 0.00014 \cdot P_{38}^2 - 0.08 \cdot V_a - 1.06 \cdot \left( \frac{V_{\text{beff}}}{V_a + V_{\text{beff}}} \right) \right] \\
&+ 2.56 + 0.03 \cdot V_a + 0.71 \cdot \left( \frac{V_{\text{beff}}}{V_a + V_{\text{beff}}} \right) + 0.012 \cdot P_{38} - 0.0001 \cdot P_{38}^2 - 0.01 \cdot P_{34}
\end{align*}
\]

where:
$|E^*| = $ Asphalt mix dynamic modulus (psi),

$P_{200}, P_4, P_{38}, P_{34}, V_a, V_{beff}$ as previously defined,

$|G_b^*| = $ Dynamic shear modulus of asphalt binder (psi),

$\delta_b = $ Phase angle of asphalt binder (degree),

**MATERIALS AND SAMPLE PREPARATION**

The mixtures considered in this study are outside the range used for the calibration of above mentioned models (TABLE 1). As matter of fact, though the database used to develop and calibrate the models covers a large number of AC mixtures (205 in the Witczak-Andrei and 192 in the Witczak-Bari (see (6) appendix CC4 and (3)), the void content $V_a$ is lower than 15.1 % and the binder content $V_{beff}$ is in the range 6-24%.

As far as aggregate grading is concerned, the values of passing of 200, 3/8 and ¾ sieves are included inside ranges of values considered by Witczak-Andrei (1999), except for 1 mix which has a passing at 3/8 sieve outside the range, as can be observed in FIGURE 1 (in Europe the sieve size is in mm on the contrary US sieve size is in inch, so to compare aggregate gradations used for porous asphalt with those used for Witczak-Fonseca and Witczak-Andrei models, linear interpolation was used). On the contrary, all the mixtures used in this study have the passing at sieve 4 outside the range considered in the Witczak-Andrei (1999) database.

![FIGURE 1 Porous asphalt aggregate grading (dashed lines), considered in experimental study, and compared with Witczak-Andrei (1999) grading band (continuous lines).](image-url)

**Aggregates**

As porous asphalts are used for surface course, the aggregates have to be a mix of basalt and limestone to assure durable friction. The mixes of aggregate have to be open graded to assure an high volume of air, and the largest particles have to be generally smaller than 20 mm because the layer has a maximum thickness of 50 mm, rarely 60 mm.
In TABLE 1, it is shown aggregate characteristics using only the parameters that you find in predictive formulas. As you can note, the value of \( P_{34} \) is generally 0\%, except for 3 mixes, where it assumes values not higher than 2\%. The aggregate passing \( P_{200} \) is composed of a mix of filler and lime, because it’s very difficult to design porous asphalt with high volume of voids without lime.

<table>
<thead>
<tr>
<th>TABLE 1 Characteristics of Porous Mixtures Used</th>
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<tbody>
<tr>
<td>Properties</td>
</tr>
<tr>
<td>mix N.</td>
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<td>12*</td>
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</table>

*mixes marked by asterisk have been considered only for the analysis of Witczak-Andrei model.

Asphalt binder

Measurements of viscosity, penetration, ductility, and softening point temperature have served quite well over the years for specifying traditional binders and to predict their properties (e.g. by Van der Poel’s nomograph) although viscoelastic behavior of asphalt is too complicated to be described by these simple empirical tests, especially for newly high performance binders, used in porous asphalt concrete. As viscoelastic properties are needed to relate physical properties to performance and to develop a performance related binder specification, in the end of 80s the Strategic Highway Research Program (SHRP) introduced new tests specification. Although newly rational binder specifications give the chance to predict asphalt concrete performance (in terms of permanent deformation, fatigue cracking, and thermal cracking) more accurately, the implementation of this system faces some drawbacks outside USA, especially in Europe, and actually both systems are yet used.

As far as predictive models of dynamic modulus of asphalt concrete are concerned, in the Witczak-Andrei model, the asphalt binder is characterized by viscosity, therefore both traditional and newly specification may be used (1). On the contrary, in the Witczak-Bari model, viscoelastic properties of asphalt binder are needed, and SHRP specifications have to be used. Therefore, both viscosity and viscoelastic parameters are measured for describing the responses of polymer-modified binder (original pen 0-70, modifier SBS) in temperature range used in the experimental tests. As the objective of this study was to verify the accuracy of previously developed predictive models when applied to estimate porous asphalt concrete complex modulus, no specific analysis was performed on models describing mechanical behavior of polymer-modified binder.
Viscosity
For estimating viscosity of asphalt at any temperature, it was assumed that linear relationship, suggested in ASTM, between logarithm of the logarithm of the viscosity $\eta$, in centipoises, and temperature $T$R, in degree Rankine, is valid (7): $\log\log(\eta) = A + VTS \cdot \log(T)$R.

To predict more accurately regression parameters, A and VTS (FIGURE 2), viscosity data (TABLE 2) were obtained by both direct measures (Brookfield device), at high temperature 140-200 °C, and viscoelastic parameters (measured by rheometer at T<40°C):

$$\eta = \left(\frac{G^*_b}{10}\right)^{4.8628} \left(\frac{1}{\sin(\delta_b)}\right).$$

Observing FIGURE 2, it’s evident that the bitumen has different behaviors at ordinary and high temperatures. But the authors considered all data obtained with Brookfield device, rheometer.
Penetration test and Ring and Ball test. The values of A and VTS, obtained in this way, are quite close to those obtained without considering rheometer results, as suggested by Witczak.

Anyway, the authors suggest for modified bitumen to don’t consider Ring and Ball test because it gives anomalous values (FIGURE 2).

**Viscoelastic parameters**

The SHRP asphalt binder specification is based on the assumption that the performance of asphalt binders can be adequately represented by thermo-rheologically simple linear viscoelastic model and measured by the *Dynamic Shear Rheometer (DSR)* test. Providing that the assumption of linear viscoelastic behavior hold, the logarithm of the modulus values $|G^*|$, measured at different temperatures, can be plotted versus the logarithm of time (or frequency) and represented by an analytical model. Some models have been proposed so far; in this study the widely accepted Christensen-Anderson-Marasteanu (CAM) model (8), was used (equation 3):

$$|G^*(\omega)| = G_g \left[ 1 + \left( \frac{\omega_c}{\omega} \right)^v \right]^w$$

where:

- $|G^*(\omega)|$ is the absolute value of complex modulus as a function of frequency $\omega$ (GPa),
- $G_g$ is the glassy modulus (log [G_g] is considered fixed at 9.1),
- $\omega_c$, crossover frequency,
- $v$, $w$ are model parameters (obtained fitting the CAM model to test data obtained from the Dynamic Shear Rheometer).

Furthermore the principle of time-temperature superposition was applied although dynamic mechanical tests performed on the used binder showed that variations of the shift factor with temperature do not follow the same rational pattern over the entire range of temperature investigated. Therefore, the superposition principle was applied in a limited range of temperature (the same used in the testes on the asphalt concrete) and resulting data, at different temperatures, was “shifted” horizontally to form a “master curve” of the polymer-modified binder used in the experimental study (FIGURE 3).

![FIGURE 3 Complex Modulus Master Curve of modified binder used in the experimental study at the reference temperature T=10°C (model parameters: $G_g=1$ GPa, $\omega_c=1.21$ rad/s, $v=0.00465$, $w=0.0429$)](image-url)
As far as the phase angle is concerned, the model (equation 4) proposed by Christensen-Anderson, that relates phase to crossover frequency and to the above mentioned \( v \) and \( w \) parameters, didn’t fit well the measured data:

\[
\delta(\omega) = \frac{90 \cdot w}{1 + \left( \frac{\omega}{\omega_c} \right)^v} \tag{4}
\]

At the same time it was found that the linear equation (5) represents well the measure of phase angle in the frequency-temperature range used in the experimental tests (FIGURE 4):

\[
\delta(\omega) = -3.626 \times 10^{-5} \cdot G^* + 69.3163 \tag{5}
\]

As the development of a mathematical model for the phase is not the main goal of this study the equation (5) was used to evaluate \( \delta(\omega) \) in the following analysis.

![FIGURE 4 Comparison between binder phase angle data and linear model proposed](image)

**FIGURE 4** Comparison between binder phase angle data and linear model proposed

**Mixtures and Sample preparation**

The mixtures were designed using Marshall Method as suggested in literature (1, 9, 10) and according to EN UNI 13108-7 (11) and EN UNI 12697-34 (12) and.

Each mixture was tested using 3 specimens. These specimens were obtained from a slab compacted with vibratory plate to the same density obtained using Marshall Method mix design.

The specimens have a prismatic shape (50x5x5cm). The EN 12697-26 gives prescriptions about tolerances for the dimensions and density of the specimens obtained cutting the slabs, in particular:

- The difference between the maximum and minimum measured value per dimension shall be 1.0 mm at the most;
- The angle between adjacent longitudinal surfaces shall not deviate from a right angle by more than 1°;
- The bulk density of each specimen, measured in accordance with the EN 12697-6, shall not differ by more than 1% from the average apparent density of the batch.
DYNAMIC MODULUS TEST AND EXPERIMENTATION
The specimens obtained as above described were tested using Four Point Bending Test (13) at several temperatures, frequencies and levels of deformation.

Four Point Bending Test Procedure
The methods used in Europe for characterizing the stiffness of bituminous mixtures are specified in standard EN 12697-26 (13). Among alternative tests permitted by European standard, in this research four point bending test on prismatic specimens is used.

The scheme and the principal concepts of test are illustrated in FIGURE 5. A prismatic specimen is subjected to four-point periodic bending with free rotation and (horizontal) translation at all load and reaction points. The applied periodic displacement is symmetrical about the zero, and sinusoidal, and the displacement amplitude shall be constant as a function of time.

FIGURE 5 Scheme of four point bending test on prismatic specimens

Prismatic specimens are subjected to four point periodic bending in their linear range. The applied periodic displacement or force is symmetrical about the zero. The amplitudes of the stress and strain are measured as a function of time, as well as the phase lag between displacement and force. The amplitude of the displacement or force shall not damage the specimen during the test. Non-linear behavior limit depends on the material but it also varies with temperature and loading frequency for a given material. Therefore, it is recommended to perform linearity tests at the highest temperature and lowest frequency.

The initial stiffness modulus shall be determined as the modulus for a load cycle between the 45th and the 100th load repetition. The stiffness modulus of the bituminous mixture shall be determined as the arithmetic mean of the values obtained from 3 specimens.

Relationships to obtain, respectively, the storage and loss components of complex modulus are:

\[
E_1 = \lambda \left( \frac{F}{D} \cos \phi + \frac{\mu}{10^3} \omega^2 \right) \quad [\text{Pa}]
\]

\[
E_2 = \lambda \frac{F}{D} \sin \phi \quad [\text{Pa}]
\]

where:

- \( F = 2P \) = the force, [N];
- \( D \) = the displacement amplitude, [mm];
- \( \phi \) = the phase angle, [rad];
- \( \omega \) = the test frequency, [Hz];
\[ \lambda = \frac{LA}{bh^2} \left(3 \cdot \frac{A^2}{4 - \frac{L^2}{L^2}} \right) \Rightarrow \lambda \approx \frac{\sqrt{3} \cdot L^3}{\pi^4 \cdot I} \] 

is the shape factor, that depends on the shape and on the specimen size;

\[ A = \frac{L - \ell}{2} = \frac{L}{3} \text{, [mm]} \]

\[ \ell = \frac{L}{3} \] 

is the distance between the two loading application points, [mm];

\[ \mu = \frac{R(x) \left( \frac{M}{\pi^4} + \frac{\eta}{R(A)} \right)}{R(x)} \Rightarrow \mu \approx \frac{M}{\sqrt{3}} \] 

is the mass factor, [g];

\[ x = \frac{L}{2} \] 

is the point where the displacement is measured, [mm];

\[ R(x) = \left[ \frac{1}{3x^2} \left( \frac{L}{3} \right)^2 \right] \Rightarrow R\left( \frac{L}{2} \right) = 36^\circ \]

\[ M = \frac{\gamma A L}{g} \] 

is the mass of the specimen, [g];

\[ \eta = 0 \] 

is the mass, [g], of the movable parts of the equipment that influence the resultant force by their inertial effects.

The device has electronic data registration equipment in order to measure the loadings and the displacements with appropriate frequencies of acquisition. To reduce acquisition errors (14), it’s suitable to elaborate the sampled values of the loading and the displacement, with linear regression, so the expressions of the functions will be known.

The symbolic expression of loading or displacement function is:

\[ F(t) = C \cdot \sin(vt + \alpha) = A \cdot \sin(vt) + B \cdot \cos(vt) \]

where:

- \( C = \) amplitude of interpolating function,
- \( \alpha = \) phase angle of interpolating function,
- \( A = C \cdot \cos \alpha, \)
- \( B = C \cdot \sin \alpha. \)

The research of the A and B constants is made using least square method; in our case, since the regression is linear in its parameters, we have a linear system of two equations in the unknowns A and B, with solution:

\[ A = \sum_{i=1}^{n} (y_i^* \cdot x_{i1}) - B \cdot \sum_{i=1}^{n} x_{i1}^2 \]

\[ B = \frac{\sum_{i=1}^{n} (y_i^* \cdot x_{i1}) - \sum_{i=1}^{n} (x_{i1}^* \cdot x_{i2}) \sum_{i=1}^{n} x_{i1}^2}{\sum_{i=1}^{n} x_{i2}^2 - \sum_{i=1}^{n} (x_{i1}^* \cdot x_{i2})^2} \]
where:

\( n \) = total number of the displacement or loading measures acquired during one cycle,

\( y_i^* \) = displacement or the loading value at the i-th instant,

\( x_{1i} = \sin(v t_i) \),

\( x_{2i} = \cos(v t_i) \),

\( v = 2\pi\omega \) [rad/s],

\( \omega \) = the test frequency [Hz];

\( t_i = \frac{T}{n} \cdot \frac{1}{n \cdot f} \) = the time [s]

Dynamic modulus data obtained for several temperatures, loading frequencies and deformation levels (see in TABLE 3). The maximum test temperature was limited to 30°C because of problems of creep of the specimens as suggested in the European standard in the case of bending tests (13).

The results from mechanical test performed in accordance to the test protocol discussed above were compared with dynamic modulus predictions by using Witczak-Andrei and Witczak-Bari models and used for their recalibration.

**TABLE 3** Temperatures, Frequencies and Deformations Used in the Experimental Study

<table>
<thead>
<tr>
<th>Temperature [°C]</th>
<th>-5; 0; 5; 10; 15; 20; 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain [µstrain]</td>
<td>25; 50; 75; 100</td>
</tr>
<tr>
<td>Frequency [Hz]</td>
<td>5; 10; 15; 20</td>
</tr>
</tbody>
</table>

**Comparison between Bending and Compression test**

Worldwide different tests to measure asphalt stiffness exist. There are significant differences in these test conditions, frequency for cyclic loading and temperature test. It is interesting and necessary for researcher and specialists to correlate results from different tests. It is important to be able to estimate which level of performances corresponds to a mix design that has already been evaluated with another test.

It’s fundamental to compare results obtained with ASTM dynamic compressive test and EN four point bending test to show that the recalibration is necessary, because the discrepancy of values depends more on the different asphalt mixes than on complex modulus tests.

In this paragraph, the authors state preliminary results of comparison because, due considerable differences of two tests, it was very difficult to obtain cylindrical and prismatic specimens with the same characteristics inside the zone of measurement.

Anyway, preliminary results show that, generally, the values of dynamic modulus obtained by using ASTM compressive test are higher than those obtained using EN bending test. Particularly, the experimentation showed that dynamic moduli obtained with ASTM compressive test are generally 25-40% greater than those obtained with EN 4 point bending test, depending on temperature and frequency loading of the test. These results are confirmed by Carbonneau et al. (15), who find that dynamic modulus obtained by using 4 point bending test is generally lower than dynamic modulus obtained with different dynamic devices.
COMPARISON OF MEASURED AND PREDICTED DYNAMIC MODULUS AND RECALIBRATION OF MODELS

As above mentioned, maximum aggregate dimension for tested mixes is less than 20 mm, therefore the authors didn’t consider the variable $P_{34}$ in recalibration procedure of predictive models examined below.

Witczak-Andrei Model (1999)

Moduli $|E^*|$ were experimentally measured on same porous asphalt concrete and compared with values obtained by Witczak-Andrei model; in FIGURE 6 predicted versus observed $|E^*|$ values for the data base of high void mixes (304 data points from 12 mixes) are plotted. The Witczak-Andrei predictive model shows a poor goodness of fit with $R^2=0.50$ and $Se/Sy=0.71$, in arithmetic scale, for the high void content mixes tested. Moreover, as shown in FIGURE 6a, the model underestimates actual dynamic moduli notably, if you consider also what the authors observed above, comparing bending and compressive tests.

![Graph comparing predicted and observed $|E^*|$ for the Witczak-Andrei model](image)

**FIGURE 6** Comparison of predicted and observed $|E^*|$ for the Witczak-Andrei model

(a) not calibrated, (b) calibrated
Carrying out an overall calibration of the model parameters, a very good agreement with the values of $|E^*|$ observed for the high voids HMA can be obtained (FIGURE 6b). The goodness of fit was evaluated both in logarithmic and arithmetic scale and the calibrated model showed an excellent correlation coefficient ($R^2=0.942$ in arithmetic scale and $R^2=0.947$ in logarithmic scale) and a very small ratio $S_e/S_y$ ($S_e/S_y=0.241$ and $S_e/S_y=0.230$ respectively in arithmetic and logarithmic scale). As matter of fact all 304 data points are around the line of equality without any trace of significant bias as shown in FIGURE 6b.

The calibrated model is showed in equation (6) below:

$$L \log |E^*|_{\text{MPa}} = -7.042650 + 0.231409P_{200} - 0.01218(P_{200})^2 - 0.01995P_4 + 0.126288V_a + 17.98494 \frac{V_{\text{beff}}}{(V_{\text{beff}} + V_a)} + \left[ 6.1131135 - 0.002562P_4 - 0.0976269P_{38} + 0.0012708026(P_{38})^2 \right] \frac{1}{1 + e^{-0.27958 - 0.61555 \log f - 0.566591 \log n}}$$

where $|E^*|$ is in MPa (the authors preferred to indicate unit of measurement at subscript to avoid mix-up, because $|E^*|$ is in psi, in the original formula).

Furthermore the residuals (gap between actual $|E^*|$ and predicted $|E^*|$) were analysed in order to verify whether the model is appropriate; as expected for a good model the hypothesis of normality of the error distribution cannot be rejected at a significance level of 5%.

**Witczak-Bari Model (2006)**

Moduli $|E^*|$ were experimentally measured on same porous asphalt concrete and compared with values obtained by Witczak and Bari predictive model; in the FIGURE 7, predicted versus observed $|E^*|$ values for the data base of high void mixes (216 data points from 9 mixes) are plotted. The Witczak-Bari $|E^*|$ predictive model shows a poor goodness of fit ($R^2=0.49$ in logarithmic scale), for the high void mix tested; however in the original data base the $R^2$ was 0.90. A model optimization was carried out as suggested by Witczak and Bari (3) and the model shown in equation (7) came out:

$$L \log |E^*|_{\text{MPa}} = -0.359 + 0.754 \left| G_b \right|^{0.052} \left[ 6.65 - 0.032 \cdot P_{200} + 0.0027 \cdot P_{200}^2 + 0.011 \cdot P_4 - 0.0001 \cdot P_4^2 + 0.006 \cdot P_{38} - 0.00014 \cdot P_{38}^2 - 0.07266 \cdot V_a - 2.0479 \left( \frac{V_{\text{beff}}}{V_a + V_{\text{beff}}} \right) \right] + 5.8868 - 0.02476 \cdot V_a - 6.8248 \left( \frac{V_{\text{beff}}}{V_a + V_{\text{beff}}} \right) + 0.012 \cdot P_{38} - 0.0001 \cdot P_{38}^2 - 0.01 \cdot P_{38}$$

where $|E^*|$ is expressed in MPa (to distinguish it from that one used in the original formula, the authors preferred to indicate unit of measurement at subscript).

The goodness of fit was evaluated both in logarithmic and arithmetic scale and the calibrated model showed an excellent correlation coefficient ($R^2=0.953$ in logarithmic scale and $R^2=0.952$ in normal scale) and a very small ratio $S_e/S_y$ ($S_e/S_y=0.216$ in logarithmic scale and $S_e/S_y=0.217$ in arithmetic scale). Furthermore the residuals (observed $|E^*|$ minus predicted $|E^*|$) were analysed in order to verify whether the model is appropriate; as expected for a good model the hypothesis of normality of the error distribution cannot be rejected at a significance level of 5%.
Kolmogorov-Smirnov test statistic = 0.06868 < critical value 0.09254
Anderson-Darling test statistic = 1.2759 < critical value 2.5018,
Chi-Squared test statistic = 12.135 < critical value 14.067.

As matter of fact all 216 data points are around the line of equality without any trace of significant bias (FIGURE 7).

It should be underline that an excellent goodness of fit ($R^2=0.89$ in arithmetic scale) may be also obtained modifying only the multiplicative constant of variables related to volumetric properties of mix ($V_a$ and $V_{b_{eff}}$) and rheological binder parameters ($|G^*|$ and $\delta$).

The TABLE 4 shows the comparison of statistics of calibrated and original models.

FIGURE 7 Comparison of predicted and observed $|E^*|$ for the Witczak-Bari model a) not calibrated, b) calibrated
TABLE 4  Summary of Statistic Analyses on Recalibrated Predictive Models

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<tr>
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</thead>
<tbody>
<tr>
<td>Total mixes</td>
<td>12</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Data Points</td>
<td>304</td>
<td>216</td>
<td></td>
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Goodness of fit in normal (arithmetic) Scale

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<tbody>
<tr>
<td>$S_e/S_y$</td>
<td>0.71</td>
<td>0.241</td>
<td>0.6364</td>
<td>0.217</td>
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<tr>
<td>$R^2$</td>
<td>0.50</td>
<td>0.942</td>
<td>0.5950</td>
<td>0.952</td>
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Goodness of fit in Logarithmic Scale

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</thead>
<tbody>
<tr>
<td>$S_e/S_y$</td>
<td>--</td>
<td>0.230</td>
<td>0.7136</td>
<td>0.216</td>
</tr>
<tr>
<td>$R^2$</td>
<td>--</td>
<td>0.947</td>
<td>0.4907</td>
<td>0.953</td>
</tr>
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</table>

CONCLUSIONS AND RECOMMENDATIONS

In this study the authors analyzed two well-known predictive dynamic modulus models, Andrei-Witczak (1999) and Witczak-Bari (2006) models, to verify if they can be used for high void content asphalts (Va>18%). Dynamic modulus of several porous asphalts was measured (304 test points) by bending test and compared with values predicted by the above mentioned models.

The result analysis shows that both models, once recalibrated, show very good agreement with the observed $|E^*|$ values for the mixes tested. Therefore the mathematical sigmoidal function form and the predictor variables used in the analysed models are appropriate for representing the dynamic modulus data of porous asphalts tested.

The recalibrated models show better goodness of fit statistics, least bias and highest accuracy, in the range tested, than the original models ($R^2$ values in arithmetic scale were 0.65 and 0.80 for original Witczak-Andrei and Witczak-Bari models respectively), even if it is well known that goodness of fit is highly dependent upon the number of observation and the range of variables in the data base.

Therefore an effort should be undertaken to expand the current database although a quite large number of aggregate grading curves, for high voids of porous mixes, were used in this study to recalibrate the models. Particularly further test results should be obtained on other types of special modified binders.

REFERENCES


