An Analysis Method for the Correlation between Catenary Irregularities and Pantograph-catenary Contact Force

Yuan Zhang
State key laboratory of rail traffic control and safety, Beijing Jiaotong University
School of Traffic and Transportation, Beijing Jiaotong University
No.3 Shang Yuan Cun, Hai Dian District, Beijing, China
Phone: 86-10-51683973
Fax: 86-10-51684081
Email: zhangyuan_111@126.com

Yon Qin
State key laboratory of rail traffic control and safety, Beijing Jiaotong University
No.3 Shang Yuan Cun, Hai Dian District, Beijing, China
Phone: 86-10-51683846
Fax: 86-10-51683846
Email: researches@163.com

Xiao-qing Cheng
State key laboratory of rail traffic control and safety, Beijing Jiaotong University
No.3 Shang Yuan Cun, Hai Dian District, Beijing, China
Phone: 86-10-51684081
Fax: 86-10-51684081
Email: 11114220@bjtu.edu.cn

Li-min Jia
State key laboratory of rail traffic control and safety, Beijing Jiaotong University
No.3 Shang Yuan Cun, Hai Dian District, Beijing, China
Phone: 86-10-51683824
Fax: 86-10-51683824
Email: teaches@163.com

Zong-yi Xing (corresponding author)
School of Mechanical Engineering, Nanjing University of Science and Technology
200 Xiao Ling Wei Street, Nanjing, China
Phone: 86-10-51683973
Fax: 86-10-51684081
Email: zhangyuan111@gmail.com

Word count: 4299
Figures and Tables: 12 (x 250) 3000
Total: 7299
Submission date: June 26, 2012
ABSTRACT

Pantograph-catenary contact force provides the main basis for evaluation of current quality collection; however, the pantograph-catenary contact force is largely affected by the catenary irregularities. To analyze the correlated relationship between catenary irregularities and pantograph-catenary contact force, a method based on NARX (Nonlinear Auto-Regressive with eXogenous input) neural networks was developed. First, to collect the test data of catenary irregularities and contact force, the pantograph/catenary dynamics model was established and dynamic simulation was conducted using MATLAB/Simulink. Second, catenary irregularities were used as the input to NARX neural network and the contact force was determined as output of the NARX neural network, in which the neural network was trained by an improved training mechanism based on the regularization algorithm. Third, the simulation results and the comparison with other algorithms indicate the validity and superiority of the proposed approach.

Key words: Catenary irregularities; Pantograph-catenary contact force; NARX neural networks; Correlation analysis

1 INTRODUCTION

With the development of the high-speed railway, electric traction has become the dominant mode providing the train power. It is the key to protecting the safety of running high-speed railways. The EMU (Electric Multiple Units) assure that the current remains in contact with the catenary stably by use of a pantograph-head sliding plate (1, 2). The catenary irregularities are an important cause of current collection performance degradation which may cause impact when the pantograph slips off the catenary. The impact would cause vibration of the pantograph catenary and reduce the current collection performance. The pantograph-catenary contact force is the basic index to measure the quality of current collection, and an inappropriate pantograph-catenary contact force is the main cause of offline pantograph and contact wire fatigue damage (3). In certain operation conditions, pantograph-catenary contact force is influenced by catenary irregularities which would be dominant in high speed, and slight irregularities may cause serious fluctuation of contact force even leading to the pantograph coming off the catenary (4,5). Therefore it is necessary to analyze the relationship of catenary irregularities and pantograph-catenary contact forces.

Some scholars studied preliminary correlation analyses concerning the catenary irregularities and pantograph-catenary contact forces. Nagasaka et al (4) designed the measurement and estimation device of catenary irregularities for analyzing the effect on Pantograph-catenary contact force. Han et al (5) analyzed the influence of rigid catenary irregularities on the contact force, and pointed out that the catenary irregularities are the main factors to determine the current collection performance of rigid contact suspension. Takamura et al (6) researched the pantograph vertical vibration which is caused by rigid suspension contact irregularities and proposed the vibration frequency formula. Usuda (7, 8, 9) presented an accurate method to measure the pantograph-catenary contact force, and discussed the relationship between the catenary abrasion and the pantograph-catenary contact force and predicted the abrasion and strain of catenary in high-speed railways. Bennet et al (10) studied the pantograph-catenary contact force detection and a relevant mechanical calculation method. Zhang (11) described the catenary irregularities with full cosine wave, and detected the irregularities which the catenary brings to the contact force in cases of a continuous harmonic wave and a single wave. Xie (12) constructed the pantograph-catenary dynamic model and analyzed the power spectrum of catenary irregularities. All of the above studies focus on the physical detection or impact analysis from single catenary
irregularities only, and the research concerning the correlation analysis between catenary irregularities and
pantograph-catenary contact force remains at the preliminary qualitative analysis phase. The quantitative
correlation analysis between the two is still lacking.

The system of catenary irregularities and pantograph-catenary contact force is a typical complex
nonlinear dynamic system. Generally it can be described approximately by simplified simultaneous differential
equations, but the result has some deviation from the actual system. The neural network can be used to describe
any nonlinear system, and has a strong self-learning and fault tolerance ability. For these reasons NARX neural
network (13) has been employed to describe the complex dynamic relationship between catenary irregularities
and pantograph-catenary contact force. Setting the input of the network are the catenary irregularities and the
output is pantograph-catenary force to correlate the analysis of the relationship between them.

This paper is organized as follows: section 1 provides some basic descriptions of catenary irregularities
and pantograph-catenary contact force, and describes the test data collection methods; section 2 presents NARX
neural network, and discusses in detail the regularization algorithm; section 3 conducts the simulation tests
based on the method proposed and analyzes the test result; Finally, section 4 makes some conclusions and the
direction of future research directions is also given.

2 CATENARY IRREGULARITIES AND PANTOGRAPH-CATENARY CONTACT FORCE

2.1 Basic Concepts

There is no clear common definition or measurement indicator for catenary irregularities. The authors of
reference (5) proposed that the rigid catenary irregularities mean that the deviation of the contact surface extends
along the current flow with the ideal smooth contact surface, and as a function of the value of the amplitude,
divided it into large and small irregularities. Researchers in reference (14) studied the catenary irregularities in
elastic suspension mode which is commonly used in high-speed railway, and defined the elastic catenary
irregularities both in the broad and narrow sense: The catenary irregularities are the deviation between the actual
geometric dimensions and the ideal geometric dimensions of the contact surface, and the definition focuses on
researching the influence of catenary abrasive hard spots, hard bend on pantograph vibration and current
collection performance; in the broad sense, besides catenary geometric factors, the factors such as the catenary
tension, the elastic uniformity, catenary structures, wire material which may cause deviation between the actual
contact force and ideal current collection value are also included to define the catenary irregularities.

Relative to the above definitions, the influence of slight irregularities on the pantograph-catenary contact
force is the only consideration in this paper. It means that in this paper the catenary irregularities are the
deviation of the catenary actual geometric state along the tangential direction from the theoretical state which is
shown in figure 1. Since the slight catenary irregularities are essentially random processes, the random noises at
certain ranges of amplitude have been employed to approximate the irregularities.

Pantograph-catenary contact force is the vertical pressure generated during the process as the
pantograph rises up to contact the catenary, which also can be interpreted as the uplift force produced by the
pantograph. In order to get a stable current performance, the electric locomotive requires a constant pressure
between the pantograph and catenary during the operation. However, in actual operation due to the catenary
irregularities and aerodynamics, the pantograph-catenary contact force keeps changing constantly.
2.2 Pantograph-catenary Coupling Model

Pantograph-catenary system is a complex dynamic system, which is usually studied as an entire system coupled to pantograph-catenary contact force. The pantograph-catenary system model has been shown in figure 2 (a). During operation, the value of pantograph-catenary contact force changes along with the stiffness of the catenary. Since the catenary stiffness is not constant and changes in each hanger span, the impact caused by the catenary on the pantograph can be described simplify by an equivalent variable stiffness spring. Based on the stiffness formula of the catenary proposed in reference (15), the contact between the pantograph and catenary can be modeled as the contact between the pantograph and a variable stiffness spring system, as shown in equation (1):

$$k(t) = k_0(1 + sf_1 + 2f_2^2 + 3f_1^2 + 4f_3^2 + 5f_4^2)$$ (1)
where: \( f_i = \cos\left(\frac{2 \pi i}{L}\right) \), \( f_z = \cos\left(\frac{2 \pi}{L} z\right) \), \( f_j = \cos\left(\frac{\pi i}{L}\right) \), \( f_\lambda = \cos\left(\frac{\pi}{L} \lambda\right) \); \( v \) is the velocity (m/s); \( L \) is the catenary hanger span (m); \( L_i \) the distance between the catenary droppers (m); \( k_0 \) is the average stiffness (N/m); \( a_1, a_2, a_3, a_4, a_5 \) are stiffness difference coefficients.

The pantograph is composed of the upper and lower frame and the slide plate, and generally its model is the equivalent mass model. Based on the principle of kinetic energy equivalence, it can be simplified the original structure into an equivalent lumped mass model. According to the number of lumped masses, the lumped mass model can be divided into unitary, binary, ternary and multi-models. The ternary model (16) was adopted in this paper, which is shown in figure 2 (b), and the dynamic equations of pantograph are as follow:

\[
m_1(\ddot{z}_1 + \dot{y}) + k_1(z_1 - z_2) + c_1(\dot{z}_1 - \dot{z}_2) + K(t)(z_1 + u) = 0 \tag{2}
\]

\[
m_2\ddot{z}_2 + k_2(z_2 - z_1) + k_3(z_2 - z_3) + c_2(\dot{z}_2 - \dot{z}_1) + c_3(\dot{z}_2 - \dot{z}_3) = 0 \tag{3}
\]

\[
m_3\ddot{z}_3 + k_3(z_3 - z_2) + c_3(\dot{z}_3) = F \tag{4}
\]

where \( K(t) \) is the catenary equivalent stiffness which is shown in equation (1); \( F \) is the static uplift force (N); \( u \) denotes the catenary irregularities; \( m_1, m_2, m_3 \) are the pantograph head equivalent mass, the upper frame equivalent mass, and the lower frame equivalent mass respectively; \( k_1, k_2 \) are the pantograph head stiffness and the stiffness between upper and lower frame respectively; \( c_1, c_2, c_3 \) are the pantograph head damping, upper and lower frame damping and the damping between the lower frame and the vehicle body respectively.

Based on the catenary and pantograph model, the catenary irregularity was introduced by the catenary dynamic uplifting quality, and the pantograph-catenary coupled dynamic model was established, and the testing data of catenary irregularities and pantograph-catenary contact force was obtained by dynamic simulation using MATLAB / Simulink.

3 NARX NEURAL NETWORKS AND REGULARIZATION ALGORITHM

3.1 NARX Neutral Network

NARX can be regarded as a BP neural network with time delay input and delayed feedback from the output to the input (17). The single-input single-output NARX neural network can be divided into four layers: input layer, time delay layer, hidden layer and output layer as shown in figure 3. Among them, the input layer is used to get the signal without any calculation; the delay layer is the multi-step delay operator of the network, which is used for time delay of the input and output feedback signals; the function of the hidden layer is to do a nonlinear process for the time delay signals with activation function; and the output layer is used to provide a linear weighting for the output of the hidden layer and to get the final network output. Setting each input delay as \( P \) and the output delay as \( Q \), the output of the \( i \)th hidden node is given by

\[
O_i = f\left[ \sum_{r=0}^{R} w_{ir}u(t - p) + \sum_{q=0}^{Q} w_{iq}v(t - q) + b_i \right] \tag{5}
\]

where, \( f \) is the activation function of the hidden layer nodes; \( w_{ir} \) is the weight between the \( i \)th hidden layer node and the \( p \)th time delay layer node of the input signal; \( u(t-r) \) is the \( i \)th time delay output of input signal \( u(t) \); \( w_{iq} \) is the weight between the \( i \)th hidden layer node and the \( q \)th time delay layer node of the output signal; \( v(t-q) \) is the
Normalized time delay output of feedback signal $y(t)$; $b_i$ is the threshold of the $i^{th}$ hidden layer node.

![Diagram of NARX Neutral Network](image)

**FIGURE 3 Structure of NARX Neutral Network**

NARX neural networks contain multi-step time delays of input and output signal which can reflect a wealth of historical information. Compared with BP or a simple recurrent neural network, NARX neural networks describe better the characteristics of dynamic time-varying systems and effectively permit the modeling of complex dynamic system. For these reasons, NARX neural networks are considered suitable for analyzing the correlation of the catenary irregularities and pantograph-catenary contact force.

### 3.2 Regularization Algorithm

In NARX neural network, the typical training algorithm contains Backpropagation Through Time (BPTT), Real Time Recursive Learning (RTRL), Dynamic BP (DBP) and Bayesian Regularization (BR). Experience shows that BPTT is not applicable to online work, the DBP algorithm is too complex and the RTRL has a lower efficiency (18). BR algorithms have a good generalization performance (19), but the implementation process is complex with many unknown parameters that lack proven method to determine them, and the calculation is extensive in order to fix the probability density function using the Bayesian formula (20, 21). A regularization algorithm is streamlined and efficient; allowing one to abandon the process of fixing the probability density function. With a regularization algorithm, the neural network will get a smaller weight after training, and the network response would have a smoothing tendency and less possibility of over fitting, Thus it was used to train the NARX neural network.

In the regularization algorithm, the network performance evaluation function is shown in equation (6).

$$ F(w) = (1 - \gamma)E_D + E_w $$

where $\gamma$ is the modifying factor, $0 \leq \gamma \leq 1$; $E_w$ is the sum of squares of the network weights and thresholds; $E_D$ is the sum of squares of the error.

Assuming a NARX neutral network with $n$ time delay layer nodes, $h$ hidden layer nodes and $m$ outputs, for the $p^{th}$ pair of input and output sample data:

$$ E_{D(p)} = \sum_{k=1}^{n} (d_{k}^{(p)} - y_{k}^{(p)})^2 $$

$$ E_{w(p)} = \sum_{k=1}^{h} b_k^2 $$

$$ E_{D(p)} = \sum_{k=1}^{n} (d_{k}^{(p)} - y_{k}^{(p)})^2 $$

$$ E_{w(p)} = \sum_{k=1}^{h} b_k^2 $$

TRB 2013 Annual Meeting

Paper revised from original submittal.
\[ E_w^{(p)} = \frac{1}{N_w} \left[ \sum_{j=1}^{n} \left( w_{jk}^{(p)} \right)^2 + \sum_{k=1}^{m} \left( w_{jk}^{(p)} \right)^2 \right] \]  

(8)

where \( d_k \) is the target output of the \( k \)th output layer node; \( y_k \) is the network output of the \( k \)th output layer node; \( N_w \) is the number of the adjustable weight of the neural network; \( w_{ij} \) is the weight between the \( i \)th time delay layer node and the \( j \)th hidden layer node; \( w_{jk} \) is the weight between the \( j \)th hidden layer node and the \( k \)th output layer node.

Using the gradient descent method to adjust the weights, and setting the hidden layer activation function \( f_h(x) = \frac{2}{1+e^{-x}} \) and the output layer transfer function \( f_o(x) = x \), and the weight adjustment of the output layer and the hidden layer, the resultant weight adjustments are shown in equations (9) and (10) as.

\[ w_{jk}^{(p)} = (1 - \gamma) \left( d_k^{(p)} - y_k^{(p)} \right) x_{lj}^{(p)} + \frac{2}{N_w} w_{jk}^{(p-1)} \]  

(9)

\[ w_{ij}^{(p)} = (1 - \gamma) \left( y_i^{(p)} \right) w_{jk}^{(p)} + \frac{2}{N_w} w_{ij}^{(p-1)} \]  

(10)

Then, the weights of the output layer and the hidden layer after adjusting become

\[ w_{jk}^{(p)} = w_{jk}^{(p-1)} \]  

(11)

\[ w_{ij}^{(p)} = w_{ij}^{(p-1)} \]  

(12)

where \( x_{lj} \) is the input of the \( j \)th hidden layer node; \( x_i \) is the output of the \( i \)th input layer node; \( \eta \) is the learning rate.

4 EXPERIMENT AND RESULTS

4.1 Data Collection and Processing

A typical elastic catenary suspension in China was taken as an illustrative example and the pantograph-catenary coupling dynamic model was simulated by MATLAB / Simulink. The parameters in catenary stiffness equation (1) are \( v=250 \text{km/h}, L = 60 \text{m}, L_1 = 8 \text{m}, k_0 = 1925 \text{N/m}, \alpha_1 = 0.0755, \alpha_2 = 0.0735, \alpha_3 = 0.1459, \alpha_4 = 0.0575, \) \( \alpha_5 = 0.0699. \) The parameters in the pantograph ternary model are \( F=90 \text{N}, m_1=6.21 \text{kg}, m_2=7 \text{kg}, m_3=12 \text{kg}, \) \( k_1=2650 \text{N/m}, k_2=10000 \text{N/m}, c_1=100 \text{N/s/m}, c_2=100 \text{N/s/m}, c_3=70 \text{N/s/m}. \) Catenary irregularities \( u \) result from random noise signal which averages zero and the amplitude range is \([-0.5 \text{mm} \text{ to } +0.5 \text{mm}].\)

Form the computer simulation model, we have collected 2000 pairs of input and output data at the simulation time of 20s for a sampling frequency of 100Hz. All pairs of the data were separated into two groups: the first group with 1300 pairs of data is employed to train the NARX neutral network and the remaining 700 data pairs are used to test the neutral network. To reflect the influence on the pantograph-catenary contact force caused by other factors, a white Gaussian noise of which the amplitude is 5% of the contact force was overlaid. The collected data are shown in figure 4; figure 4(a) portrays the catenary irregularities data and figure 4(b) the pantograph-catenary contact force data. In order to improve the learning efficiency and to speed up the convergence of the neural network, all of the input data and output data were normalized with the following functions.
\( x^{\text{scal}} = \frac{x}{x_{\max}} \frac{x_{\min}}{x_{\min}} \)  

where \( x, x_{\max} \) and \( x_{\min} \) are the original, the maximum and the minimum values respectively, and \( x^{\text{scal}} \) is the value which has been processed.

\[
RMSE(y, y_m) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y(i) - y_{M}(i))^2} 
\]

\[
R(y, y_m) = \frac{\sum_{i=1}^{N} (y_i - \overline{y})(y_{M,i} - \overline{y}_M)}{\sqrt{\sum_{i=1}^{N} (y_i - \overline{y})^2 \sum_{i=1}^{N} (y_{M,i} - \overline{y}_M)^2}} 
\]

where, \( y \) is the target outputs, \( y_{M} \) is the neural network outputs; \( N \) is the number of the data samples; \( \overline{y} \) and \( \overline{y}_M \) are the averages of the \( y \) and \( y_{M} \) samples, respectively.

First of all, to indicate the complex relationship between the catenary irregularities and the Pantograph:catenary contact force, the experiment was carried out based on the BP neural network with the Levenberg-Marquardt (LM) algorithm and the result is shown in TABLE 1. The training and the testing RMSE of the BP neural network are 0.1393 and 0.2401 respectively and the correlation coefficient \( R \) is 0.4274 and 0.1845 respectively. At the same time, figure 5 shows the comparison of the target outputs and the BP neural
network output. Among them, figure 5(a) shows the comparison of the training data samples while figure 5(b) shows the comparison of the testing data samples. It can be seen that the BP neutral network cannot describe the complex relationship between the catenary irregularities and the pantograph-catenary contact force.

<table>
<thead>
<tr>
<th>TABLE 1 the Performance Index of Each Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models</td>
</tr>
<tr>
<td>Training RMSE</td>
</tr>
<tr>
<td>Testing RMSE</td>
</tr>
<tr>
<td>Training R</td>
</tr>
<tr>
<td>Testing R</td>
</tr>
</tbody>
</table>

In order to demonstrate the effectiveness of the NARX neural network, both the NARX neural network with a regularization algorithm (referred to as NARX-R) and the Elman neural network with the LM algorithm (referred to as Elman-LM) were implemented. Based on the expert experience and trial and error method, for NARX-R, the number of input and output delay steps was determined as 45 and the number of hidden layer nodes was set to 17. With the Elman-LM, the number of hidden layer nodes was set at 20.

Figure 6 shows the curve of mean-squared training error using the iterative process of Elman-LM. After 300 epochs training, the training error became stable. Figure 7 shows the training RMSE of NARX-R changes with the iterations, which shows that the error was no longer reduced after 450 epochs. Therefore, the number of iterations of Elman-LM and NARX-R were initialized to 300 and 500 respectively.
The number of iterations
Mean squared error

FIGURE 6 Training Process of Elman Neural Network

The number of iterations
RMSE

FIGURE 7 Training Process of NARX Neural Network

The RMSE and R of each neural network are listed in Table 1: the training and testing RMSE of Elman-LM are 0.1089 and 0.1189, respectively; the training and testing R are 0.7058 and 0.6988, respectively; the training and testing RMSE of NARX-R are 0.0533 and 0.1100, respectively; and the training and testing R are 0.9384 and 0.8029, respectively. Figure 8 and 9 respectively illustrate the correlation of the target outputs and the neural network outputs. Among them, figures 8 (a) and 9 (a) are training data-dependent and figures 8 (b) and 9 (b) are testing data-dependent. All results demonstrate that both the testing and training RMSE of the NARX-R are less than by the Elman-LM and both the testing and training R of NARX-R are larger than Elman-LM, which means that the accuracy of NARX-R is better than that of Elman-LM and that NARX-R is better suited to analyze the complex relationship between catenary irregularities and pantograph-catenary contact force.
For delineating the performance of NAEX-R and Elman-LM more directly, the comparisons of the target outputs and the neural network outputs of Elman-LM and NARX-R are shown in figures 10 and 11 in which figures 10(a) and 11(a) are for the training data, and figures 10(b) and 11(b) are for the testing data. It is thus clear that comparing with Elman-LM, the neural network outputs of NARX-R displays a better conformity with the target outputs and can follow the tendency of the target outputs better.

In summary, the NARX-R method proposed in this paper can fit the characteristics of the complex dynamic system better, and accurately achieve the correlation analysis of the catenary irregularities and pantograph-catenary contact force.
FIGURE 10 Comparisons of the Target Outputs and Elman Neural Network Outputs

FIGURE 11 Comparisons of the Target Outputs and NARX Neural Network Outputs

5 CONCLUSIONS

Based on the NARX neural network, a correlation analysis method is developed to analyze the complex
dynamic relation between the catenary irregularities and the pantograph-catenary contact force. In order to train
the NARX neural network, a pantograph-catenary coupling dynamic model is constructed to obtain the
experimental data of catenary irregularities and pantograph-catenary contact force and a regularization
algorithm is adopted. The simulation results and comparisons with other neural networks demonstrate the
effectiveness and validity of the proposed method. Further research efforts will be made toward including other
kinds of catenary irregularities and improvements of the structure of the NARX neural network and optimization
of the algorithm to enhance network performance. In addition, the experimental data collected in this paper is
obtained by using computer simulation model, so the real measurements of catenary irregularities and the
pantograph-catenary contact force will be needed to further verify the validity and improve the applicability of
the method.

ACKNOWLEDGEMENT
This research was sponsored by National Key Technology R&D Program of China (No. 2011BAG01B05) and
National High-tech R&D Program of China (863 Program, No. 2011AA110501) and the State Key Laboratory
of Rail Traffic Control and Safety (No. RCS2010ZZ002) of Beijing Jiaotong University. Their supports are
gratefully acknowledged.

REFERENCE
(3) K. Lee. Analysis of dynamic contact between overhead wire and pantograph of a high-speed electric train.
Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, Vol.221,
(4) S. Nagasaka, M. Aboshi. Measurement and Estimation of Contact Wire Unevenness. Quarterly Report of
(6) T. Takemura, Y. Fujii, M. Shimizu. Characteristics of over-head rigid conductor line having T-type cross
(7) T. Usuda. The Pantograph Contact Force Measurement Method in Overhead Catenary System. World
Accessed September 9, 2011
2011, pp.214.
(9) T. Usuda. Estimation of Wear and Strain of Contact Wire Using Contact Force of Pantograph. Quarterly
(10) Bennet, J., Montesinos, J., Cuartero, F., Rojo, T. Arias. E Advanced Algorithm to Calculate Mechanical
(11) W. H. Zhang, G. M. Mei, L. Q. Chen. Analysis of the influence of catenary’s sag and irregularity upon the
Chinese)
(12) J. Xie, Z. G. Liu, Z. W. Han. Pantograph and Overhead Contact Line Coupling Dynamic Model Simulation
and Analysis of Imbalance of Overhead Contact Line. Electric Railway, No. 6, 2009, pp. 23-26,29. (In
Chinese)
(13) Siegelmann H. T. Computational capabilities of recurrent NARX neural networks. IEEE Transactions on


