Assigning User Class Link Flows Uniquely

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The standard method for predicting traffic flows on urban road networks, static user-equilibrium traffic assignment, is based on the principle that drivers seek their own least cost routes from their origins to their destinations. This principle corresponds to a user-equilibrium state in which all used routes have equal costs and no unused route has a lower cost for every origin-destination pair. This problem can be formulated as a convex optimization problem with linear constraints, and solved with an iterative algorithm. Although the total flows on links of the urban road network are uniquely determined by this formulation, multi-class link flows are not. An additional assumption, the condition of proportionality, that the proportion of travelers on each of the two alternative, equal-cost segments (sequences of links) should be the same regardless of their origin or their destination, may be imposed to determine these flows uniquely.

Two multi-class assignments to the Chicago regional network were performed in which the order of the trip matrices was specified as car followed by truck, and truck followed by car; each was iterated towards a user-equilibrium with a relative gap less than 1E-8 with the travel forecasting software system, Visum. The total link flows for the two assignments are effectively equal; however, substantial differences exist between the class link flows determined by the arbitrary ordering of the matrices in the two assignments. Post-processing to impose the condition of proportionality on the class O-D flows removed about 90% of these differences. Analyses of class link flows of cars and trucks before and after imposing the condition of proportionality for one of the assignments, car followed by truck, reveal that about half of all links experience differences in user class flows, ranging up to +/-300 vph. The largest differences in class link flows occur on links with class flows less than 2,000 vph. These findings offer insights into the magnitude of the differences arising from the non-uniqueness of class link flows, which links are subject to such differences, and the importance of imposing the condition of proportionality in multi-class traffic assignments for travel forecasting practice.

Key words: static user-equilibrium traffic assignment; multi-class link flows; condition of proportionality; order of assigned trip matrices
INTRODUCTION

The standard method for predicting traffic flows on congested urban road networks, static user-equilibrium traffic assignment, is based on a criterion first proposed by Wardrop (1) that drivers seek their own least cost routes from their origins to destinations. This concept corresponds to a user-equilibrium (UE) state in which all used routes have equal costs and no unused route has a lower cost for every origin-destination pair. This problem was first formulated as a convex optimization problem with linear constraints by Beckmann et al. (2).

A property of Beckmann’s formulation, and hence of all algorithms devised for its solution, is that user class link flows (e.g., cars and trucks) and route flows are not uniquely determined. The reason is that link travel times are functions of total link flows, and not functions of route or class flows. Therefore, flows can be swapped among routes or classes, leaving the user-equilibrium total link flows unchanged. If one wishes to determine class link flows and route flows uniquely, an additional assumption is required. One plausible assumption is the condition of proportionality, namely that the proportion of travelers on each of two alternative, equal-cost route segments should be the same regardless of their origin or their destination. This condition is illustrated with an example in the second section.

The condition of proportionality can be applied to solutions produced with any algorithm defined on a bush-based representation of the road network. These include the LUCE algorithm in Visum (3) and the OUE algorithm in TransCAD (4). The proportionality condition also led Bar-Gera (5) to design a new algorithm, Traffic Assignment by Paired Alternative Segments (TAPAS) to compute route flow and two-class link flow solutions. For this paper, LUCE was applied to determine class link and route flows on the Chicago regional network; these flows were then adjusted for proportionality.

The magnitude of differences in class link flows between the solution to the standard traffic assignment problem and a solution with the condition of proportionality imposed is unknown. The objective of this paper is to address this issue by analyzing the effect of the condition of proportionality on the class link flows. Following a description of trip matrices and network data in the third section, link flows for cars and trucks are examined for a pair of alternative segments in the fourth section. Differences in class link flows, with and without proportionality, are then analyzed in relation to class link flows with proportionality, average total link flows, volume-to-capacity ratios and congested speeds for the entire road network in Figures 4-6. Next, the effect of the order of assigning the trip matrices in the initial solution to the assignment is examined for all links in the network in Figure 7. Finally, the class O-D flows with and without proportionality for two orderings of trip matrices are examined for a typical pair of alternative segments in the fifth section. Conclusions and recommendations for practitioners are offered in the last section.

LITERATURE REVIEW

Solution methods for the standard traffic assignment problem, in which link travel times/costs depend only on their own flows, have advanced steadily since 1952. Early methods were based on the intuitive assumption that drivers seek the shortest route to their destinations. Until the mid-1970s, many practitioners were unaware of Beckmann’s UE formulation and its implications for designing algorithms that converge towards the UE state. Examples of early methods are the non-convergent, iterative capacity-restraint method of the Bureau of Public Roads (6) and the incremental method devised for TRANPLAN (7), which does converge towards the UE state, albeit very slowly. Patriksson (8) and Ortúzar and Willumsen (9) provide further details about static traffic assignment.

Convergent assignment methods based on Beckmann’s formulation emerged in the late 1960s and early 1970s in Ph.D. thesis research in operations research (10, 11, 12, 13). The first convergent method to be widely applied was based on the linearization method of Frank and Wolfe (14). One of the originators of this assignment algorithm was LeBlanc; another was Evans (15), who extended the formulation to include trip distribution, and rigorously analyzed the mathematical properties of a partial linearization algorithm. Their algorithms, which solved the assignment problem in terms of link flows, are
called link-based. The link-based method rather quickly replaced the older heuristic methods during the 1980s, when software systems for mini- and micro-computers began to be offered. Today, most commercial travel forecasting software systems (CUBE, EMME, SATURN and TransCAD) include a link-based solution method. In contrast, the route-based method of Bothner and Lutter (16) was implemented in Visum. A drawback of all of these methods, however, is that the assignment problem cannot be solved precisely with reasonable computational effort.

The precision of the solution needed for practice depends on its use. One of the main applications of travel forecasts is to compare scenarios. Such comparisons are only valid if the precision of each solution is substantially better than the differences among the scenarios. Boyce et al. (20) explored the effects of solution precision on scenario differences, recommending that the relative gap, a standard measure of convergence, should be less than 1E-4.

Refinements to these solution methods continued through the 1990s; when Bar-Gera (17) proposed an origin-based assignment algorithm, the situation began to improve. His method included several advances; one was the organization of the search on an acyclic, origin-based subnetwork defined for each origin zone, termed a bush by Dial (18). Bar-Gera’s algorithm was able to achieve precise solutions of the assignment problem for the first time, although the computation times were relatively long. Dial (19) proposed his own bush-based algorithm, which solved the assignment problem precisely and more quickly. LUCE and OUE, with post-processing for proportionality, as well as TAPAS, represent the current state of the art.

The non-uniqueness of class link and route flows under the UE assumption may be illustrated with a simple example. Suppose that the total link flows shown in Figure 1 represent a perfect UE solution with identical travel times on a pair of alternative segments, [2, 3, 5] and [2, 4, 5]. How many travelers from each origin use each segment? Three solutions for the flows on each of the four routes in this network are shown in Table 1, each solution corresponding exactly to the total link flows. If one wants to know how many travelers on link [2, 3] come from each origin, an additional behavioral assumption is required. A plausible assumption is the condition of proportionality, namely that the proportion of travelers on each of the two alternative segments should be the same regardless of their origins or destinations. This condition is observed in the solution h* in Table 1, as a ratio of 1 to 3 is found for routes 1 and 2 (25 to 75) as well as for routes 3 and 4 (15 to 45), corresponding to the total link flows (40 to 120).

![FIGURE 1  Example of O-D flows over a pair of alternative segments.](Source: Boyce et al. (21, p. 7))

<table>
<thead>
<tr>
<th>Zone pair</th>
<th>Route</th>
<th>Description</th>
<th>h*</th>
<th>h₁</th>
<th>h₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-C</td>
<td>1</td>
<td>A-1-2-3-5-C</td>
<td>25</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>A-C</td>
<td>2</td>
<td>A-1-2-4-5-C</td>
<td>75</td>
<td>60</td>
<td>100</td>
</tr>
<tr>
<td>B-C</td>
<td>3</td>
<td>B-1-2-3-5-C</td>
<td>15</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>B-C</td>
<td>4</td>
<td>B-1-2-4-5-C</td>
<td>45</td>
<td>60</td>
<td>20</td>
</tr>
</tbody>
</table>

Source: Boyce et al. (21, p. 7; 22)
Proportionality also applies to multi-class assignments. If travelers from origin B belong to two different classes, for example 40 cars and 20 trucks in passenger car equivalents, then there are several possibilities. In one case only one class uses both alternatives. For example, trucks might be prohibited from using link [2,3]. In this event all trucks use route 4 only, and no additional assumption is needed on how the two classes use the two segments. Then, the condition of proportionality applies only to cars from origins A and B. In a second case the generalized cost of travel is equal on both segments for one class, but different for the other class. Since travelers only use least cost routes under the UE assumption, this situation is the same as the case of prohibited links. In a third case both segments have equal costs for both classes of travelers. Then the condition of proportionality can be applied to class flows as well, so there would be 10 cars and 5 trucks on route 3, and 30 cars and 15 trucks on route 4.

Four reasons to adopt the condition of proportionality are: 1) it is a reasonable condition that is easily understood; 2) it offers consistent treatment of travelers in a forecast, which is important for equity; 3) it results in stable solutions with respect to model inputs; and 4) satisfaction of the condition of proportionality can be tested computationally for any solution. These properties contribute to a solution’s usefulness, especially as the only alternative is to choose arbitrarily one of the many alternative UE solutions.

The remainder of the paper seeks to assess the extent of the arbitrariness of class link flows through examples and charts illustrating how link flows differ under various assumptions. In particular, the effect of the arbitrary ordering of the trip matrices in the assignment is considered. The advantages of imposing proportionality in professional practice are emphasized.

TRIP MATRICES AND REPRESENTATION OF THE ROAD NETWORK

Results presented below are based on the assignment of car and truck matrices representing the morning peak period (6:30 - 8:30 am) of the Chicago region in 1990. The car matrix was created with an origin-destination-mode choice function, given a fixed truck trip matrix. The Chicago regional zone system for 1990 consisted of 1790 zones (Figure 2a); the road network had 12,982 nodes and 39,018 links (Figure 2b). Total regional person trips by car and transit were estimated to be 1.51 million persons per hour, resulting in 0.98 million vehicles per hour (vph); total truck trips were 0.45 million passenger-car equivalents per hour (pceph). Exogenous estimates of the total weekday person flows departing from and arriving at each zone during the two-hour peak period were based on weekday estimates by the Chicago Area Transportation Study.

Origin-destination person flows by mode were determined by a negative exponential (logit) function, \( d_{pq}^m = R_p S_q \exp(-\beta c_{pq}^m) \), where \( d_{pq}^m \) is the origin-destination-mode flow in persons per hour from origin \( p \) to destination \( q \) by mode \( m \) (car, transit); \( R_p, S_q \) are balancing factors ensuring that the origin and destination constraints on total departing and arriving flows are met; \( c_{pq}^m \) is the origin-destination generalized cost by mode \( m \) denominated in minutes; and \( \beta \) is a cost sensitivity parameter. Use of this function means that origin-destination flows are positive and real-valued, with many flows being less than one person per hour. Further details are given by Bar-Gera and Boyce (23). The generalized travel costs on which the car flows were based are the endogenous UE times and travel distances consistent with the model-determined car flows, the exogenous truck trip matrix, and fixed travel times and fares for transit.

A key parameter of the logit function is the sensitivity of travelers to generalized costs. A larger value of the parameter \( \beta \) means travelers are more sensitive to travel times, distances and fares, whereas a smaller value means they are less sensitive. For smaller values, trips lengthen in time and distance, which increases congestion over the road network and shifts travelers to transit, as compared with larger values. In the results presented here, a relatively large value (0.20) was used. For this value the mean travel time for interzonal car travel is about 20 minutes, with a corresponding mean interzonal transit travel time of about 27 minutes.
In the City of Chicago, two expressways have car-only express lanes. In addition, trucks cannot use the Lake Shore Drive, a 24 km multi-lane, grade-separated roadway along the shore of Lake Michigan. Trucks are also restricted from a boulevard system in the City of Chicago and from certain arterial roads in suburban neighborhoods. Altogether, trucks are restricted from 562 links of the road network.

Given these two trip tables for cars and trucks, two assignments were solved with Visum for generalized link costs defined as travel time only. In the initial solution of one assignment, the car matrix was assigned first, followed by the truck matrix; then the solution was iterated to a relative gap of less than 1E-8 for a road network with the noted restrictions on truck use. In the other case, the matrices were assigned in the opposite order. Then, both arrays of class link flows were adjusted so as to impose the condition of proportionality. The analysis in the fifth section compares these two assignments for a pair of alternative segments with regard to class link flows.

FIGURE 2  Chicago regional zone system and road network.  
Source: Chicago Area Transportation Study

EFFECTS OF PROPORTIONALITY ON CLASS LINK FLOWS

In this section, differences in class link flows, with and without proportionality, are shown in Figure 3 for two links, an expressway link and an adjacent arterial link. Then, findings for class link flows from the assignment with trip matrices in the order of car, then truck, are presented. Differences in class link flows with and without proportionality are shown with charts and histograms in Figures 4-6. Relationships of class link flow differences to volume-to-capacity ratio and congested speed are analyzed. Finally, the effects of the order of the trip matrices in the assignment, and the condition of proportionality, are examined in Figure 7.

To illustrate the effect of proportionality on class link flows, consider two adjacent links (solid arrows) and a related pair of alternative segments (dashed sequences of arrows) in northwest Chicago, shown in Figure 3. The diverge and merge nodes of the pair of alternative segments are shown as points A and B. The class link flows without and with proportionality for cars and trucks are shown in Table 2. Truck link flows are given in units of passenger-car equivalents per hour (pceph) units. These flows suggest the effect of imposing proportionality on class link flows can be substantial. For the expressway link, the difference (delta) in car and truck flows with and without proportionality is 203 vph, or 12% of the car flow with proportionality. For the arterial link, the difference in car and truck flows with and without proportionality is 199, or 17% of the car flow with proportionality. The total flows on each link are shown in the last column; the link capacities are 5,400 and 2,040 vph, respectively.
TABLE 2  Flows on Two Links with and without Proportionality

<table>
<thead>
<tr>
<th>roadway link (solid arrow):</th>
<th>car flow (vph)</th>
<th>truck flow (pceph)</th>
<th>delta =</th>
<th>delta/with-car</th>
<th>delta/with-trk</th>
<th>total flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>without</td>
<td>with</td>
<td>without</td>
<td>with</td>
<td>vph</td>
<td>%</td>
</tr>
<tr>
<td>Edens Expressway</td>
<td>1,942</td>
<td>1,739</td>
<td>4,748</td>
<td>4,950</td>
<td>203</td>
<td>12</td>
</tr>
<tr>
<td>Cicero Avenue</td>
<td>946</td>
<td>1,145</td>
<td>1,292</td>
<td>1,093</td>
<td>199</td>
<td>17</td>
</tr>
</tbody>
</table>

Flows in this table were computed with Visum. Flows are rounded to whole numbers and may not sum to total flows.
The class flows with proportionality over the two segments are shown in Table 3 together with the segment proportions for class and total flows. Among nearly 7,266 pairs of segments in a TAPAS solution, the expressway link is included 101 segments, the arterial link in 45 segments, and both links in 11 pairs of alternative segments. The properties of the proportionality adjustment are as follows: 1) only links included in a pair of alternative segments can be adjusted for proportionality; 2) proportionality post-processing adjusts the proportion of route and class flows over each segment, but does not change the total flow of a link; 3) the proportion of class flows comprising a link’s total flow is a result of adjustments over all PASs that include that link.

**TABLE 3  Flows on Two Segments with Proportionality**

<table>
<thead>
<tr>
<th>segment</th>
<th>car flow (vph)</th>
<th>truck flow (pceph)</th>
<th>proportion by segment (%)</th>
<th>segment flow (vph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edens Expressway</td>
<td>173</td>
<td>216</td>
<td>86</td>
<td>389</td>
</tr>
<tr>
<td>Cicero Avenue</td>
<td>28</td>
<td>35</td>
<td>14</td>
<td>62</td>
</tr>
</tbody>
</table>

Flows in this table were computed with TAPAS (5). Flows are rounded to whole numbers, and may not sum to totals.

In order to extend this example to the entire network, differences in class link flows, with and without proportionality, versus the class link flow with proportionality for cars and trucks are shown in Figure 4. The vertical axes are truncated at +/-200, which excludes several outliers. The car link flow horizontal axis ends at 7,000 vph; however, truck link flows extend to nearly 12,000 pceph, but with few differences in link flows from imposing proportionality beyond 7,000 vph. Therefore, Figure 4b is truncated at 7,000 vph for comparability with Figure 4a. These two charts clearly show that the largest effect of imposing proportionality pertains to links with class flows less than 2,000 vph.

The Chicago regional network consists of 35,436 links, plus 3,582 centroid connectors. For links that may be used by trucks as well as cars, 19,743 links (57%) have zero difference in class link flows with and without proportionality. Of these links with zero flow difference, 4,509 links (23% of the zero flow difference links) are not used by either class, and primarily lie at the periphery of the Chicago region. Zero class flow differences suggest that the link is not included in any pair of alternative segments; therefore, their class flows are uniquely determined.

The number of links with a given difference in car link flow with and without proportionality is shown in Figure 5a by link flow difference intervals: car link flow difference equals the link flow with proportionality minus the link flow without proportionality. The vertical axis is the logarithm of the number of links in the link flow difference interval. About one-half of all links have no link flow difference, one-third have differences between 0 and +/-10, one sixth have differences between +/-10 and +/-100, and nearly 300 have differences exceeding +/-100. The histogram for truck link flow differences (not shown) is the exact mirror image of the car flow histogram. Since the total flow on each link is uniquely determined by the assignment, any difference in truck link flow is equal and opposite in sign to the difference in car link flow.

Figure 5b shows the average total link flow of links with differences in car link flows for the same intervals defined above. The 300 links with the highest car link flow differences exceeding +/-100 have average total flows on the order of 1,600 vph. In contrast over 15,000 links with zero car flow difference have an average total flow of less than 300 vph.

Figures 6a and 6b present car link flow differences for 30,365 links by volume-to-capacity ratio and link congested speed, showing that the differences are symmetric with respect to the sign of the link flow difference. Figure 6a suggests that the differences are unimodally distributed with respect to volume-to-capacity ratio. Relatively few links have ratios that may be considered to be excessive for capacities defined as level of service D; nevertheless, the network may be regarded as relatively congested. Figure 6b shows that the differences are clustered by congested speed, that is the speed at the link’s total flow, reflecting the frequency distribution of links by free flow speed. Both plots are truncated at +/-200 vph.
In the course of comparing the class link flows without proportionality to the flows adjusted for proportionality, we noticed that the arbitrary choice of initially assigning the car matrix before the truck matrix resulted in substantially different class segment flows from the opposite choice of assigning truck before car. The analysis that led to this finding is presented for a pair of alternative segments in the fifth section. These two assignment orderings are described below as ‘car/truck’ and ‘truck/car.’

Since the total flow between each O-D pair is unchanged by the order of assigning the trip matrices, and since the total link flows are uniquely determined by each assignment, the total link flows from the two solutions should be equal, subject to differences in convergence. Comparison of the total link flows for 30,365 links (excluding centroid connectors and links with zero flow in both solutions) showed very small differences in flow for 16,366 links (54% of all links with flow). The total absolute flow difference (TAFD) is 629 vph; the root mean square difference (RMSD) is 0.26. These measures of difference in convergence provide a basis for comparing differences in class link flows between the two assignments.
To compare and visualize these results, class link flows from the two assignments before adjustment for proportionality are plotted in Figure 7a for car and Figure 7b for truck. The TAFD for both cars and trucks is 337,800 vph, or 537 times the difference in convergence stated above; the RMSD is about 33.2, or 130 times the difference in convergence. These differences reflect both the effect of the order of the trip matrices and the non-uniqueness of the class link flow solutions.

Figure 7c for car and Figure 7d for truck show the differences in the class link flows between the two assignments after adjustment for proportionality. The TAFD between the two adjusted assignments was reduced to 35,700 or 57 times the difference in convergence; the RMSD is 4.5 or 17 times the difference in convergence. The differences reflect the overall convergence of two assignments and deviations from proportionality following the post-processing adjustment of each assignment. Computational procedures currently available in commercial software systems do not permit more precise convergence. Further refinement of computational precision presents an ongoing challenge for software developers.

We understand that some travel forecasting software developers realize that the specification of the order of the trip matrices in an assignment affects the class link flows; however, this property may not be widely appreciated by practitioners. Similar effects might be introduced by renumbering zones or by large-scale coding changes in the network associated with scenario analyses. Practitioners should be aware that such changes could result in differences in class link flows that should not be attributed to differences in scenario inputs.

**CLASS O-D FLOWS OVER A PAIR OF ALTERNATIVE SEGMENTS**

The last section examined the differences in class link flows with and without the condition of proportionality being imposed on two multi-class assignments. In order to explore these effects of proportionality in more detail, consider a typical pair of alternative segments located near North Avenue in Chicago, two miles north of the Central Area shown in Figure 2a. The objective of this analysis is to illustrate how arbitrary class link flows can be if proportionality is not imposed. The example was originally motivated by the construction of a new bridge on North Avenue connecting nodes 6380 and 6389. Suppose that an analyst wished to examine the truck flows using this bridge in a two-class assignment of cars and trucks. In the user-equilibrium solution, the North Avenue bridge is a link in Segment 1.
FIGURE 7  Effects of order of assignment and proportionality on user class link flows.

The total flows over the two segments are shown in Table 4 for two assignments. The top half of Table 4 shows the total segment flows when the specified order of assignment is car/truck. The bottom half shows the total segment flows when the specified order of assignment is truck/car. The left side of the table shows the total flows without proportionality. The right side shows the total flows with proportionality, demonstrating that the two assignments have the same solution.
The upper left side shows the flows over the two segments without proportionality with order car/truck: 93% of car flows used Segment 1, compared to only 26% of truck flows. The lower left side with the order truck/car shows only 55% of cars flows used Segment 1, compared to 92% of truck flows. Hence the differences in flows by class are dramatically different for the two assignments. After proportionality is imposed, shown on the right side of Table 4, the car and truck flows each have 69% on Segment 1, an increase in the Segment 1 truck flows of 89 vph (160%) for the first order, and a decrease of 50 vph (26%) for the second order. In the lower left half of Table 3, the opposite result may be observed.

**TABLE 4 Total Segment Flows without and with Proportionality by Order of Assignment**

<table>
<thead>
<tr>
<th></th>
<th>order of trip matrices in the assignment: car/truck</th>
<th></th>
<th>order of trip matrices in the assignment: truck/car</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>flows without proportionality vph</td>
<td>flows with proportionality vph</td>
<td></td>
</tr>
<tr>
<td>segment</td>
<td>1 2   1/total (%)</td>
<td>1 2   1/total (%)</td>
<td>class flow</td>
</tr>
<tr>
<td>cars</td>
<td>338  25  93</td>
<td>249  114  69</td>
<td>363</td>
</tr>
<tr>
<td>trucks</td>
<td>56   155  26</td>
<td>145  66   69</td>
<td>211</td>
</tr>
<tr>
<td>total</td>
<td>394  181  69</td>
<td>394  181  69</td>
<td>574</td>
</tr>
</tbody>
</table>

Flows are rounded to whole numbers; totals may not sum due to rounding.

Table 5 shows the numbers of O-D pairs that comprise each segment flow by class. The column, either/both, indicates how many O-D pairs are found on either segment, and how many are found on both segments. As shown, the numbers are substantially different without proportionality, but the same with proportionality. The differences are especially large for Segment 2. The number of O-D pairs with proportionality shows the shift in flows from one segment to both segments: 670 pairs for cars and 256 pairs for trucks.
TABLE 5  Number of O-D Pairs without and with Proportionality by Order of Assignment

<table>
<thead>
<tr>
<th>order of trip matrices in the assignment: car/truck</th>
<th>O-D pairs without proportionality</th>
<th>O-D pairs with proportionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>segment</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>cars</td>
<td>668</td>
<td>392</td>
</tr>
<tr>
<td>trucks</td>
<td>204</td>
<td>245</td>
</tr>
<tr>
<td>total</td>
<td>678</td>
<td>483</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>order of trip matrices in the assignment: truck/car</th>
<th>O-D pairs without proportionality</th>
<th>O-D pairs with proportionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>segment</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>cars</td>
<td>624</td>
<td>641</td>
</tr>
<tr>
<td>trucks</td>
<td>256</td>
<td>131</td>
</tr>
<tr>
<td>total</td>
<td>649</td>
<td>655</td>
</tr>
</tbody>
</table>

CONCLUSIONS

The effects of imposing the condition of proportionality on car and truck link flows from two assignments on a large-scale network of the Chicago region were compared in terms of: 1) class flows over a pair of alternative segments; 2) class link flows over the entire network; 3) effects of the ordering of the trip matrices in the assignment on the class link flows. Differences in class link flows with and without proportionality range up to ±300 vph. Relative differences in segment class flows for the case examined are much more substantial, the truck flows on one segment being more than four times larger after adjustment for proportionality.

Whether these differences are large enough to warrant concern in applications is a matter for practitioners to judge. To be assured that class link flows are comparable in scenario analyses, especially in the case of special purpose facilities such as truck routes, practitioners would seem to be well-advised to consider imposing proportionality on their multi-class traffic assignments. Class link flows without proportionality are partially determined by the ordering of the class trip matrices. The designation of one matrix as ‘primary’ by some practitioners may suggest that one order is more correct than another; in fact, such order-related differences occur arbitrarily.

The imposition of the condition of proportionality has the potential to improve the usefulness of traffic assignments in transportation planning practice by determining multi-class link flows uniquely. Even if observed class link flows are not found to conform to proportionality in reality, at least the predicted flows are consistent across all O-D pairs. In this sense, assignments adjusted for proportionality offer a superior solution. Ongoing research is analyzing new data on observed expressway flows for proportionality. At the time this paper was prepared, two commercial software systems (TransCAD and Visum) offered a capability to impose proportionality through post-processing of assignment solutions.

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