Risk Assessment of the Stability Properties of the Dynamic Network Loading

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Abstract:
Over the past decades Dynamic Traffic Assignment models are increasingly used for online and offline applications of various urban environments for the successful and efficient deployment of traffic control strategies and the evaluation of traffic management schemes and policies. The stability of the dynamic network loading procedure is crucial for achieving the most accurate representations of traffic phenomena in congested urban networks, where the increase of congestion level often creates problems of discontinuity and system failure especially when the demand levels are temporally unstable and severe.

In this paper a stability analysis is proposed in the context of evaluating the properties of the final resulting dynamic network loading solution through the evolution of time. The scope of this analysis is to provide practitioners and researchers a useful tool for the temporal statistical analysis and risk assessment of the dynamic representation of traffic conditions. The methodology of the proposed stability analysis is implemented on two reference networks. The results for the statistical analysis and the evaluation of the stability properties are discussed over an extensive sensitivity analysis of the simulation step to the selected stability measures.

Keywords: Dynamic Traffic Assignment, Mesoscopic, Stability, Stability Measures, Risk Assessment
INTRODUCTION AND BACKGROUND

The increasing interest in Dynamic Traffic Assignment (DTA) models was stimulated by the fast evolution of Intelligence Transportation Systems (ITS) and Advanced Traveler Information Systems (ATIS) over the past decades. DTA models, introduced by Yagar (1), Robillard (2), and Merchant and Newhauser (3), among others, represent the temporal extension and disaggregation of the classical static assignment problem. The static assignment problem assumes steady traffic conditions over the study period whereas its dynamic counterpart captures more realistic time dependent traffic conditions. Thus, the aim of a DTA model is to determine the network flow patterns given the Origin-Destination (O-D) demand, the network structure and the link performance functions incorporating the time dependency in both the supply and the demand attributes (4). An extensive review of the DTA models and the formulation, characteristics, solution methodologies, limitations and practical applications is provided by Chiu et al. (5) and Peeta and Ziliaskopoulos (6).

DTA models are categorized in literature into four main groups: mathematical programming, optimal control, variational inequality and simulation based. Another classification of DTA models, which takes into account the level of detail and traffic representation, categorizes the models into macroscopic, mesoscopic and microscopic. In this paper the case of a simulated-based mesoscopic traffic simulator is considered. Some of the frequently used DTA mesoscopic traffic models include CONTRAM (7), DYNASMART (8), DYNAMIT (9), METROPOLIS (10), DYNAMEQ (11), VISTA (12). Significant ongoing research is performed over the past years in the field of Open-Source DTA software and traffic flow simulation engines, such as DTALite (13), DynusT (14), TRANSIMS (15), MITSIM (16), SUMO (17). Also Tammenga et al. (18) proposed OpenTraffic (19), a simulation platform which enables the collaboration of academics from different geographic areas and disciplines to work together.

Stable and robust time dependent network states are of great importance in the successful deployment of traffic control and operational strategies aiming at the minimization of network disruptions and ensuring the existence of network equilibrium through time. The usual fluctuations of traffic demand and network characteristics are the most common source of creation of unstable transient conditions which often produce questionable modeling results especially for online procedures or dynamic representations of large-scale networks.

The implementation of a DTA model on a large-scale network causes important challenges to the modeling procedure, such as the manipulation of network and demand data, the modeling of turning movements, the efficient computation of link travel times and the handling of complex path data (20).

The necessity to restrict and minimize the time dependent system risks and vulnerabilities stimulates the need for valid statistical analyses of the network performance and representation of traffic conditions and therefore of the final DTA solution. One of the main issues affecting the performance of a DTA model is the stability of the solution. An equilibrium network state is supposed to be stable when even an arbitrarily small perturbation to the system will not cause it to diverge from the equilibrium state (21).

The importance of stability analysis of the network equilibrium has been addressed by many researchers in the past as a convergence condition to a stable solution. The notion of stability in the context of a DTA model implies that all solutions are bounded and converge to the time-dependent desirable states, providing a stable solution which minimizes or limits the deterioration of the system performance (6). Watling (21) and Peeta and Yang (4) provide a detailed review and classification of previous stability analyses on the convergence of the DTA solution to equilibrium in the context of within-day dynamics (22-27).

The stability analysis of the day-to-day dynamic system (28) is based on comparing point properties of the DTA solution under perturbation to an equilibrium solution. In many cases researchers have proposed measures for the stability analysis of their DTA models and solutions. Some of these approaches are reviewed in the next section.

This paper focuses on four main objectives. The first addresses the necessity for promoting stability analysis in the context of evaluating the performance of the final resulting solution through the
evolution of time. It is noted that the proposed stability analysis is not concerned with the convergence of
the DTA procedure or the system’s equilibrium point seeking. Secondly, a proposal is made for certain
stability measures in the above context. Thirdly, an examination of the influence of the simulation step in
the stability of the DTA solution is performed, through a sensitivity analysis on the examined stability
measures and standard performance indices. Finally, a thorough statistical analysis of the stability
measures is made and a proposition for a risk assessment through an ordinal classification and evaluation
of the more stable solution is put forward. The risk assessment as discussed in this paper is part of the
stability analysis, where the calculated stability measures are used to evaluate the risk of the dynamic
loading solution to be unstable during the simulation horizon. Therefore, the risk assessment procedure is
evaluating the possibility that an arbitrarily small perturbation to the system during the simulation horizon
will cause the system to diverge from the equilibrium state. It is expected that this analysis will be useful
to practitioners and researchers interested in evaluating the system’s fluctuations through time and its
stability properties.

The paper is organized as follows: the next section presents a more detailed review of stability
measures proposed in the literature and sets out the methodology for the stability analysis. In the
following section the test networks and the computational experiments are described. The results are
presented at the following section. Finally, the main conclusions are summarized in the last section with
some remarks on future research.

STABILITY ANALYSIS

This section discusses some of the most common stability measure found in the literature, then proposes
some extensions to the stability perspective and an outline of the proposed methodology for the stability
analysis in terms of risk assessment is given.

Notation

Consider a network represented by a connected graph \( G[A, N] \), where \( A \) are the set of links and \( N \) are
the set of nodes. The set of origin and destination nodes is \( R (R \subset N) \) and \( S (S \subset N) \), respectively, and
the set of reasonable path connecting an origin \( r (r \in R) \) and a destination \( s (s \in S) \) is \( K_{rs} \). The origin –
destination (O-D) demand between an origin \( r \) and a destination \( s \) travelling in a path \( k (k \in K_{rs}) \) is
\( q_{rsk}^d \) during the \( d \)-th time step, \( \tau_d \) (where \( T = \sum \tau_d \)) with travel time is \( u_{rs,k}^d \). The minimum travel
time between the O-D pair, \( rs \), is \( u_{rs,\min}^d \). The vehicular flow at link \( a (a \in A) \) during the \( d \)-th time
step, \( \tau_d \), is \( x_a^d \) and the average speed of that link is \( s_a^d \).

Stability Measures

The stability measures and criteria reviewed in this section is a collection of metrics based on the
concepts of stability, efficiency, convergence and sensitivity describing and quantifying the performance
and the evolution of the network’s states.

Iida (29) proposed as a stability measure of travel times of the given DTA solution, the travel
time reliability, which is affected by fluctuations of traffic flow. Under this perspective, the variability of
travel time (link-based or path-based) is used by many researchers as a stability and performance
measure.

Taylor (30) proposed stability criteria as part of one stopping criterion in the iterative process of
the DTA procedure implemented in the CONTRAM software in order to compare the results of each time
step between consecutive iterations. Three of the proposed stability criteria include the RMS (Root Mean
Square) Change, the Average Absolute Difference and the Percentage Absolute Difference in Link Flows, where for every iteration \( m \), the differences in link flows during each time step are taken into account throughout the simulation period in order to determine the stability of the solution provided by the \( m \)-th iteration with regards to the \((m-1)\)-th iteration.

The RMS Change in Link Flows:

\[
\text{RMS}(G) = \sqrt{\frac{1}{A \cdot T} \sum_{a=1}^{A} \sum_{d=1}^{T} \left( X_{a,m}^{r_d} - X_{a,m-1}^{r_d} \right)^2}
\]  

Average Absolute Difference in Link Flows:

\[
\text{AAD}(G) = \frac{1}{A \cdot T} \sum_{a=1}^{A} \sum_{d=1}^{T} \left| X_{a,m}^{r_d} - X_{a,m-1}^{r_d} \right|
\]

Percentage Relative AAD in Link Flows:

\[
\text{PR} - \text{AAD}(G) = \frac{100}{1} \frac{\sum_{a=1}^{A} \sum_{d=1}^{T} \left| X_{a,m}^{r_d} - X_{a,m-1}^{r_d} \right|}{X_{a,m}^{r_d}}
\]

Another stability criterion proposed by Taylor (30) is based on the total perceived travel cost \( c \) of the users of the network between consecutive iterations throughout the simulation period.

Percentage Relative Total Perceived Cost:

\[
\text{TPC}(G) = 100 \frac{|c_m - c_{m-1}|}{c_{m-1}}
\]

Florian et al. (31) proposed a measure inspired from the static assignment which is used for the qualification of the given solution in respect to the network equilibrium. It expresses the difference between the total travel time experienced and the total travel time that would have been experienced if all vehicles had a travel time (over each time step) equal to that of the current shortest path.

Gap Of The Routes’ Travel Times:

\[
\text{Gap}(G) = \sum_{rs}^{\text{RS}} \sum_{k}^{K_{rs}} u_{rs,k}^d \cdot q_{rs,k}^d - u_{rs,min} \sum_{k}^{K_{rs}} q_{rs,k}^d
\]

Chiu et al. (5) proposed several stability measures for determining the convergence of the DTA procedure and for characterizing the solution, (a) a relative gap, reflecting the difference of the average and minimum route cost, (b) a comparison of the time varying spatially averaged network speed which is strongly influenced by the length of the links and (c) a comparison of the number of vehicles inside the network and waiting to enter.

The Relative Gap:

\[
\text{RGap}(G) = \sum_{rs}^{\text{RS}} \sum_{k}^{K_{rs}} \left( u_{rs,min}^d \cdot \frac{T \sum_{k}^{K_{rs}} q_{rs,k}^d}{T \sum_{i}^{T} \sum_{k}^{K_{rs}} q_{rs,k}^d} \right) - \sum_{rs}^{\text{RS}} \sum_{k}^{K_{rs}} u_{rs,k}^d \cdot q_{rs,k}^d
\]
The RGap reflects the difference between the minimum route cost and the average route cost relative to minimum cost, as a weighted average across all O-D pairs, for each time step across the entire assignment period, providing a temporal distribution of these differences throughout the simulation horizon. The Time Varying Spatially Averaged Network Speed is strongly related to the network representation and conditions and in many cases may not be applicable for a stability analysis.

In the context of this paper, where the stability analysis is limited on the final DTA solution, a proposal is made for transformation of the Average Absolute Difference in Link Flows originally proposed by Taylor (30) in order to reflect the differences between consecutive time steps and not between the consecutive iterations as in Eq.1, Eq.2 and Eq.3.

\[ \text{Time-Dependent Average Absolute Difference in Link Flows: } \text{TDAAD}(G)^{\tau_d} = \frac{1}{A} \sum_{a=1}^{A} \left| X_a^{\tau_d} - X_a^{\tau_d-1} \right| \]  

The motivation for the proposed stability measure is to attain an understanding of the temporal network loading. This stability measure depicts the time-varying network loading providing insights without taking into consideration the path statistics, based only on the time-dependent link flows.

### Proposed Stability Analysis

As mentioned earlier, the main focus of this paper is the necessity for promoting stability analysis in the context of evaluating the performance of the final resulting solution through the evolution of time and providing a risk assessment for the stability of the network. The steps for the proposed stability analysis are the following:

A first step is the selection and calculation of the stability measures that represent better the modeling scope of the project for providing more sensitivity to the factors affecting the decision process where the DTA solution will be used. It is noted that the stability analysis is concerned with the final DTA solution which has resulted by the iterative process of the DTA model. Thus, the stability measures are calculated for each time step of the simulated horizon.

A second step involves the statistical analysis of the performance of the network and the stability measures. The coefficient of variation is calculated as part of a risk assessment procedure. This statistical measure reflects the dispersion of the data series around the mean and it is commonly used for determining the volatility of investments. In the context of the DTA solution, the risk assessment is focused on whether the DTA network loading has the ability to cope with unexpected events which will increase the uncertainty in the forecasted network loading. The temporal analysis of the calculated stability measures can provide insights into the time-windows which appear to have a greater potential of exhibiting instability. If the practitioner or the researcher is interested in sensitivity testing or comparing of different runs, then it is crucial to perform an ANOVA analysis to determine the statistical significance of the results and the calculated stability measures.

Finally, in order for the user to select the more stable solution, it is recommended to perform an ordinal classification of the results for each run and each stability measure and evaluate each run by combining the ordinal classification results.

### DESCRIPTION OF THE COMPUTATIONAL EXPERIMENTS

The computational experiments of the stability analysis of the DTA solution were performed on two medium-scale reference network data sets, which are available in the website maintained by Bar-Gera (32). The first network is the well-known Sioux Falls in South Dakota which is a small size network proven to be excellent for coding and debugging applications. The second is the Anaheim, CA network which is a representation of a medium-sized city. The mesoscopic DTA procedure were modeled using the DTALite simulation-assignment open-source model available at (13).
The objective of computational experiments was twofold: (a) to present the methodology for the proposed stability analysis and (b) to perform a sensitivity analysis of the influence of the time step to the temporal stability of the DTA solution. The computational experiments for each test network consisted of multiple runs with the same model setup, network conditions and temporal and spatial demand distribution, but with different simulation and departure time step sizes. The sensitivity analysis of the influence of the time step size was based on 5, 10, 15, 20, 30-minute simulation and departure step sizes. Here, we hypothesize that the finer time resolution provides more chances in decision points in the DTA assignment procedure. Even so, the objective of performing the current stability analysis is to evaluate if the chosen time resolution is causing instability to the system and to assess the risk for any potential instability.

**Test Networks**

**Sioux Falls, South Dakota**

The Sioux Falls test network consists of 24 nodes, 76 links and 528 O-D pairs. The link and demand parameters were based on the data provided by Bar-Gera (32). The static demand data (39,676 trips) which represents the peak hour, was extended to a 3-hour peak period (59,960 trips) and dynamically distributed through the entire simulation period.

**Anaheim, California**

The Anaheim test network consists of 416 nodes, 914 links and 1406 O-D pairs. The network and demand data were also based on the data provided by Bar-Gera (32). The static demand data (104,694 trips) which represents the peak hour was also extended to a 3-hour peak period (226,051 trips) and dynamically distributed through the entire simulation period.

**Model Setup**

The DTALite software is a freeware dynamic traffic assignment simulation model developed independently by a subcontractor of SHRP 2 C05 TRB research program (Understanding the contribution of Operations, Technology, and Design to Meeting Highway Capacity Needs) and released under a GNU General Public License in January 2010. DTALite is a mesoscopic simulation-based DTA model which uses Newell’s traffic flow model (32-35).

The size of the time step in the DTA procedure plays a significant role on the accuracy of the results and the computational time which is very crucial, especially for online applications. The size of time steps in DTA applications typically varies between 5-min and 20-min. In this paper a sensitivity analysis of the size of the time step in the stability of the network was performed with 5, 10, 15, 20, 30 minutes simulation and departure steps for the OD-based routing for the decomposition of the 180 min simulation horizon. The vehicle loading distribution was identical for each experiment and the procedure for its estimation is described in the following section.

**Demand Distribution**

The resolution of the required time dependent trip matrix used as an input in the DTA procedure depends on the desired modeling accuracy and/or the variability of the available data. There is extensive research in the literature (e.g. 36, 37) regarding the estimation of dynamic O-D trip matrices from observed link flows or O-D travel surveys.

A common method for disaggregating the total O-D matrix into finer time resolutions is to use factors derived from the temporal profile of the demand (5). In this paper this method was used for deriving the time dependent O-D matrix from the static peak hour matrix, which is not based on actual
data, but it can be applied in a real application with minor adjustments. Since there are no observed data for the two test networks, the following assumptions were made (a) the temporal profile of the demand follows a log-normal distribution for the 3-hours peak period, (b) the peak hour is the first hour and corresponds to the 11% of the daily demand (c) the starting demand (prior to the 3-hour peak period) corresponds to the 2% of the daily demand (d) the vehicles are uniformly distributed within 15-min simulation steps in order to generate the vehicles’ flow rates. The compatibility between the tests was ensured by using the same demand distribution and the same assumptions and methodology for creating a disaggregated O-D matrix from the original static data.

The vehicle loading distribution was created on a basis of 15-min steps and remained the same for all experiments in order to ensure that the sensitivity analysis performed was influenced strictly by the simulation step and not to the demand distribution. The decomposition of the simulation horizon into 15-min steps for calculating the detailed vehicle distribution was chosen as a common practice for data collection in this step so as to better replicate actual conditions and challenges.

The steps for creating the final demand distribution are the following:

- The static peak hour matrices were expanded to three hours matrix in order to represent the extended morning peak period 07:00 – 10:00 (420 – 600 min)
- The demand was decomposed into 15-min steps based on a log-normal distribution
- The vehicles’ flow rates were calculated based on uniform rates within the 15-min steps, in order to generate vehicle loadings for the different simulation steps.

RESULTS

In this section the results of the computational experiments are grouped into four sub-sections: the summary statistics of the computational experiments, the stability measures, the statistical analysis of the results and the ordinal classification and risk assessment of the stability.

Summary Statistics

Fig.1 presents the graphical representation of the relative differences of travel times for both network runs reveals the instability of the performance measure during the peak period of demand loading, and more notably, as the time step size increase the relative differences become more significant.

**FIGURE 1** Percentages of Relative Differences of Average Travel Time per Simulation Step

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The average and the standard deviation of the Average Travel Times (in minutes), which is the most common performance measure, can be found in the legend of the Fig.1. Based on the ANOVA statistical analysis, the average travel time for both networks presents significantly different results between: 10-min and 20-min, 10-min and 30-min, 15-min and 20-min, 15-min and 30-min and finally 20-min and 30-min. Thus, it is evident that there is some statistically significant sensitivity with regard to the simulation step size for both networks.

**Stability Measures**

In the current analysis three stability measures were selected: the Relative Gap (Eq.6) and the Time Varying Spatially Averaged Network Speed proposed (Eq.7) by Chiu et al. (5) and the Time-Dependent Average Absolute Difference in Link Flows (Eq.8) as described in previous section. The reason behind this selection is based on the underlying meaning of these three measures. The Relative Gap (RGap) reflects the difference between the minimum route cost and the average route cost relative to minimum cost across all O-D demand pairs, providing a better insight into the path selection procedure and the paths’ statistics. The Time Varying Spatially Averaged Network Speed (avS) reflects the performance of the network averaged over all links (and not paths as in RGap) and it is strongly correlated with the network coding and representation. Finally, the Time-Dependent Average Absolute Difference in Link Flows (TDAAD) reflects the distribution of link volumes across the network not only spatially but also temporally, providing the risk assessment perspective of the analysis.

The temporal analyses of the RGap (Fig. 2) for both test networks appear to have similar distribution, although the phenomena in the Anaheim network appear to be much more intensive. The instability of the RGap it is more evident in the Anaheim network when the 30-min and the 10-min simulation step are used. In the Sioux Falls runs the instability of the solution is more intensive as the simulation step increases especially during the most heavily demand-wise time period for the network loading procedure.

**FIGURE 2 Temporal Distribution of Relative Gap (RGap) per Simulation Step**

The temporal analyses (Fig.3) of the Time Varying Spatially Averaged Network Speed (avS) present a different distribution for the two network setups. It is apparent that the congestion effects are not wearing off in the more heavily congested network of Anaheim. The test runs in the Anaheim network are less different with each other, whereas the Sioux Falls runs exhibit differences during the peak period.
The results for the Sioux Falls network with simulation step of 30-min, which appear to be significantly different from the other test runs during the first 20 minutes of the simulation period, should be tested against actual data for their representation of real conditions, and should not only be based on the stability analysis in order to find the “best” solution. Since the runs have been performed on test networks, this validation of results cannot be actually performed, so this data set is excluded from the rest of the comparisons.

![Graph: Temporal Distribution of Spatially Averaged Network Speed (avS) per Simulation Step](image)

The temporal analyses (Fig.4) of the Time-Dependent Average Absolute Difference (TDAAD) in link flows present significant differences between the two network’s runs which is mainly a result of the traffic building up and the relevant discharge of the congestion effects. The temporal distribution of this

![Graph: Temporal Distribution of Time-Dependent Average Absolute Difference in Link Flows (TDAAD) per Simulation Step](image)
stability measure reflects the relative differences between consecutive time steps, so as the differences are
more intensive, the TDAAD is appearing to have greater values. The runs at the Sioux Falls network
appear to be less stable as the simulation step increases. The distributions of this stability measure in the
Anaheim runs appear to be more consistent with each other, apart from the run of the 30-min simulation
step.

The proposed TDAAD stability measure may provide valuable insights into the stability of the
temporal network loading. The main advantages of the proposed measure are (a) it incorporates the
relative differences between consecutive time steps which provides the temporal aspect of the analysis
and (b) it uses the link volumes of the assignment and the performance of the system, whereas the RGap
uses the path statistics which are biased from one time step to another and avS uses the speed which is
strongly related to the network coding.

Statistical Analysis of Results (ANOVA)

The next step in the stability analysis is the statistical analysis of the final loading, as part of the
sensitivity analysis, by using the ANOVA test to compare the means between pairs of test runs in order to
identify the significant differences. This step is important when one considers the computational needs,
especially in a large-scale network or for online applications, the data requirements for calibrating and
validating the model, etc. Although, the ANOVA analysis would typically consider multiple repetitions
for the same case, in this paper a limited range is used only for presentation purposes.

Table 1 presents the results from the ANOVA tests for the RGap and the TDAAD. The
differences of the avS for both networks’ runs were not statistically significant based on the ANOVA
analysis for the entire simulation period.

<table>
<thead>
<tr>
<th>TABLE 1 Significant Differences of Means between Test Runs</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sioux Falls – RGap</strong></td>
</tr>
<tr>
<td>Step</td>
</tr>
<tr>
<td>5min</td>
</tr>
<tr>
<td>10min</td>
</tr>
<tr>
<td>15min</td>
</tr>
<tr>
<td>20min</td>
</tr>
<tr>
<td>30min</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Sioux Falls – TDAAD</strong></th>
<th><strong>Anaheim – TDAAD</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Step</td>
<td>5min</td>
</tr>
<tr>
<td>5min</td>
<td>na</td>
</tr>
<tr>
<td>10min</td>
<td>x</td>
</tr>
<tr>
<td>15min</td>
<td>x</td>
</tr>
<tr>
<td>20min</td>
<td>x</td>
</tr>
<tr>
<td>30min</td>
<td>na</td>
</tr>
</tbody>
</table>

NB. x: denotes the statistically significant difference, -: denotes that there is no statistically significant difference,
na: not applicable

The differences of means between test runs have more pairs with significantly different results in
Sioux Falls than in Anaheim for both stability criteria (RGap and TDAAD), which can partially be
explained as a weakness with regards to the dynamic stability of the Sioux Falls network, since the
sensitivity analysis produces significantly different results.
Risk Assessment

The analysis of the coefficient of variation provides the risk assessment perspective of the stability analysis and it can be used for evaluating a single run or for comparing runs with each other. As it can be seen from the graphical representation of the Coefficient of Variation in Fig.5, most of the runs are not exceeding the risk limit of 100%, which is generally considered to indicate an unacceptable risk limit. The variation of the avS is less than of the other two measures which behave similarly with ranges 60%-100%.

![Sioux Falls Stability Measures' Coefficient of Variation](image1)

![Anaheim Stability Measures' Coefficient of Variation](image2)

**FIGURE 5** Stability Measures’ Coefficient of Variation

It is proposed to use a combination of stability measure in order to evaluate the risk of instability of a solution and thus removing any subjectivity that one stability measure might include in its calculation. The combinational evaluation of the risk of instability, it is performed through an ordinal classification of the coefficients of variations for each stability measure for all runs. The lower rating indicates less risky solution in terms of instability.

**TABLE 2** Ordinal Classification of Stability Measures

<table>
<thead>
<tr>
<th>Simulation step</th>
<th>5-min</th>
<th>10-min</th>
<th>15-min</th>
<th>20-min</th>
<th>25-min</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGap</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>TDAAD</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>avS</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Total rating</td>
<td>5 (2)</td>
<td>9 (5)</td>
<td>10 (8)</td>
<td>6 (5)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulation step</th>
<th>5-min</th>
<th>10-min</th>
<th>15-min</th>
<th>20-min</th>
<th>30-min</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGap</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>TDAAD</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>avS</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Total rating</td>
<td>11 (8)</td>
<td>9 (8)</td>
<td>6 (4)</td>
<td>7 (2)</td>
<td>12 (8)</td>
</tr>
</tbody>
</table>

**NB.** The (secondary) rating in the parenthesis is the summation of the RGap and the TDAAD only.
The ordinal classification of the test runs and the final classification for both networks is presented in Table 2. The least risky with regards to instability is the run with the 5-min simulation step for the Sioux Falls network and for the Anaheim network the runs with 15-min and 20-min steps. Although the secondary rating of the final classification, which takes into account only the RGap and the TDAAD scoring, is almost the same, the resolution of the analysis is less if the avS measure is excluded.

CONCLUSIONS

In this paper, the necessity for performing stability analysis of the DTA solution is been discussed as part of a risk assessment procedure through the evolution of time. In this framework, we proposed stability measures and a methodology for evaluating or comparing the stability of a DTA solution. The methodology is based on calculating selected stability measures that describe the scope of the analysis and on performing statistical analysis for providing a risk assessment for the DTA solution. In addition, an examination was made of the influence of the simulation step in the stability of the DTA solution, through a sensitivity analysis on the selected stability measures and standard performance.

The computational experiments were performed for two referenced networks involved the calculation of two commonly used stability measures and one new measure proposed in this paper. The statistical analysis of the three stability measures for five different simulation steps sizes provided valuable information for the behavior of selected measures in two different network and demand setup. The stability analysis concluded with the combination risk assessment of the instability for the test runs.

The advantages of the proposed stability analysis include:

- The identification of time dependent risk behavior of the stability properties of the DTA solution
- The evaluation of the stability which is time dependent
- The combinational evaluation of the stability measures which takes into account different aspects and properties of the DTA procedure or the network setup.

The main contribution of this work is the proposed stability analysis which can be a useful tool for practitioners and researchers interested in evaluating the system’s fluctuations through time and its stability properties. It should be noted that the proposed stability analysis is not a calibration procedure, but an assessment of the final solution produced by the already calibrated DTA module.

This study can be expanded to include the assessment and comparison of the behavior of other stability measures as a sensitivity analysis of the stability measures with different models, networks and demand conditions. Future work will include the sensitivity analysis of the stability of the network setup (e.g. higher or lower level of reserved capacity, dense or sparse network coding, small, medium or large scale network) and of the chosen demand distribution (e.g. constant, normal, lognormal). Also, in an extended stability analysis a much more detailed statistical analysis with ANOVA would consider multiple repetitions for the same cases for a more extensive analysis.

Additionally, the risk assessment can be extended to provide the valuable information on time windows as to when there is a higher risk for unstable traffic conditions and thus a greater risk of misrepresentation of the traffic conditions. Furthermore, the aim of the risk assessment and the stability analysis would not only be to find the vulnerable time windows of the DTA solution, but also to find ways to improve or sustain a level of stability and reliability both spatially and temporally.

REFERENCES


