Logic Scoring of Preference (LSP) Application to Transportation Investment Portfolio Optimization: A Case Study in Colorado Springs

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Abstract

The evaluation and prioritization of transportation investments at the portfolio level presents a complex decision-making problem for state and regional planning organizations. Historically, transportation investment decisions have been dealt with as a series of stand-alone problems to be resolved using straight-forward engineering solutions. In this context, improvement needs and project solutions were identified based on simple criteria, such as traffic congestion levels. Investment portfolio optimization was accomplished by listing projects in order of most to least congestion reducing, and then allocating funding to projects by rank until funding is exhausted. Recently, increasing awareness of the complex interdependencies among transportation, land-use, social, economic and ecological systems has fostered implementation of multi-criteria analysis (MCA) investment prioritization approaches that incorporate increasingly more complex goals and metrics. The simplest MCA decision model is the Weighted Sum Model (WSM). More rigorous methods, such as the Analytical Hierarchy Process (AHP) and the Technique for Ordered Preference by Similarity to Ideal Solution (TOPSIS), provide increased functionality, and support prioritization that is driven by asset performance and financial return in addition to engineering criteria. This paper examines the suitability of three decision models for the optimization of transportation investment priorities across full programs/portfolios. A Logic Scoring of Preference (LSP) approach is contrasted to the WSM approach that is currently used by the Pikes Peak Area Council of Governments (PPACG), as well as to an enhanced linear programming optimization (OPT) algorithm approach. Functionality, advantages, and disadvantages of each method are discussed, and potential enhancements are identified.
1 INTRODUCTION

Selection of projects for inclusion in fiscally constrained long range plans and short term improvement programs has historically been completed using engineering methods, using straightforward engineering criteria such as crash and/or congestion reduction. Projects were typically ranked and prioritized for funding in order of most to least improvement, with performance for cost occasionally considered in project selection. With project rank thus identified, projects were funded in priority order until available funding was exhausted (rank and cut). Recently, as both awareness of, and requirements to consider other criteria have increased, there has been an increased use of multi-criteria analysis (MCA) methods. MCA methodology has proven attractive for addressing these emerging considerations because it provides the necessary platform to evaluate individual projects using a variety of transportation and non-transportation asset performance measures. The MCA methodologies most commonly applied to transportation problems include: the Weighted Sum Model (WSM), the Analytic Hierarchy Process (AHP) (1) and the Technique for Ordered Preference by Similarity to Ideal Solution (TOPSIS) (2). Each of these MCA methods is designed to provide decision-makers with the ability to make the best possible individual investment decisions based on past, present and future predicted information. Transportation MCA applications include: project prioritization/selection (3, 4, 5 and 6), performance monitoring (7), and evaluation of both economic development linkages (8) and transportation sustainability (9).

To date similar rigor has been applied only on a limited basis to optimizing the performance of the full portfolio of potential transportation projects. Rigorously analyzing portfolio performance is common in other fields, as examples from stock portfolio/mutual fund analysis (10) and internet/computer facilities investment portfolio analysis (11 and 12). Application of this approach to the optimization of transportation investment represents an emerging focus in which transportation investment is no longer viewed as a series of stand-alone projects to address specific issues, but rather as investment in an integrated system that is also characterized by complex interdependencies with related land-use, economic, ecological, and social systems.

Transportation investment portfolio optimization can be viewed, in its simplest form, as selecting the best fiscally constrained combination of projects from a finite set of available projects using adopted goals and metrics for either the 20+ year Region Transportation Plan (RTP) or the 4+ year Transportation Improvement Program (TIP). This requires identification of a “best set of projects” that, when implemented, will simultaneously minimize negative and maximize positive total final outcomes, as measured by adopted performance metrics. To find this “best set of projects” optimal solution, sets of projects drawn from the larger set of projects must be systematically analyzed using a formal decision process.

The first and second order decision models presented in this paper apply metaheuristic methodologies (13) to transportation investment portfolio optimization. In computer science, metaheuristics is a computational method of finding an optimal solution by iteratively trying to improve a future condition using some predetermined measures of quality (performance metrics). Metaheuristic computational methods include: combinatorial optimization, evolutionary algorithms, dynamic programming, and stochastic optimization. While the most common metaheuristic method is stochastic optimization, the searching use of random variables is not suited for transportation portfolio optimization. Similarly, dynamic programming refers to simplifying a complicated problem by breaking it down into...
simpler sub problems that can be solved once, in a recursive manner. The interrelationships between
goals, and how each individual project impacts those interrelated goals means that dynamic programming
is ill-suited to portfolio optimization. Instead, combinatorial optimization processes that find an optimal
set from within a finite set of projects should be used. Evolutionary Algorithms are also useful, but their
added complexity make them computationally challenging for operational implementation. Due to the
complexity of the systems involved in the portfolio analysis it is likely that there is more than one
“optimal” portfolio. Identifying the trade-offs between different optimal potential portfolios may be a
second goal when setting up and solving transportation portfolio optimization.

2 PROBLEM STATEMENT

The Pikes Peak Area Council of Governments’ (PPACG’s) 2035 Moving Forward Update long
range transportation plan (LRTP) was developed using a collaborative process with non-traditional
resource agency participation (e.g. U.S. Fish & Wildlife Service, National Park Service). During the
visioning phase of the LRTP update, 17 performance-based evaluation criteria were identified as the basis
for project selection for inclusion in the fiscally-constrained LRTP, and weights were established for each
of the evaluation criteria. Each project was scored on the evaluation criteria using consistent,
predetermined metrics. A simple, WSM-based decision model was then used to calculate weighted scores
for 139 projects and select a shortlist of projects for inclusion in the LRTP. Using a rank and cut process,
modified only as necessitated by funding eligibility requirements, 27 projects were selected. After the
fact, the process that was used was viewed by some stakeholders as insufficiently transparent and as
inflexible. Deficiencies cited included the inability to account for synergies among projects or to respond
to unique circumstances or needs. The small number of projects “funded” was also viewed unfavorably
by some stakeholders. This paper explores alternative transportation portfolio optimization methodologies
that might be implemented to improve upon the decision process used by PPACG for the 2035 LRTP.

3 ALTERNATIVE DECISION MODELS

3.1 Zero Order Decision Model: Weighted Sum Model (WSM)

The zero order decision model used by the PPACG for 2035 LRTP fiscally constrained project
selection is commonly known as the Weighted Sum Model (WSM). In the WSM, each project \( (i) \) receives
a total weighted score \( (V_i) \) as the sum of each criteria score \( (S_j) \) for project \( (i) \) weighted by an a priori
weight \( (W_j) \), such that the project with the highest resulting total score, or max \( (V_i) \) is afforded highest
priority directly. Priority for the rest of the project set is set directly as well, according to rank per the
value of \( V_i \). This can be written in short hand form as equation \[1\] below.

\[
V_i = \sum_{j=1}^{J_{\text{max}}} S_{ij} W_j
\]

[Equation 1]

Where: \( J_{\text{max}} = \) the total number of scores and associated weights for the \( i^{\text{th}} \) project.

The simplicity of this method is attractive, especially when communicating with the public, but it contains
significant pitfalls. First, the set of weights may have a high degree of arbitrariness, as the choice of a
weight is already in itself a decision, but not necessarily an outcome of a formal decision process.
In order to have weights mean the same across projects, they must also be based on a formal set of criteria. This is a very difficult task to achieve when many decision makers are involved and unknown biases can influence given weight choices. Thus, an inclusive decision process that can be applied to criteria selection, weight choices and ultimate transportation improvement portfolio selection and optimization is needed if the preferences are to be valid. The PPACG WSM application utilizes criteria and weights developed through an inclusive process, but one that lacks formality and transparency needed to make it flexible or defensible.

Using the WSM, it is difficult to cross prioritize or “bundle” best choices of projects because performance scores \( S_j \) are tied to individual projects and not be able to go across projects as they should. That is, a performance \( j \) for project \( i \) may be different and of un-measurable value for a project \( i+1 \) which may, or may not contain the \( j \) criterion. In this case it is in fact completely random and naïve what \( V_i \) really is and means to the decision process. Thus, the weighted average may give an initial “feeling” of choice and preference, but it is insufficient to give a global and adjustable score without some logical filters.

### 3.2 First Order Decision Model Approach: Linear Programming Optimization Algorithm (OPT)

A significant improvement over the zero order, WSM decision model used for the PPACG 2035 LRTP can be achieved by a first order linear programming optimization algorithm (OPT) proposed by An and Zheng (14). Application of OPT for the 139 PPACG 2035 LRTP Update projects produced increases in total benefit scores (15,232 vs. 6,051) and number of projects selected (133 vs. 27) as compared to zero order the WSM-based approach used by PPACG for the 2035 LRTP. These results confirm that the OPT would be effective in maximizing both number of projects selected and overall benefit. However, neither the need to account for synergies among projects (if you build Project A would it make Project B more or less beneficial), nor the danger of precluding a project from selection based on cost alone (is the project critical; could it be phased) would be addressed directly by OPT.

The proposed OPT decision model would introduce restricted formal logic functions (FLF) to provide a “yes/no” bias to the selection criteria as described below in equations [2 and 3].

\[
x_m = \begin{cases} 
1, & \text{if } C_m \leq B \\
0, & \text{if } C_m > B
\end{cases} \quad [\text{Equation 2}]
\]

\[
x_n = \begin{cases} 
1, & \text{if } C_n \leq |B - \sum x_mC_m| \\
0, & \text{if } C_n > |B - \sum x_mC_m|
\end{cases} \quad [\text{Equation 3}]
\]

Where: \( B \) is the total allowed budget for all projects and the definition of \( C_m \) and \( C_n \) are the costs associated to the \( i^{th} \) project under two constraints that in a single strike “filter out” that project from the selection set, if either it exceeds the budget, or there is no budget left.

Applying the first OPT formal logic function, as represented by equation [2], if the cost of an individual project is more than the funds available \( (B) \), the project is immediately deselected \( (X_m = 0) \). At first this seems reasonable; however cost negotiations, phasing accommodations or extreme necessity (e.g. to replace a fallen bridge) could be blind-sided on the very first pass by application of this zero-bandwidth binary filter. Expansion of the OPT method such as formulations 3 and 4 as described in the
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NCHRP 590 report (15), have improved the OPT functionality by creating extra constraints in a Linear Programming environment whilst maintaining a zero-bandwidth logic filter.

The second function, as represented by equation [3], acts as an “empty bucket” filter. This filter excludes any project whose incremental cost, $C_n$, exceeds what is left after other, higher priority projects have been funded. Again, at first this seems reasonable, but if a critical project is not selected or costs more than funds available, there is no method to de-select an already selected project to include this critical project. This scheme also lacks a decision process that weighs in cross-linkages among projects with a common methodology that is discriminating of many aspects of the decision process. In most cases scores across links are intrinsically non-quantitative and yet need enough of a quantitative core to be objective and practical.

After the implementation of the two formal logic functions, a linear programming optimization algorithm (OPT) couples the constraints to these two formal logic functions $X_m$ and $X_n$. Finally, the constraints imposed by the total budget, until it is exhausted, are evaluated by doing a binary search that relaxes when the maximum number of projects is funded and the budget is met. Equations [4 and 5], below, show the linear programming hyperspace.

$$\text{Max} \left\{ \sum_i^{\text{max}} \sum_j^{\text{max}} X_i S_j W_j \right\} = \text{Max} \left\{ \sum_i^{\text{max}} (CP_{ij} \ V_i) \right\}$$  \hspace{1cm} \text{[Equation 4]}

Where: $CP_{ij} = \text{chosen projects and } V_j \text{ is like } V_i \text{ but only of selected projects, such that:}$

$$\sum_i C_i X_i \leq B$$ \hspace{1cm} \text{[Equation 5]}

The products of $CP_{ij}$ and $V_i$ when at a maximum, represent all the chosen projects and the weighted sum of such projects. However, as described above, $X_m$ and $X_n$, filter out projects with respect to constraints within narrow logic criteria. The fundamental limitation intrinsic in the first order model is the use of formal logic functions that provide zero bandwidth to the local decision and filter out projects without allowing the degrees of freedom that are inherently needed in any decision process. Thus, the zero order and first order models are mathematically descriptive of mappings that could be seen as objective and formal, but that fall short of the expected breadth and depth that decision-makers require of an analytical approach to decision making that the public can accept as fair and careful of special circumstances. Such mathematical descriptions may create a sense of modeling but do not constitute formal models. Thus, the optimization process (OPT) is a support system for decision-making. While the analyses results are useful guides to the final decision, they are not substitutes for the final judgment. On seeing the optimized results, decision-makers are likely to consider constraints and objectives that were not included in the analyses (thus making the model blind to these variables). It is the decision-maker’s final selection – grounded in their paradigm but informed by the analyses – that is funded, implemented and managed.

3.3 A Second Order Decision Model Approach: Logic Scoring of Preference (LSP)

The central technique used as the framework for the second order model is the Logic Scoring of Preference (LSP) method for decision processes. The LSP method described in the seminal work of J. J. Dujmovic (11) was originally developed for evaluation and selection of complex networks. The key point of this method is the expansion of the bandwidth of the logic functions. Thus, the “yes/no” measure
becomes percentages of preference from 0 – 100%. Furthermore, the preferences can be directly linked to a quantitative measure of “cost/benefit” ratio.

Starting with equation [1] from the WSM in which the \( i \)th project total score was:

\[
V_i = \sum_{j}^{J_{\text{max}}} S_{ij} W_j
\]

The LSP \( i \)th project total score is generalized as:

\[
V_i (G(S_{ij})) = \sum_{i}^{I_{\text{max}}} \sum_{j}^{J_{\text{max}}} G(S_{ij})
\]  \[\text{Equation 6}\]

In this formulation, \( S_{ij} \) is still a performance criterion score, but the weights are generated by a mapping function \( G(S_{ij}) \) between each performance criterion of the \( i \)th project, generating first an elementary preference, \( EP_j \) which is represented by the preference aggregation structures shown in Figure 1.

\[
E_i = (G(S_{i1}) (EP_1)^r + G(S_{i2}) (EP_2)^r + \ldots + G(S_{ik}) (EP_k)^r
\]  \[\text{Equation 7}\]

For the zero-order model (Weighted Sum Model) with constant weights, \( G(S_{ij}) = W_j \), where \( r \) is equal to 1. Also, the elementary preference is normalized such that \( 0 < EP_j < 1 \) (or 0-100%). Thus, the preference aggregate for the \( i \)th project is:

\[
E_i = W_1 EP_1 + \ldots + W_{J_{\text{max}}} EP_{J_{\text{max}}} = \sum_{j}^{J_{\text{max}}} (EP_j) W_j
\]  \[\text{Equation 8}\]

Equation [8] is identical to equation [1] for \( V_i \), provided that \( S_{ij} \) is now a normalized score \( EP_j \) for the \( i \)th project. However, there is a fundamental conceptual adjustment from \( S_{ij} \) to \( EP_j \). Firstly, \( EP_j \) is a preliminary score, not yet subjected to a logical function. Also, the power index \( r \) will be described as a function of the logical function bandwidth, \( d \). This is the core of the LSP methodology: the logical function bandwidth in LSP varies from 0 to 1, whilst the formal logic has only two values (0, 1). Thus, in a sense the bandwidth of formal logic is zero and of LSP infinite.
**Figure 2** shows how the formal logic function (FLF) of the first order OPT model is expanded in the bandwidth such that the aggregation preference for the $i^{th}$ project varies between “very good” or $d = 1$, and “very bad” or $d = 0$. The value of $r$ depends on $d$ such that a value of $r$ can be chosen from objectively calculated results where the formal logic values of “yes/no” or “0/1” are expanded to degrees of preference. It is clear that $d$ is the indicator of the average position between $E_{\text{max}}$ and $E_{\text{min}}$.

\[
\begin{align*}
d = 1 & \quad E_{\text{max}} = \max (E_1, E_2, \ldots E_k) = \text{"Positive Ideal Solution (PIS)"} \\
d = 0.5 & \quad e_0 = \text{neutral "preference"} \\
d = 0 & \quad E_{\text{min}} = \min (E_1, E_2, \ldots E_k) = \text{"Negative Ideal Solution (NIS)"}
\end{align*}
\]

**FIGURE 2 Expanded Second Order Preference Aggregation Structure**

In the second order LSP decision model, various values of $r$ are used to create degrees of logic such as “quasi-AND” and “quasi-OR.” In the decision literature, AND is a “conjunction,” meaning that $E_1$ and $E_2$ and…$E_k$ are all necessary and need to be “added” to the aggregate preference. These are “must-have” performance criteria. It is here that one can see that the expansion of the logic bandwidth spread the values of the preferences, allowing a better decision space. The formal logic function of the first order model does not allow for the value of $E_j$ to be a number between 0 and 1. Thus, the OPT model is very restrictive and may filter out projects too early by having only $X_j = (0, 1)$. The range of degrees of logic function created by the values of $r$ includes: AND ($r = -\infty$), HARMONIC MEAN ($r = -1$), ARITHMETIC MEAN ($r = 1$), MEAN SQUARE SUM ($r = 2$), and OR ($r = +\infty$).

How the value of “$r$” leads to the tabulated quasi logic functions is well explained by references (11 and 15). Here we will use these tabulated functions to create “structured decision circuits” which are set as the standards for the decision processes. In this manner the a priori transparency of the circuits will make the results for every project readily comparable. There are five types of circuits as described in reference (11). These are:

1) CPA (Conjunction with partial absorption)  
2) Quasi-AND (Quasi-conjunction)  
3) Neutrality (A or arithmetic mean)  
4) Quasi-OR (Quasi disjunction)  
5) DPA (Disjunction with partial absorption).

The reader should readily see that circuit type 3 (Neutrality - A or arithmetic mean) has already been discussed. In circuit type 1 (CPA - Conjunction with partial absorption), a conjunction shows that “$E_1$ AND $E_2$, AND … $E_k$ means that the preferences are mandatory. **Figure 3** below is an example for circuit (2, “C - +” - Quasi-AND) with the simplified assumption of simple weights for $S_j (= W_j$). In this example, the values specified for $r$ and $d$ are specified for a set of three performance criteria/scores. Circuit types 4 and 5 will be explained later.
For each project, thus, for simplicity, in $S_i$, the $i$ is dropped. Thus, the MDE for the elementary preference ($E_0$) in a quasi-conjunction (where simultaneity is required for performance criteria) is:

$$E_0 = \left( \sum \frac{W_j}{S_j^r} \right)^{1/r} = (0.5 S_1^{-0.028} + 0.3 S_2^{-0.028} + 0.2 S_3^{-0.028})^{-1/0.028}$$

The combined or aggregate preference for this example ($S_1=.7, S_2=.8$ and $S_3=.8$) is $E_0=73\%$ or 0.7, closer to $d=1$, or the conjunction of the $i^{th}$ project is a strong preference, close to 100\% (or $d=1$). Where as in the first order model this would be $X_n$ or $X_m$ where it would be simply 100\% (or $d=1$). From this example, the value of the LSP method is clear. Since $E_0$ is 73\% and not 100\%, the bandwidth leaves 27\% for other projects to move ahead of this one. A similarly revealing example for Quasi-OR is presented in reference (11). It is this sliding scale within LSP that yields an immediate advantage over the first order model (where all exponents would be just “1” or $r=1$).

The Quasi-logic circuits (QLC) are the basic units for optimization. In this paper only the Quasi-AND function, as shown in Figure 4, will be used for Pikes Peak area data, as the benchmark for all initially mandatory preferences. However, for completeness and to show the richness of the LSP method, the circuit diagram for the Disjunction function is also shown. The circuit diagram for the Quasi-AND function shows the ability to add “Optional” preferences, while the circuit diagram for the Disjunction function shows that “Sufficient” and “Desired” preference specifications can be added for that QLC function. Decision networks with the use of QLCs provide greater flexibility than either the WSM or OPT methods for development of a Formal Decision Process. QLC first does an “A” model; then it adds the large bandwidth logic to automatically drive the preference.

The set of $\{S_j\}$ or $\{S_{ij}\}$ can be connected to allow qualitative measures such as mandatory, optional, sufficient, and desired can be added to the decision process. Thus, every project starts with the purely mathematical weighted sum (box A) and the logic filtering is contained in the “Quasi” boxes yielding more degrees of freedom and bandwidth to choices. Optimization of the decision is then an integral part of exciting these circuits until the constraints are met. The validity of this method can be justified in terms of mathematical formalisms, such as “Threshold Logic” or “Preference Neural Networks” (16). The second set of weights, as shown in Figure 4, allows another degree of freedom that is used to spread these qualitative preferences.
As discussed above, LSP allows a larger bandwidth of preferences by expanding the rigid filtering of formal logic (0, 1) to a more practical infinite set of “percentage” scores for the of preferences around the neutral value of a weighted sum scheme. This methodology is repeated for all projects within the same Decision Network. As an extension of this approach that is not used in this paper, functional descriptions for $G(S_{ij})$ could also be developed in lieu of simple “weights.” With final “scoring” complete, budgetary constraints and the risk associated with determining the best cost/benefit ratios can be evaluated as the second part of the decision process. This can be done within a single project circuit or by using all projects in a Decision Network.

4.1 Cost and Budget Constraints

There are many ways to quantify the budget constraints such that decisions are focused only on projects that fulfill the following criteria: (1) projects that are not too expensive, so that the largest number of projects are selected for inclusion in the Regional Transportation Plan; and (2) that all chosen projects have a maximum “aggregate preference” ($\max(E_0)$) and best cost/benefit ratio. This is another great advantage of the LSP process. It actually allows one more degree of freedom to the decision process – that is, after “all the votes are in,” or all aggregate preferences are chosen, the preferences can be weighted further based on budgetary conditions, and clear benefit/cost ratios of each project or by the aggregate.
In this paper we use the three cost models presented by J. J. Dujmovic, and generalize them with a novel “best use of money” law for the Global Criterion.

The simplest rule for “Cost/Benefit” is given by equation [9], below (the linear model).

$$Q = \frac{E}{C} \quad \text{for the } i^{th} \text{ project}$$  \[\text{Equation 9}\]

Where: $E$ is the preference score for the $i^{th}$ project, $C$ is its associated cost, and $Q^{-1}$ = “cost/benefit” ratio.

Thus, the function $Q$ means that the score of the $j^{th}$ criterion, resulting from a combination of these quasi-logic functions, versus its cost, is the preference score-to-cost ratio, or inverse cost/benefit ratio. We use $Q = (\text{cost/benefit})^{-1}$ to stay consistent with the quantitative methods to follow. Figure 5 shows a project ($i=1$), with four performance criteria ($E_j = 1, 2, 3, 4$) and their associated costs ($C_j$).

Finally it is obvious that the preliminary budget constraint is:

$$\sum_{i=1}^{n} C_i \leq B$$  \[\text{Equation 10}\]

Where: $B$ is the total budget and $i = 1, …, n$ are for all chosen projects.
However, equation \[10\] can also be used in different ways within a single project. It is clear that we can also use the aggregate “Q” for the \(i^{th}\) project, and the aggregate preferences to show that the best cost/benefit ratio for total budget allocation has been met when equation \[11\] is satisfied.

\[
\sum_{i=1}^{n} \frac{E_i}{Q_i} \leq B \quad \text{[Equation 11]}
\]

Equation \[11\] shows that the usually qualitative cost/benefit analysis is now quantitatively linked to the budget constraints, and the optimization constraints of the first order model are now met with such quantitative decision parameters embedded in the logic system used.

In today’s economy, to maximize the number of projects within budget is difficult to attain. Thus, decision-makers need another degree of freedom to allocate resources marginally to each project using a performance parameter that can “spread the money around” as much as possible and at the same time, show the benefit of one project versus another. Possible cross project bundling and other partial or time-dependent budget factors that are also allowed here are considerations that must be supported in the evaluation of funding allocation alternatives.

The required second layer final tuning can be achieved by introducing a final preference score \(p\), to the \(Q\) score as shown in equation \[12\] below.

\[
Q_k = p \left( \frac{E_k}{E_{\text{max}}} \right) + (1-p) \left( \frac{C_{\text{min}}}{C_k} \right),\ k=1,\ldots, n \quad \text{[Equation 12]}
\]

Where: \(0 \leq p \leq 1\).

In this case, \(k\) can mean the \(i^{th}\) project or just the \(j^{th}\) criterion taking a larger role (or not) because the cost \(C_k\) may exceed an a priori determined minimum cost that the agency or decision-makers are willing to pay. This is a way to quantify the phrases “this is just too expensive at this time to have this performance criterion,” and “not now.” Thus, if such a situation occurs, a new \(E\) versus \(C\) curve and a new preference-based \(Q\) can go up or down depending on the purely resource allocation preferences at the final level of decision.

The parameter \(p\) is also an “importance parameter” – that is, just how important this or that project is in this budgeting year is the bias on \(Q\) by \(p\). Thus, \(Q\) can be by project \(Q_i\) or the Global Criterion. Equally important to know is just how much project \(i\) (or criterion \(j\)) takes money from the budget or adds to the cost of enforcing such a criterion. This is accomplished by:

\[
Q_k = pE_k + (1-p) \left( \frac{C_{\text{max}} - C_k}{C_{\text{max}}} \right) \quad \text{[Equation 13]}
\]

Where: \(k = i\) for projects or \(k = j\) for criteria within a project

For \(k = i\), the \(Q\)-index for the project goes up or down depending on the difference \(C_{\text{max}} - C_k\), or how much the cost of a project takes from the budget. Thus, in this case \(C_{\text{max}} = B\). In the case \(k = j\), the maximum accepted cost of enforcing criterion \(j\) of project \(i\), shows how much that criterion pushes \(Q\) up.
or down. Thus, it allows the ability to modify this criterion (in this case $C_{\text{max}} < B$). In the Colorado Springs data, this step is omitted as $p$ is not yet used in the examples given.

Another quantitative correction to the decision process is that all costs can be associated with the present value ($PV$) of the total cost over time. Thus,

$$C_{\text{present}} = \frac{C_{\text{future}}}{(1 + i\%)^n}$$

[Equation 14]

Where: $C_{\text{present}} = \text{cost this budget year}$

$C_{\text{future}} = \text{cost after conclusion}$

$i\% = \text{rate of inflation}$

$n = \text{the number of time cycles (months, years, etc.)}$

Thus, all $C_i$ can be analyzed as cost-over-time. This is also a useful measure for $Q_k$ if one simply waits and makes no decisions at the current budget versus make the decision and know the future cost. Risk is introduced in the analysis in two ways:

(1) The risk of not making a decision or delaying a project as $C_i$ increases over time, and

(2) The risks of pitching the wrong projects or choosing projects with diminished synergy — “burning the budget” with insignificant projects

For case (1), calculation of two or more values of $Q$ over different $PV$s suffices. For case (2), the importance index can be used by sorting $Q$ values — that is run a binary search of $Q_{t+1} - Q_t = \Delta Q$ (where: the $t$-indices mean different times) and define the performance, index as:

$$p = \frac{Q_{\text{min}}}{\Delta Q}$$

[Equation 15]

Where: $Q_{\text{min}}$ means the smallest Q or best cost/benefit. It also means that now the final priority (or performance) index, $p$, is a function of time.

### 4.2 Law of Best Effectiveness

The Law of Best Effectiveness in the Decision Process is described here as the “Decision Power.” It is possible to define just how responsive (effective) the set of decisions were that made one budgetary/project choice better than another. In this paper, “Decision Power” is shown as the curve of $Q$ versus $C$. Assuming that the combined $Q$, whether biased by $p$-indices or not, still follow a curve described generally by:

$$Q_i = E_i C_i^{-\alpha}$$

[Equation 16]

Where $\alpha$ is now the effectiveness parameter, which before was only equal to “1.” This parameter can be used as the only parameter needed for optimization. The higher the value of the $\alpha$, the better is the choice. Further, using a log-log plot of the equation, the slope is equal to $\alpha$. 

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Since in a log-log plot, the $y$-axis is $y = \ln Q$, then when $x = \ln B$, $y = 0$, or $Q = 1$. Thus, if all $Q_i$ fall on the line or close to it, the budget is met with the best cost/benefit ratio. The budget is completely met and the best set of projects is funded when:

$$\alpha = \frac{b}{\ln B} \quad \text{[Equation 17]}$$

Where: $b = \ln Q_0$, and $Q_0$ = slope of the $E_k$ versus $C_k$ plot for the total number of projects.

In non-linear plots, $Q_0$ is the derivative of the function approximating $E$ versus $C$.

**5 APPLICATION of SIMPLIFIED LSP MODEL TO COLORADO SPRINGS DATA**

Data for nine sample PPACG 2035 LRTP projects was used to run an Excel platform LSP decision model using the Quasi-AND logic function. For this test adopted weighting and normalized scores for nine projects, for which costs ranged from $12,000 to $80,000,000, were processed in a step-wise fashion, using separate, linked worksheets. Data input and processing was completed for five selection criteria at a time, and then linked in a final LSP calculation step. The test project set included roadway, transit and non-motorized mode improvements, demonstrating the necessary broad applicability of the model across modes and project types. The results from test application, shown in Figure 6, demonstrate the feasibility of applying LSP for transportation investment portfolio optimization as it has been applied to other complex optimization problems. However, the simple Quasi-AND test does not begin to tap the potential of an LSP-based decision model to address complex interactions among projects with respect to benefit. The flexibility available within LSP to incorporate less rigid logic functions, such as the Disjunction function, will support development of a logic network that provides the full functionality needed for transportation investment portfolio optimization.

![Benefit/Cost Ratios for Selected Projects](image)

**FIGURE 6 Results of LSP Quasi-AND Application Test for PPACG 2035 LRTP Projects**
6 CONCLUSIONS

The zero order WSM decision model commonly used for transportation investment project selection is essentially an individual project evaluation tool that lacks the functionality required for transportation investment portfolio optimization. The inability of the Weighted Sum Model’s formulation to adapt to address exceptional circumstances or to factor in project interactions were concerns during PPACG 2035 LRTP development that marred an otherwise exceptional collaborative planning process.

Application of a proposed enhance decision model using a linear programming optimization algorithm to the full 2035 LRTP project set demonstrated significant advantage in maximizing total benefit associated with selected projects as well as the number of selected projects. However, the OPT decision model does not have adequate “bandwidth” to provide flexibility needed to accommodate funding of high cost projects, funding of “critical” projects, nor does it have functionality to address project interactions in a way that is needed for full transportation investment portfolio optimization. A carefully crafted LSP decision network can provide the framework for a broader bandwidth decision model that can include full functionality needed for transportation investment portfolio optimization. Although this approach represents an improved approach for transportation investment, it is an approach that has been successfully implemented for computer system and investment portfolio applications.
REFERENCES


