Disruption Response Planning for an Urban Mass Rapid Transit Network

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ABSTRACT

Given many cities’ growing dependence on public rail transport, simple network disruptions can lead to widespread confusion and significant productivity loss to the society. Therefore, a systematic approach to develop efficient disruption response and minimize the negative impacts is required. In this paper, we develop a planning procedure to supplement a degraded urban mass rapid transit network through intelligent introduction of shuttle bus services in the disrupted area. The proposed method includes two important mechanisms, namely (1) using column generation to identify all beneficial bus routes, including those which might not be intuitively found, and (2) using a path-based multi-commodity flow formulation to select the best among these candidate bus routes. Finally, the method is applied to two disruption case studies defined using real-world data; the corresponding results confirmed the practicality of the proposed approach: (1) the procedure can be carried out efficiently, (2) introducing bus routes to the naive bridging services can easily yield significant improvement on commuters’ travel delay, (3) the distribution of commuters’ travel delay is improved considerably with an optimized response, and (4) many realistic operating constraints can be handled in planning process.

Keywords: Transport disruption, Response planning, Network disruption, Mass rapid transit, Public transport
INTRODUCTION
As cities and demand for mobility grows, having an efficient public transport network is becoming more important. Being a shared mode of transportation service, public transport can carry more passengers more efficiently than private transport, thus reducing both per unit transportation cost and environmental impact. In Singapore, a city with a population of 5 million, for example, around 4 million public transport (i.e. bus and mass rapid transit (MRT) trips were made on any given day in 2011.

This heavy reliance on public transport placed all related infrastructure under enormous strains. Simple service disruptions can easily lead to unacceptable outcomes affecting a large number of commuters, especially for infrastructure with high utilization rate, such as the MRT tracks and stations. Large disruption events will not only cause inconvenience and travel delay for commuters; commuters can lose confidence in public transport and be encouraged to use private transport in the long run. Take for example the 16th Dec 2012 disruption in Singapore’s MRT: train service at 11 stations were disrupted for 5 hours and more than 100,000 commuters were affected.

Besides minimizing the number of disruptions through better engineering, maintenance and operational planning, it is critical to put in place an effective network disruption response to minimize the severity of potential impacts. However, while much research efforts have been dedicated to the strategic planning and operational management of the routine operations of the public transport networks (1), significantly less effort has focus on the response planning for potential disruptions. Figure 1 shows an integrated planning framework covering both traditional and disruption response planning. As can be seen, while both planning components deal with the design of service lines and frequencies, the performance measures used are different. Traditional planning is concerned with objectives such as service quality and operational cost over a long term, while disruption response planning focuses on minimizing the impacts over a much shorter period, usually no longer than several hours. Consequently, models and techniques developed for routine planning are often not applicable when planning for disaster response. Instead, a major challenge of the latter is: how best to introduce new, albeit temporary, services to complement the original undisrupted part of the network, so that the overall travel demand can be addressed with minimum negative impacts.

In this paper, we propose a disruption response planning procedure to supplement a degraded urban MRT network through intelligent introduction of shuttle bus services in the disrupted area. The remaining paper is organized as follows: in Section 3, relevant papers in the literature are reviewed. In Section 4, the disruption planning problem will be described; the column generation based planning approach will be developed. Finally, the results of applying the method on two case studies based on real-world data are presented in Section 5 while Section 6 concludes the paper.

LITERATURE REVIEW
Traditional planning for public transport (bus, railway, tram and underground) concerns solving the decision problems arising at the planning stage of public transport from strategic level to operational level, such as network design, line planning and timetabling. Various models and methods based on optimization and operations research techniques have been developed in the literature. Readers may refer to (1, 2, 3) for comprehensive reviews. The network design step deals with the configuration of new transportation system to achieve specific objectives. Ceder and Wilson (4) presented a two-level methodological approach to the bus network design problem considering just the passenger viewpoint as well as both the passenger and operator impacts. Melkote and Daskin
FIGURE 1 Integrated planning framework for public transport

(5) developed an integrated optimization model which considers simultaneously the decisions of network design with facility (infrastructure of public transport) locations. The integrated problem was modelled as a special case of network design problem and realistically sized problems could be solved very efficiently. Mauttone and Urquhart (6) developed a route set construction algorithm, called pair insertion algorithm, for the transit network design problem considering the convenience from the perspectives of both users and operators. With the determined network structure, the following step is the line planning problem which is to design line routes and their frequencies according to the travel demand. Schobel and Scholl (7) presented integer programming models for the line planning problem with the objective of minimizing the travel time over all commuters including penalties for transfers. A solution approach using Dantzig-Wolfe decomposition was also proposed. Borndorfer et al. (8) developed a new multi-commodity flow model for line planning and devised a column generation solution approach that allows to generate lines dynamically. Timetabling is the following decision problem which is to establish the schedules and frequencies of detailed transport services with the objective of minimizing passenger waiting times both at stops and transfers (9, 10).

Reliability and robustness of public transport services are receiving more and more attention as any disturbances (e.g., special events or accidents) in the transportation network may significantly reduce the service quality or even cause disruption. In light of this, preventive planning concepts and models have been proposed by researchers when designing public transport services which is robust to disturbances, such as (11, 12, 13, 14).

As an alternative approach to robust planning, disruption response planning focuses on devising responsive measures for potential transportation system disruptions, which are able to alleviate disruption consequences. However, very few studies on disruption response planning for public transport can be identified in literature. As the most relevant study, Meyer and Belobaba (15) examined the contingency planning processes and identified important characteristics in the planning process. To the best of our knowledge, no efforts has been dedicated to applying mathematical optimization techniques for the disruption response planning of public transport. This
paper particularly models and solves the response planning problem for disruptions in an urban MRT network.

**DISRUPTION RESPONSE PLANNING PROBLEM**

In this section, a detailed description of the disruption response planning problem is first presented. Then, the planning process is developed in the following steps: (1) defining a network representation of the integrated MRT and bus service network, (2) finding the response bus route candidates, (3) bus route selection from candidate list, and (4) determining number of buses required on each route.

**Problem description**

Consider the response planning for the scenario in which a partial section of an MRT line fails to function properly. Shuttle bus services should be employed to recover the transportation service along the disrupted section of the MRT line, so that affected commuters are able to continue their journey with minimum disruption. The standard response that are usually taken by MRT operators is to deploy shuttle buses running in parallel to the disrupted MRT section. Introducing the standard response buses is the basic responsive way of recovering transportation service at the disrupted MRT stations. However, this is usually not the most effective responsive manner since there is no consideration of commuters’ travel pattern for the particular disrupted scenario. Therefore, it is necessary to identify more effective shuttle bus services in order to reduce travel delay for commuters. One example of alternative recovery measures is to deploy express shuttle bus services that travel across the disrupted region. By taking such express bus services, some commuters may encounter less travel delay, and those heading beyond the disrupted area may pass the disrupted MRT section and continue their journey by MRT with minimum disruption.

Figure 2 shows an example of a disruption event in which the train links between Stations $s_1$ and $s_2$ are halted. The standard response introduces shuttle bus service linking up all the stations between and including $s_1$ and $s_2$, as depicted by the six red broken arrows. Besides this standard response, other shuffling services, such as Route 1 to Route 4 as shown here, may be introduced. Depending on the travel demand, a combination of these routes can be a more cost-effective response; it is the objective of our research to develop the methodology to identify these candidate routes, and find the combination.

Formally, the response planning problem is to design the response shuttle-bus services to construct a temporary integrated MRT-bus service network with an objective of minimizing the total travel time delay of all commuters, given a disrupted urban train network and commuters’ travel demand.

**Network representation with transfer consideration**

The integrated MRT and bus service network is defined by a directed graph $G(N, A)$ with a particular consideration of the transfers between the two transportation modes. Figure 3 shows an illustrative example of the graph with three MRT stations belonging to two MRT lines. The node set $N$ consists of two subsets: MRT nodes $N_1$ and bus nodes $N_2$ which are labeled as MRT and bus, respectively. Note that non-interchange stations correspond to one MRT node and one bus node (e.g., Station A and C in Figure 3) while interchange stations are associated with one bus node and multiple MRT nodes (e.g., Station B). Each node $i \in N$ is defined as a $(\rho^+(i), \rho^-(i))$ tuple where $\rho^+(i)$ indicates the station name and $\rho^-(i)$ corresponds to the transportation modes:
bus or MRT lines. Similarly, the arc set $A$ consists of three subsets: subset of train arcs $A_1$ running between two neighboring train nodes, subset of bus arcs $A_2$ connecting two bus nodes, and subset of transfer arcs $A_3$ linking the train nodes and bus nodes sharing the same station name. Note that:

1. $A = A_1 \cup A_2 \cup A_3$;
2. $A_1 \cap A_2 = \emptyset$, $A_2 \cap A_3 = \emptyset$, $A_1 \cap A_3 = \emptyset$;
3. $\rho^+(i) \neq \rho^+(j), \rho^-(i) = \rho^-(j)$ $\forall (i, j) \in A_1 \cup A_2$;
4. $\rho^+(i) = \rho^+(j), \rho^-(i) \neq \rho^-(j)$ $\forall (i, j) \in A_3$.

With the above defined directed graph, we are able to identify for each commuter the shortest path he/she takes. The transfer arcs $A_3$ help to capture the transfers between different transportation modes and MRT lines. Note that commuters’ waiting at bus stops is not modelled in the network representation, since the waiting time highly depends on the response speed of MRT operators. As a result, the commuters’ travel time and travel delay mentioned in the following do not include waiting time for shuttle buses.

**Generation of bus route candidates**

The step of bus route generation aims at finding effective shuttle bus routes in addition to the standard bridging route according to the travel demand of the disruption scenario. For operational convenience, we assume the generated bus routes should start from Station $S_1$ or $S_2$ and terminate...
FIGURE 3 A graph representation of the integrated MRT and bus service network

at the same station. Figure 3 shows four examples of bus routes starting from station $S_1$: express bus service Route A running in parallel to the disrupted MRT section; direct bus service Route B linking $S_1$ with another neighboring station; and two more complex service Routes C and D passing through two stations and returning back to $S_1$. We remark that it may not be possible to enumerate all potential bus routes especially when the MRT network is large. One reason is that the non-intuitive bus routes (C and D) are very likely to be missing. Another reason is that the total number of bus routes is of the order of $n!$ where $n$ is the number of MRT stations that are considered to run bus services. Therefore, we employ column generation algorithm to dynamically generate a set of bus route candidates consisting of both intuitive and non-intuitive routes.

FIGURE 4 Examples of bus route candidates

Column generation is a efficient algorithm for solving large-scale linear programs where
the number of decision variables are much more than that of constraints. The basic idea of tackling
linear programs with such a special structure of constraint matrix is to iteratively solve a restricted
problem only involving a subset of decision variables (*restricted master problem*) and generate
beneficial decision variables by a sub-problem (*pricing sub-problem*) dynamically. The iterative
procedure of solving the restricted master problem and generating new decision variables termin-
ates when the objective function value of the restricted master problem cannot get improved with
introduction of any more decision variables. Therefore, only a small subset of decision variables is
generated and included in the restricted master problem. The benefit of computational efficiency
comes from the restricted master problem and the pricing sub-problem which are much easier to
tackle than the original problem.

Before presenting the column generation algorithm, the following notations for input data
are defined:

**Sets:**

- $R$ : set of bus service routes, initially $R$ only includes the standard bus bridging route
  indexed by 0, i.e., $R = \{0\}$
- $K$ : set of commuters grouped by their origin and destination

**Parameters:**

- $\alpha_{ij}^r$: 1 if link $(i, j) \in A_2$ is covered by route $r \in R$; and 0 otherwise
- $o_k$: origin MRT station of commuter group $k \in K$
- $d_k$: destination MRT station of commuter group $k \in K$
- $q_k$: number of commuters in group $k \in K$
- $c_{ij}$: travel time associated with link $(i, j) \in A$
- $c_0^k$: travel time of commuter group $k \in K$ under non-disrupted condition

Note that the commuters’ travel demand is represented by parameters $o_k, d_k$ and $q_k$ which
could be estimated from historical data. In some cities with implementation of automated fare
collection system like Singapore, the travel demand could be well obtained from the smart card
data. Link travel time parameter $c_{ij}$ should be pre-calibrated for all the MRT, bus and transfer
links. The original travel time for commuters under non-disrupted condition can be obtained by
finding the shortest path in the original MRT network.

20 **Restricted master problem**

In the restricted master problem, we define the following two sets of decision variables:

- $x_{ij}^k : \in \{0, 1\}, \forall (i, j) \in A, \forall k \in K$. 1 if commuter group $k$ uses link $(i, j)$; and 0
  otherwise.

- $y_r : \in \{0, 1\}, \forall r \in R$. 1 if bus service route $r$ is employed; and 0 otherwise.

Then, the restricted master problem is formulated as follows:

\[
[RMP] \quad \text{minimize} \quad \sum_{k \in K} \left( \sum_{(i,j) \in A} c_{ij} x_{ij}^k - c_0^k \right) \\
\text{subject to} \quad \sum_{(i,j) \in A} x_{ij}^k = 1 \quad \forall k \in K | \rho^+(i) = o_k, \rho^-(i) = MRT
\]
Objective function (1) minimizes the total travel delay over all commuter groups. Constraints (2) - (6) are the flow conservation constraints for each commuter group at the source node, sink node and other nodes, respectively. Constraints (2) ensure that there is a unit flow starting at the origin node corresponding to the origin MRT station for each commuter group while Constraints (3) prevent any flow going back to the origin nodes. Different from the constraints defined with respect to the origin nodes, commuters terminates at any one node (bus node or MRT node) corresponding to their destination, as ensured by Constraints (4). Constraints (3) and (5) are imposed to prevent any flow going back to the origin nodes and destination nodes, respectively. Constraints (7) guarantee that bus arcs that are not covered by employed bus services cannot be used. Constraints (8)-(9) define the domain of the decision variables. Note that the binary decision variables are relaxed to be continuous as the restricted master problem only intended to solve the linear relaxation of the formulation.

**Pricing sub-problem**

Let \( \pi_{ij}^k \leq 0 \ \forall (i,j) \in A_2, \forall k \in K \) be the dual variables associated with Constraints (7) of the restricted master problem. Therefore, the reduced cost of bus route \( r \in R \) is:

\[
\tilde{c}_r = \sum_{k \in K} \sum_{(i,j) \in A_2} \pi_{ij}^k \alpha_{ij}^r
\]

The pricing sub-problem should identify such bus route with minimum \( \tilde{c}_r \). If it is found to be negative, the bus route found should be added into the bus route set \( R \). We introduce the decision
variable $z_{ij}, \forall (i, j) \in A_2$ which takes 1 if the bus link $(i, j)$ is covered by the new generated bus route in the pricing sub-problem and 0 otherwise. Therefore, the reduced cost could be updated as:

$$
\tilde{c}_r = \sum_{k \in K} \sum_{(i,j) \in A_2} \pi_{ij}^k z_{ij}
$$

Then, the pricing sub-problem could be formulated as follows:

**[PSP]** minimize \[ \sum_{k \in K} \sum_{(i,j) \in A_2} \pi_{ij}^k z_{ij} \] subject to \[ \sum_{(i,j) \in A_2} z_{ij} = 1 \] \[ \rho^+(i) = s_1 \text{ or } s_2, \rho^-(i) = \text{bus} \] (13)

\[ \sum_{(i,j) \in A_2} z_{ij} - \sum_{(j,i) \in A_2} z_{ij} = 0 \] \[ \forall i \in N \] (14)

\[ \sum_{(i,j) \in A_2} c_{ij} z_{ij} \leq L^1_{\text{max}} \] (15)

\[ \sum_{(i,j) \in A_2} z_{ij} \leq L^2_{\text{max}} \] (16)

\[ z_{ij} \in \{0, 1\} \quad \forall (i, j) \in A_2 \] (17)

The objective function (12) minimizes the reduced cost of the generated bus route. Constraint (13) ensures that the bus route should start from the two starting stations $s_1$ or $s_2$. The flow conservation restriction should be observed at all nodes, as ensured by Constraints (14). In addition, two operational restrictions are also imposed for the generated bus routes: Constraint (15) ensures that the total travel time of any bus route should not exceed $L^1_{\text{max}}$ and Constraint (16) guarantees that the generated bus route does not visit more than $L^2_{\text{max}}$ stations. The above two constraints are introduced for the sake of operational efficiency, since bus routes with long travel distance or many stops may not be efficient for disruption response. The domain of the decision variable $z_{ij}$ is defined by Constraint (17).

We remark that the pricing sub-problem is a shortest path problem with negative link cost. Thus, sub-tours may exist in the obtained bus route which should be discarded. In order to avoid this, a post-generation checking procedure is introduced. If any sub-tour exists, we add the following cuts and re-run [PSP] until no sub-tours exist:

$$
\sum_{(i,j) \in S^t} (1 - z^t_{ij}) \geq 1
$$

(18)

where $S^t$ is the set of bus links comprising the sub-tours in the $t^{th}$ run. With the additional constraint when solving [PSP] in $(t+1)^{th}$ run, the sub-tours in the previous $t$ runs could be avoided.

If the objective value of the pricing sub-problem $\tilde{c}_r < 0$, we add the bus route $r^*$ into the bus route set $R$ and re-run the restricted master problem. The optimal solution $z_{ij}$ could be used to initialize the parameter $\alpha_{ij}^r$ for the new generated bus route $r^*$. Otherwise, no beneficial bus route can be found and thus the column generation procedure terminates.
The above restricted master problem and pricing sub-problem are solved iteratively in the column generation procedure until no bus routes with negative reduced cost can be generated. Note that the column generation procedure should be conducted for both of Station $s_1$ and $s_2$. By solving the pricing sub-problem, we observe that Constraint (16) is tight in most of the cases. In other words, the pricing sub-problem tends to generate bus routes passing as many stations as possible. Based on this observation, we run the column generation procedure with a set of values $[a, b]$ for the parameter $L_{max}^2$ aiming at introducing diversity in terms of the number of passing stations for the generated bus routes. The overall column generation procedure for the bus route generation is summarized by Algorithm 1.

**Algorithm 1**: Column generation procedure of bus route generation

1: **Input**: $G(N, A)$ and all parameters
2: **Output**: a set of bus route candidates $R$
3: Initialize $R = \{0\}$, $\pi \leftarrow 0$;
4: **For** $s_1$ and $s_2$
5: **For** $L_{max}^2 \in [a, b]$
6: solve the restricted master problem [RMP];
7: update dual variables $\pi$;
8: solve the pricing sub-problem [PSP];
9: **If** sub-tour exists
10: add new constraint (18);
11: **goto** Line 8;
12: **End if**
13: **If** the objective function value of [PSP] is negative
14: obtain the new bus route $r^*$;
15: $R \leftarrow R \cup \{r^*\}$;
16: **Else**
17: **break**;
18: **End if**
19: **End for**
20: **End for**

**Selection of Bus routes**

With the set of generated bus routes $R$, the bus route selection step is to find the most efficient combination of bus routes subject to certain operational constraints. One typical constraint is that the number of shuttle bus routes starting from a station should not exceed $N_{max}^1$. Otherwise, commuters may get confused if there are too many choices. We also impose an upper limit on the total number of employed bus routes $N_{max}^2$ to reflect the resource constraint of the MRT operator for the disruption response. The bus route selection is modelled as by a path-based multi-commodity flow problem. We define parameter $c_{kr}$ as the travel time of commuter group $k \in K$ travelling from origin to destination by taking bus route $r \in R$. Let parameters $\mu_{r1}$ and $\mu_{r2}$ be 1 if bus route...
$r \in R$ passes by stations $s_1$ and $s_2$, respectively, and 0 otherwise. The following decision variables are defined for the formulation of the bus route selection:

- $\delta_r \in \{0, 1\}, \forall r \in R$. 1 if bus route $r$ is employed; and 0 otherwise.
- $\lambda_{kr} \in \{0, 1\}, \forall k \in K, \forall r \in R$. 1 if commuter group $k$ takes bus route $r$; and 0 otherwise.

Then, the selection of bus routes is modelled as follows:

\[
\text{minimize} \quad \sum_{k \in K} \sum_{r \in R} q_k (c_{kr} - c_0^k) \lambda_{kr} \\
\text{subject to} \quad \sum_{r \in R} \lambda_{kr} = 1 \quad \forall k \in K \quad (20)
\]

\[
\lambda_{kr} \leq \delta_r \quad \forall k \in K, \forall r \in R \quad (21)
\]

\[
\sum_{r \in R} \mu_{r1} \delta_r \leq N^1_{max} \quad (22)
\]

\[
\sum_{r \in R} \mu_{r2} \delta_r \leq N^2_{max} \quad (23)
\]

\[
\sum_{r \in R} \delta_r \leq N^2_{max} \quad (24)
\]

\[
\delta_0 = 1 \quad (25)
\]

\[
\delta_r \in \{0, 1\} \quad \forall r \in R \quad (26)
\]

\[
\lambda_{kr} \in \{0, 1\} \quad \forall k \in K, \forall r \in R \quad (27)
\]

Objective function (19) minimizes the travel delay over all commuter groups. For each commuter group, one bus service route should be selected as ensured by Constraints (20). Constraints (21) guarantee that only deployed bus service routes could be used by commuters. The upper limit restriction on the employed bus service routes passing by station $s_1$ and $s_2$ are imposed by Constraint (22)-(23). The overall resource capacity of the disruption planning is defined by Constraint (24). Constraint (25) ensures that the standard response bus service must be introduced. The domains of the decision variables $\delta_r$ and $\lambda_{kr}$ are defined by Constraints (26) and (27).

**Determination of bus deployment**

With the selected bus routes, the final step is to determine the number of buses that need to be deployed on each route. By solving the above bus route selection formulation, we are able to get the ridership on each section of the selected bus routes. Let $Q^i_r$ be the total number of commuters using section $i$ of bus route $r$. Therefore, the number of buses that should be deployed on bus route $r$ is:

\[
N_r = \max \{ Q^i_r \} / N_0 \quad \forall r \in R \quad (28)
\]

where $N_0$ is the capacity of a bus.
CASE STUDY

In this section we test the performance of the proposed approach for disruption response planning by two disruption cases based on a city’s MRT network.

Disruption cases

Table 1 shows the details of the two disruption cases: number of disrupted stations and links, and size of the graph defined for the integrated MRT-bus network. The non-disrupted MRT network consists of four MRT lines and three Light Rapid Transit (LRT) lines with 109 stations and 236 directed links. Case 1 is a minor disruption scenario in which only one MRT station and its connecting four links are disrupted, while Case 2 is a major disruption scenario in which 7 stations along a MRT line and their connecting links are disrupted. Note that the number of bus nodes $|N_2|$ is less than the number of MRT stations 109 since we only define bus nodes for the stations that are located near the disrupted region. As a result, we reduce the size of bus arcs significantly and thus improve the computational efficiency of solving the pricing sub-problem. The commuters’ travel demand of the two disruption cases was estimated from historical smart card data during morning peak hours.

<table>
<thead>
<tr>
<th>Disruption cases</th>
<th>Disruption scale</th>
<th>Size of graph $G(N, A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of stations</td>
<td>No. of links</td>
</tr>
<tr>
<td>Case 1 (minor)</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Case 2 (major)</td>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

The parameters related with the operational restrictions of bus routes are set as follow:

- Maximum travel time of bus routes: $L_{max}^1 = 40$ minutes;
- Maximum stops of bus routes: $L_{max}^2 \in [2, 3]$.

Effectiveness of additional bus services

To assess the effectiveness of the proposed response planning approach for MRT disruption, we compare the response approach of employing additional bus services against the standard response of running only bridging service. Figure 5 shows the sensitivity analysis of average travel delay with respect to the maximum number of additional bus routes associated with each end station $s_1$ and $s_2$. Note that the average travel delay with a given number of additional bus routes $n$ could be obtained by setting parameter $N_{max}^1 = n + 1$ and solving program [SEP]. Note that the constraint related with parameter $N_{max}^2$ is not imposed in this section as we only test the sensitivity of parameter $N_{max}^1$.

In Figure 5, the average travel delay with 0 additional bus route corresponds to the response performance by running standard bridging service. It is obvious that introduction of additional bus services yields significant decrease of average travel delay: it decreases by 28.7% from 18.4 to 13.1 minutes for the minor disruption case, and decreases by 49.1% from 20.7 to 10.6 minutes for the major disruption case. Another observation is that the reduction of travel delay is contributed by the employment of first few additional bus services while introducing more only generates
FIGURE 5 Sensitivity analysis of average travel delay with respect to the number of additional bus routes

In the following case study, we set the parameter $N_{max}^1 = 4$ and $N_{max}^2 = 6$ since the contribution from more bus routes is marginal. Performance of the solution approach

Table 2 shows the computational results of the solution approach developed for the disruption response planning problem. As can be seen, both test cases can be solved very efficiently (less than 1 minute). The column generation procedure successfully identifies 50 and 70 efficient bus service routes for Case 1 and Case 2, respectively. Regarding the distribution of computational efforts over each step, the parameter initialization step for the bus route selection takes up most of the computational time as we solve a series of shortest path models for setting the parameter $c_{kr}$. Another observation is that the column generation for finding efficient bus routes is sensitive to the size of set of bus node $N_2$ and set of bus arcs $A_2$. This confirms the necessity of defining bus nodes for MRT stations that are only in the disrupted region instead of for all MRT stations. The bus route selection step costs minimum computational time thanks to the developed path-based multi-commodity flow formulation.

Optimal bus service routes

The optimal bus service routes of the two test cases obtained from the proposed response planning approach are shown in Figures 6 and 7, respectively. For Case 1, the selected additional bus services include an express service linking station A1 and A3/C5, Route 2 connecting A1 with two
TABLE 2 Computational results of the solution approach.

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (minor)</th>
<th>Case 2 (major)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Running time (sec):</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>column generation for generating bus routes</td>
<td>1.8</td>
<td>7.2</td>
</tr>
<tr>
<td>parameter initialization for route selection</td>
<td>18.2</td>
<td>27.3</td>
</tr>
<tr>
<td>bus route selection</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>total</td>
<td>20.1</td>
<td>34.7</td>
</tr>
<tr>
<td><strong>Number of generated bus routes:</strong></td>
<td>50</td>
<td>70</td>
</tr>
</tbody>
</table>

most favorite stations A5/B5 and A4 (about 11% and 17% of all commuters are heading towards the two stations), and Route 3 providing a shortcut between Line A and B. For Case 2, the three selected bus routes are all providing inter-line services: Route 1 connecting Line B with A and C, Route 2 connecting Line B with C and D, and Route 3 linking Line B with A. As can be seen, two of the three additional bus routes in Case 1 and all three ones in Case 2 belong to non-intuitive bus routes as shown in Figure 4. This demonstrates the advantage of the developed column generation approach in finding effective yet non-intuitive bus routes.

![Figure 6 Optimal bus service routes of Case 1](image)

8 Distribution of travel delay
9 In addition to the analysis of average travel delay, we further investigate the distribution of travel delay over all the commuters, and compare the distribution under standard response and our proposed response approach. Figure 8 shows the cumulative distribution of travel delay with and without direct bus services for the two test cases.
10 As can be seen from Figure 8, the introduction of additional 3 bus routes improves the distribution of travel delay significantly. For Case 1, the travel delay of all commuters are less...
FIGURE 7  Optimal bus service routes of Case 2

FIGURE 8  Cumulative distribution of travel delay with and without additional bus services
than 20 minutes under optimized response condition while about 65% commuters encounter more than 20 minutes under the standard response condition. Similarly, for Case 2 about 20% of the commuters have less than 15 minutes travel delay under the standard response condition while the additional bus services improve the percentile to about 98%.

CONCLUSION

This paper studied and proposed a procedure to manage the disruption response planning problem in an urban MRT network. The naive standard response of running bus bridging service along the disrupted train services, while necessary, is shown to be insufficient and can be improved upon. Furthermore, the proposed approach is demand-responsive: depending on the time of day and the travel requirements of the commuters, different bus services can be deployed so as to address the challenge more effectively. The proposed approach has two important mechanisms: (1) a column generation procedure to dynamically generate demand-responsive bus routes; and (2) a path-based multi-commodity network flow formulation to identify the most effective combination of these candidate bus routes. The method is applied to two disruption scenarios using real-world data; the corresponding results confirmed the practicality of the proposed approach: (1) the procedure can be carried out efficiently, (2) introducing bus routes to the naive bridging services can easily yield significant improvement on commuters’ travel delay, (3) the distribution of commuters’ travel delay is improved considerably with an optimized response, and (4) realistic operating constraints can be handled in the planning process.

REFERENCES


