Comparison of Pavement Network Management Tools based on Linear and Non-Linear Optimization Methods

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ABSTRACT

Transportation officials at the state and local governments must make the best use of available budget to maintain the existing street network under the current budget environment. Mathematical optimization models can help to identify the optimal maintenance and rehabilitation strategy. This paper compares a linear versus a non-linear optimization model for pavement network management. Both models use the Markov transitional probabilities to represent pavement network deterioration, and both have been implemented as spreadsheet tools for use by municipalities. These tools allow users to estimate the minimum budget required to maintain a pavement network to achieve a desired network condition. For a given budget, the tools can be used to determine the budget allocation required to achieve the best network condition. The algorithms for both models are presented, and the results produced from the two models are compared.
INTRODUCTION

How to best utilize available funds to maintain and improve street network condition is a challenging task facing many local transportation officials. The City of Toledo has experienced deteriorating pavement network condition in recent years, particularly in its residential street network, due to delayed maintenance and reductions in its capital improvement budget. A tool aimed at identifying and optimizing the maintenance and rehabilitation strategy to improve the city’s pavement condition has been developed. This tool uses the Markov transition probability concept \((J)\) to represent pavement network deterioration, and uses either linear or non-linear programming \((J)\) to model the optimization problems of: (1) finding the minimum budget required to achieve the desired level of pavement network condition, or (2) deciding the best budget allocation between maintenance and rehabilitation to achieve the highest possible network condition for a given budget.

The tool is implemented in Microsoft Excel and uses the Excel’s optimization function, Solver, to solve the optimization problem. With Excel Solver, an optimal (maximum or minimum) value for a formula in one cell — called the objective cell — can be determined, subject to constraints, or limits, on the values of other formula cells on a worksheet. The Solver works with a group of cells, called decision variables, to participate in computing the formulas in the objective and constraint cells. The Solver adjusts the values in the decision variable cells to satisfy the limits on constraint cells and produce the optimal value for the objective cell.”\(^{(2)}\)

The Excel Solver uses the Simplex Method to solve linear optimization problems. For smooth non-linear optimization problems, it uses the Generalized Reduced Gradient (GRG) method, and for non-smooth nonlinear optimization problems, it uses the Evolutionary method \(^{(3)}\). By integrating the Markov transition probability model with the Solver’s linear and non-linear optimization solutions, a linear and a non-linear optimization models were formulated in the spreadsheet.

In this study, the residential street pavement network condition data from the City of Toledo were analyzed and optimized by both the linear and nonlinear models, and the resulting treatment polices were compared. Both the GRG and Evolutionary methods were used to solve the nonlinear model. The same results were produced, but the Evolutionary method took significantly longer time to solve the problems.

LITERATURE REVIEW

Linear programming and non-linear programming are the two main types of algorithms utilized by researchers in developing pavement management optimization models. In linear programming models, a key assumption is that all functions including objective and constraint function are linear, while in non-linear programming, this assumption does not hold \(^{(1)}\).

In previously proposed optimization models, Golabi et al. \(^{(4)}\), and Chen et al. \(^{(5)}\), utilized linear programming, Abaza and Ashur \(^{(6)}\) developed their model based on nonlinear programming. Pavement condition prediction model is an important component of a pavement optimization model. There are two main types of prediction models: deterministic models and probabilistic models. According to Butt et al. \(^{(7)}\), the pavement deterioration rates are often “uncertain”. Therefore, the probabilistic model based on the Markov process is the most frequently used approach \(^{(4, 5, 6)}\).
The linear and non-linear models have the same objective, which is to minimize the total repair cost of the pavement network to achieve a desired future condition level. They also shared common assumptions and constraints, although expressed in different formats. The total mileage of the studied pavement network is assumed to remain constant. This assumption is reflected in the Sum-to-One constraints. The pavement network is divided into five pavement condition states (1, Excellent; 2, Good; 3, Fair; 4, Poor; 5, Very Poor). Pavements in each condition state may be treated by one or more of the following five repair treatments: 0), Do Nothing; 1), Crack Sealing; 2), Thin Overlay; 3), Resurfacing; 4), Reconstruction. The non-linear model only allows one single treatment policy for the whole study period in terms of the percentage of the network in each condition state receiving treatment, while the linear model allows a different treatment policy for each different analysis year. Table 1 contains the treatments options eligible for each condition class defined in this study.

### TABLE 1 Allowable Treatments

<table>
<thead>
<tr>
<th>Condition</th>
<th>Do Nothing</th>
<th>Crack Sealing</th>
<th>Thin Overlay</th>
<th>Resurfacing</th>
<th>Reconstruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excellent</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Good</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td></td>
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<tr>
<td>Fair</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td>Yes</td>
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<td>Poor</td>
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<tr>
<td>Very Poor</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The state transition constraint is the key component in both models. It integrates the network condition deterioration trend to the underlying optimization problem, and determines the pavement condition for the following year.

The non-negativity constraints ensure that all variables in the optimization model are non-negative. The target condition constraints are used to specify the desired proportion of pavement in each condition state. For example, pavements in Poor and Very Poor conditions are considered as deficient, so certain limits, say, no more than 10% and 5% respectively, may be set as the target condition constraints.

If the desired target condition of the pavement network is better than the existing condition, both the linear and non-linear models tend to demand a large amount of treatments to achieve the desired condition level in the beginning of the analysis period. Therefore, the initial several years would require very high, sometimes unrealistic, budgets. To prevent this, the budget constraints can be used to ensure that the required budgets recommended by the model will not exceed the maximum possible budget each year, thus providing a more balanced treatments distribution over the entire study time period, and smooth out the initial high budget demands.
The linear model is set up in such a way that each analysis year may have a different treatment policy, which is defined as the percentages of pavement network receiving various treatments, including the Do Nothing treatment.

In the objective function equation (1), \( Y_{ik'k} \) is the decision variable representing the proportion of pavement in condition state \( i \) with last treatment \( k' \) receiving recommended repair treatment \( k \) in year \( t \).

The state transition constraints equation (2) is applied in the model starting at the second analysis year, the proportion of pavement in condition state \( j \) with last treatment \( k' \) in year \( t \) is derived from two parts of pavements in different condition states in year \( t-1 \): one part with last treatment \( k' \) received no new treatment and the other part received new treatment \( k' \).

Equation (3) is the non-negative constraints. Equation (4) is the sum-to-one constraint that reflects the assumption that the total network size remains constant.

The target condition constraints equation (5) ensures the proportion of pavement in certain condition states is in a defined range for the final year.

The budget constraints equation (6) is used to ensure that the required budgets recommended by the optimized solution do not exceed the maximum available budget each year.

Based on the above discussion, the linear model for pavement maintenance and rehabilitation optimization is formulated as follows:

Minimize

\[
\sum_{t=1}^{T} \sum_{k'=1}^{K} \sum_{i=1}^{I} \sum_{k=0}^{K} Y_{ik'k} \cdot C_{ik'k} \tag{1}
\]

Subject to

State transition constraints:

\[
\sum_{k=0}^{K} Y_{ik'k} = \sum_{i=1}^{I} \sum_{k=1}^{K} Y_{(i-1)kk'} \cdot P_{k'ij} + \sum_{i=1}^{I} \sum_{k'=0}^{K} Y_{(i-1)kk'} \cdot DN_{k'ij}
\]

for all \( t = 2, \ldots, T; k' = 1, \ldots, K; j = 1, \ldots, I \);

Non-negativity constraints:

\[
Y_{ik'k} \geq 0 \quad \text{for all } t = 1, \ldots, T; k' = 1, \ldots, K; i = 1, \ldots, I; k = 0, \ldots, K \tag{3}
\]

Sum-to-one constraints:

\[
\sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{k=0}^{K} Y_{ik'k} = 1 \quad \text{for all } t = 1, \ldots, T \tag{4}
\]

Target condition constraints:

\[
S_{ij} \leq \varepsilon_{ij} \quad \text{for selected } j \tag{5}
\]

Budget constraints:
\[
\sum_{t=1}^{T} \sum_{k'=1}^{K} \sum_{i=1}^{S} \sum_{k=0}^{K} Y_{ik'k} \cdot C_{ik'k} \cdot L \leq B_t \quad \text{for all } t = 1, \ldots, T
\] (6)

Where
\[Y_{ik'k} \text{ = proportion of pavement in state } i \text{ with last treatment } k' \text{ receiving new treatment } k \text{ in year } t.\]
\[C_{ik'k} \text{ = unit cost of applying treatment } k \text{ in year } t \text{ to pavement in state } i \text{ with last treatment } k'.\]
\[P_{kj} = \text{probability that pavement receiving new treatment } k \text{ transit from state } i \text{ to state } j.\]
\[DN_{k'j} = \text{probability that pavement with last treatment } k' \text{ receiving no new treatment (Do Nothing) moves from state } i \text{ to state } j.\]
\[S_{ij} = \text{Proportion of pavement in state } j \text{ at year at final year } T\]
\[\varepsilon_{ij} = \text{upper limit of proportion of pavement in condition } i \text{ in year } t.\]
\[L = \text{total length of entire pavement network}\]
\[B_t = \text{maximum available budget in year } t.\]
\[T = \text{number of analysis years}\]
\[K = \text{number of repair treatment types}\]

**NON-LINEAR MODEL ALGORITHM**

In the non-linear model, all years in the analysis period have the same treatment policy. In other words, the same percentages of the pavement network will receive a particular treatment every year. This offers the maintenance agency a simple and consistent treatments policy for implementation. In the objective function equation (7), \(X_j\) is the proportion of pavement in state \(j\) receiving the yearly treatments.

The state transition constraints equation (8) is applied in the model starting at the second analysis year, the proportion of pavement in condition state \(j\) in year \(t\) is derived from two parts of pavements in different condition states (\(i=1,2,\ldots,5\)) in year \(t-1\): one part received no treatment and the other part received the yearly treatment improvement.

Equation (9) is the non-negativity constraints. Equation (10) represents the sum-to-one constraints, which is used to ensure the overall network size remains constant, by adding the proportion of the pavement received no treatment and the proportion received a new treatment in all condition states. Equation (11) is the target condition constraints. Equation (12) is the budget constraints.

Based on the above discussion, the non-linear programming model for pavement maintenance and rehabilitation optimization is formulated as follows:

Minimize
\[
\sum_{t=1}^{T} \sum_{j=1}^{5} S_{ij} X_j L C_j
\] (7)
Subject to

State transition constraints:
\[ S_{t,j} = \sum_{i=1}^{5} S_{t-1,i} \left\{ (1 - X_i)DN_{ij} + X_iP_{ij} \right\} \] for all \( t = 2, \ldots, T; j = 1, 2, \ldots, 5; \) (8)

Non-negativity constraints:
\[ X_i \geq 0 \quad \text{for all } i = 1, \ldots, 5 \] (9)

Sum to one constraints:
\[ \sum_{k=0}^{4} X_{ik} = 1 \quad \text{for all } i = 1, \ldots, 5 \] (10)

Target condition constraints:
\[ S_{T,j} \leq \varepsilon_{T,j} \quad \text{for selected } j \] (11)

Budget constraints:
\[ \sum_{j=1}^{5} S_{t,j}L C_j \leq B_t \quad \text{for } t = 1, \ldots, T \] (12)

Where
\[ S_{t,j} = \text{Proportion of pavement in state } j \text{ at year } t \]
\[ X_i = \text{Proportion of pavement in state } i \text{ receiving treatment} \]
\[ T = \text{Number of analysis years} \]
\[ L = \text{Total length of entire pavement network} \]
\[ C_j = \text{Unit cost of applying treatment to pavement in state } j \]
\[ DN_{ij} = \text{Probability that pavement receiving no treatment (Do Nothing) moves from state } i \text{ to state } j \]
\[ P_{ij} = \text{Probability that pavement receiving new treatment transit from state } i \text{ to state } j \]
\[ \varepsilon_{T,j} = \text{Upper limit of proportion of pavement in condition } j \text{ in final year } T \]
\[ B_t = \text{Maximum available budget in year } t. \]

LINEAR AND NON-LINEAR OPTIMIZATION COMPARISIONS

Two case studies are conducted to compare the minimum budget required by the two models to manage a pavement network. The City of Toledo residential street network is used as an example.
Case I

Model Input and Constraints

The required input includes: (1) existing network condition distribution, (2) analysis period in years, (3) target network condition, and (4) unit cost of different repair treatments. The pavement deterioration trend is an optional input as a default trend is provided. Additional constraints such as annual budget limits can be added.

The residential street network at the City of Toledo has a total of 744.98 lane miles. The network condition can be described by the percentages of network in the following five condition states: Excellent, Good, Fair, Poor, and Very Poor. Table 2 shows the estimated residential street network condition at 2013. The analysis period is 10 years, starting from year 2013.

The Target network condition is the desired network condition, which can be specified by the user in terms of the percentages of network in each of the condition states. In this case study, the Target is to limit the amount of very poor pavements to its current level of 18%.

The unit costs per lane mile for each repair treatment option used in the analysis are: Crack Sealing: $2,500; Thin Overlay: $100,000; Resurfacing: $200,000; and Reconstruction: $600,000.

Table 3 shows the Markov transitional probability matrix which represents how a typical pavement deteriorates naturally without any repair. In Figure 1, the bottom curve that deteriorates from a score of 95 to 55 in 35 years illustrates this deterioration trend.

Table 4 shows the assumed probability that a pavement will be in various condition states after different repair treatments. In Figure 1, the corresponding effectiveness of various repair treatment options are shown.

<table>
<thead>
<tr>
<th>TABLE 2 Projected 2013 Pavement Condition</th>
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<tbody>
<tr>
<td>Lane-miles</td>
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<tr>
<td>---</td>
</tr>
<tr>
<td>Excellent</td>
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<tr>
<td>Percentage</td>
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</table>

<table>
<thead>
<tr>
<th>TABLE 3 Markov Transition Matrix (Do Nothing)</th>
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</thead>
<tbody>
<tr>
<td>Excellent</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Excellent</td>
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<tr>
<td>Good</td>
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<tr>
<td>Fair</td>
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<tr>
<td>Poor</td>
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<tr>
<td>Very Poor</td>
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</tbody>
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<table>
<thead>
<tr>
<th>TABLE 4 Post Repair Condition Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excellent</td>
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<td>---</td>
</tr>
<tr>
<td>Crack Sealing</td>
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<tr>
<td>Thin Overlay</td>
</tr>
<tr>
<td>Resurfacing</td>
</tr>
<tr>
<td>Reconstruction</td>
</tr>
</tbody>
</table>
FIGURE 1 Deterioration trend.

The objective is to find the minimum budget required to limit the percentage of pavement network in ‘very poor’ condition state to 18%, under two scenarios: first, without annual budget limit, and second, with a maximum annual budget of $15 million.

Model Outputs:

With the aforementioned inputs and constraints, the linear and non-linear model each generated one set of optimized treatment policy and resulted pavement condition without budget constraint. Figure 2 (a) shows budget comparison between linear and non-linear methods. Figure 2 (b) and Figure 2 (c) show the similar pavement conditions projected by the two models respectively, as results of similar treatment policy.

It can be seen that the two models produced nearly identical results under no annual budget limit scenario. Both require a total of about $75 million for the 10 year-period, with a significant portion, over $20 million needed in the initial year.

Since such a heavily front-loaded budget requirement is not likely practical, a maximum annual budget limit of $15 million is added as a constraint.
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With annual budget constraint of $15 million, to limit the percentage of very poor pavement to 18%, the two models project different budget requirements. The linear model results show a total of about $79 million is required for the 10 year-period, or on average, $7.9 million per year. The solution of the non-linear model requires a total of about $108 million for the 10 year-period, or $10.8 million per year. That is 37% more than linear method. Figure 3 (a) shows the budget comparison between the two. Figure 3 (b) and Figure 3 (c) show the pavement conditions projected by the linear and non-linear models respectively.

Results from both models again show that more budgets are required in the initial year(s), because the projected network conditions in later years are based on the network condition in initial year through the Markov transition matrix. The total cost to maintain the network for the entire analysis period would be lower when the network is in better condition to begin with. Therefore, the solutions from the optimization models recommend spending as much budget as possible in the beginning of the analysis period to bring the network to a very good condition level, and save money in later years’ maintenance and repair costs. This lowers the overall cost in the long run, but the initially high cost and uneven budget requirements may not be practical in most government agencies.

The results of the linear and non-linear models also differ in one important aspect. That is, the linear model produces a different treatment strategy for each year during the analysis period, while the non-linear model generates one treatment strategy for the entire analysis period. A treatment strategy is expressed as the proportions of pavements in various condition states to receive certain treatment. For example, assuming pavements in Very Poor condition requires reconstruction, not all Very Poor pavements can be reconstructed due to the high cost may exceed the available budget. Instead, only a portion of the Very Poor pavements may be reconstructed. Because the linear model can vary treatment strategy from one year to the next, it is more flexible and efficient, and hence results in significant less overall budget. The drawback of this flexibility is more significant fluctuation of the required budget from year to year.
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FIGURE 3 Case I comparison results with budget constraint: (a) Case I budget comparison with budget constraint, (b) Case I linear model projected pavement conditions with budget constraint, (c) Case I non-linear model projected pavement conditions with budget constraint.

Case II

Model inputs and constraints:

In this case, the objective is to find the minimum budget required to reduce the percentage of “very poor” pavements in the network from 18% to 10% over the 10 year time period, under two scenarios: first, without annual budget limit, and second, with a maximum annual budget of $20 million. All the other inputs and constraints remain the same as in Case I.

Model Outputs:

Figure 4 (a) shows the required budget comparison between the linear and non-linear models. Even though the required budgets produced by the linear model vary far more significantly during the analysis period, both models require a total of about $111 million for the 10 year-period, or $11.1 million per year, on average. Figure 4 (b) and Figure 4 (c) show the network pavement conditions projected by the two models are very similar.
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(a)

(b)
In the second scenario, a maximum annual budget limit of $20 million is added as a constraint. The resulting linear model projects a total of about $112 million for the 10 year-period, or $11.2 million per year, which is only slightly higher than no annual budget limit scenario. Figure 5 (a) shows the linear method budget requirement trend under this scenario. Figure 5 (b) shows the pavement conditions projected by the linear model.

The non-linear model cannot find a solution under this scenario, because the non-linear model lacks the flexibility of the linear model in varying the proportion of network receiving different treatment each year, i.e. non-linear model offers only one set of treatment policy for all the analysis year in terms of the percentage of the network in each condition state receiving treatment.
FIGURE 5 Case II results with budget constraint: (a) Case II linear model budget trend with budget constraint, (b) Case II linear model projected pavement conditions with budget constraint.
CONCLUSIONS

The spreadsheet optimization tool described in this paper can be used by local transportation officials to estimate the budget requirement to maintain or improve their pavement networks. From the analysis and comparison of the two Cases with and without annual budget constraint, it can be seen that without annual budget constraint, the two models generated nearly identical results: almost the same total budget requirement, similar treatment plans and projected pavement conditions. However, when the annual budget constraint is added, the linear model produces a lower required budget than the non-linear model. If the annual budget constraint added is somewhat stringent, the Solver may not be able to find a solution for the non-linear model. This is due to the non-linear model offers only one set of treatment policy for all the analysis year in terms of the percentage of the network in each condition state receiving treatment, while the linear model has the flexibility in varying the proportion of network receiving different treatment each year. Based on the case study results, the non-linear model appears to produce a more conservative solution and requiring more budgets on average. However, the non-linear model produces one consistent repair treatment policy which may be easier for agencies to adopt.
REFERENCES


