Optimization of transit operation strategies: A Case Study of Guangzhou, China

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Abstract

This paper aims to optimize transit operation strategies including fare structure and service frequency to obtain demand-supply equilibrium in transit systems such as bus, BRT and metro. Transit fare structure and service level have a significant impact on passenger mode choice and system welfare. Considering these impacts in objective function, we propose an optimization model with constraints on fare control, capacity, budget and flow to meet both the agencies’ and users’ expectation to transit services. The penalty function method is adopted to simplify the optimization model into a general programming model with linear constraints. The Genetic Algorithm (GA) and the Simulated Annealing algorithm (SA) are used to obtain near-optimum solutions. Finally, the optimal transit operation strategies are applied to a real-scale network of Guangzhou to test the model and the suggested algorithms.

Keywords: transit; operation strategy; optimization
1 Introduction

Past research on transit operation strategies often focuses on the pricing of one transit mode and neglects the interactions among different transit modes through their fare structures. In a macroscopic view, if the demand for transit is high enough, each transit mode in a network will carry a large number of passengers, sometimes up to the capacity, which motivates the transit operators to optimize transit operation strategies that affect the transit demand assignment. On the other hand, if the demand for transit is low, some transit mode operators will have to merge with others or close their service. Therefore, it is critical to optimize transit operation strategies for multiple modes to obtain demand-supply equilibrium under different levels of demand.

Classical models assume that competition occurs either in the price dimension (Bertrand competition) or in the service dimension (Cournot competition). This paper aims to optimize the transit operation strategies on demand distribution in transit competition markets to improve the level of service. The transit operation strategies include approaches on both transit fare structure and service frequency where transit fare structure can be defined as transit fares and correlations across multiple transit modes. The conventional fare structure indicates differential fare in one mode. In this paper, the fare structure used is the fares of many transit modes parallel each other, which presents the fare ratio relationship among them.

Previous studies on transit fare structures mainly focused on the equity issues (Robert Cervero, 1990). In this paper, we concentrate on the relationship among different transit modes in terms of fare structure in a systematic perspective. Several decades of research on transit pricing has provided clear insights into pricing methods, effects on revenue, demand and equity. Seminal working on transit pricing has begun with Mohring (1972) and Turvey and Mohring (1975). They improved the former models that contributed to provide both theoretical and practical insights into the problem of designing optimal fares. During 1970s and early 1980s, the U.S. Urban Mass Transportation Administration (UMTA) established a Pricing Division within its Office of Service and Methods Demonstration. This institution held a series of conferences, workshops, and research projects on transit pricing during this period. After that, the main issue of the transit pricing research gradually turned to its influence on demand, operation, and equity (Robert Cervero, 1990).

The effect of varying transit fare structure on its demand in a multimodal transit system is obvious when competition or cooperation exist between modes (Konstantina Gkritza etc., 2011). In most cases, passengers are likely to be less sensitive to the changes in fare level than those in travel time. For instance, Robert Cervero (1990) showed that passengers are approximately twice as sensitive to the changes in travel time as they are to the changes in fares. We note that the effect from transit fare on transit demand can be represented by elasticity. Fare elasticity and the effect from changes in fare on demand have attracted significant attention in the literature, including Goodwin (1992), Dargay and Hanly (2002), Hensher (2008), and Neil Paulley et al. (2006). The values of fare elasticity with different periods including short run, medium run, and long run were studied by Gilbert and Jalilian (1991). Therefore, the consideration of fare elasticity is essential in our paper.

This paper concentrates on the effects of transit operation strategies on demand side to obtain supply-demand equilibrium. Little research has studied the transit fare structure defined in this paper. However, the conventional fare structures, i.e., differential fares are formulated according to the length or time period of trip instead of flat fare, have been mostly focused on equity issues (Robert Cervero, 1990). Thus, the conventional fare structures need to improve transit equity, particularly for the groups of low-income or physical disabilities. On the other hand, the transit fare structure is an effective way to resolve the transit demand assignment problems since mode choice is easily affected by the changes in fare. Proost and Dender (2008) used a numerical model of the urban transportation sector to calculate the optimal transport fare structure and its effects on the transport equilibrium and welfare.

This paper proposes programing models to find transit supply-demand equilibrium by optimizing operation strategies including fare structure and service frequency. We define an objective function that composed of direct and indirect cost of operators and transit users to evaluate the performance of optimal transit operation strategies. Furthermore, three levels of transit demand are considered in numerical example and discussion, and the Genetic Algorithm (GA) and Simulated Annealing algorithm (SA) will be applied to validate the results obtained from a real-scale network of Guang Zhou.

The remainder of this paper is organized as follows: Section 2 presents a literature review of
optimization problems of transit operation strategies; Section 3 proposes the optimization model; the
solution algorithms are discussed in Section 4; numerical results on a real-scale network are reported in
Section 5; Section 6 concludes.

2 Optimization Model
As mentioned above, the changes of transit relative fares and service frequency will lead to demand
redistribution, namely the optimal transit operation strategies may shift passengers on crowded modes to
other transit modes with surplus carrying capacity.

In this paper, multi-modal and one-period network models will be considered in optimization model to
deal with the problem about transit operation strategies. The multi-modal is comprised of bus, BRT and
metro; one period means that the total transit demand is a constant in calculations but reassigned on three
types according to fare structures and transit service level.

2.1 Problem Statement
In order to optimize the transit operation strategies and evaluate the influence of it on the distribution of
demand, we consider three transit systems: bus transit system; BRT transit system; and metro transit
system.

Assuming that the competition relationships exist in all of the transit systems, different transit
operators are independent mutually in operation. Therefore, the current transit operation strategies may
often lead to the problem of demand disequilibrium in public transport network, as can be attributed to the
loss of the action of transit fare-demand elasticity. We assume that all transit links can be run by
pedestrians who try to reach bus, BRT stops or metro stations; however, the walking time to transit stations
is ignored since we mainly concentrate on the impacts of transit operation strategies on demand distribution.
The topological configuration of bus, BRT and metro networks are deemed to be known, and hold no
redesign in a period as well. The transit lines used between one origin and a destination are parallel to
respond to the competition relationship.

In this paper, the transit fares and service frequencies are to be variables in optimization function.
Furthermore, three levels of transit demand are used to simulate the impacts of transit operation strategies
on a real-scale network.

2.2 Constraints
Constraints of optimization model of transit operation strategies are described as interaction between transit
partners. There are three partners in urban public transport system, including managers (government),
operators and passengers. Everyone plays a different role in decision-making of transit operation strategies,
which may determine transit structures and the level of service provided.

Competition has been introduced and accepted widely by many developed and developing countries
after UK bus market reform (Beesley and Glaister, 1985). During that reform, the role of government is a
regulator that enforces standards of service quality and behavior; the transit companies are responsible for
operator strategies in term of policies to pursue their objectives of maximum revenue; and the riders choose
transit modes randomly according to the fares, travel time and level of services provided by operators.
These three partners compete and cooperate in the urban transit market, and their objectives can be
captured by the following constraints.

2.2.1 Fare Constrains
In competition transit market, transit fares are regulated by manager. Transit manager is usually
government or a council authorized by government. They formulate the rules to set service level and fares
by introducing comprehensive competitive tendering of the contracts thus defined. This was known as
“competition for the market” (Gwilliam, Nash, Mackie, 1985). The constrain of transit fare is presented by:

\[ f_{i,\min} \leq f_i \leq f_{i,\max} \]

where \( f_{i,\min} \) and \( f_{i,\max} \) represent the upper and lower limit of transit fare respectively for mode \( i \).
2.2.2 Capacity and Budget Constrains

(1) Capacity constraint
There might be two types of capacity constraints. Firstly, the actual numbers of travelers in a period cannot be above the actual seat capacity supplied by the operator. Secondly, the number of seats supplied by the operator cannot exceed the maximum seat capacity given by the fleet of vehicles the operator controls in the period (Pedersen, 2003). In this paper, the second type is used for capacity constraint, which can be expressed as follow (Gallo et al., 2011):

\[ q_{i,k} \leq TCap_{i,k} \quad \forall i,k \]

where \(TCap_{i,k}\) is the capacity of transit line \(k\) of mode \(i\); \(q_{i,k}\) is the flow of transit line \(k\) of mode \(i\).

(2) Budget constraint
The maximum available number of transit fleet is used for budget constraint in our paper, which can be expressed as:

\[ \sum_{i,k} \text{fleeting}(rt_{i,k},F_{i,k}) \leq NT_{i,k} \]

where the function \(\text{fleeting}(\cdot)\) indicates the available number of transit mode fleet rounded up to the next integer for the transit line \(k\); \(rt_{i,k}\) is the route time of line \(k\) on mode \(i\); \(F_{i,k}\) represents transit service frequency of line \(k\) on mode \(i\); and \(NT_{i,k}\) is the maximum available number of line \(k\) with transit fleet. Therefore the budget constraint can be expressed as \(F_{i,k} \leq NT_{i,k}/(2 \cdot rt_{i,k})\).

2.2.3 Flow Constrain
Flow constrain describes the assignment of transit demand on each mode, which establishes a bridge between transit operation strategies and demand assignment by ridership mode choice. The models of transit assignment could be classified into two categories by demand: static (frequency-based) and dynamic (schedule-based) transit assignment models (Hamdouch etc., 2011). For simplicity, the static transit assignment model is used in the evaluation of the transit operation strategy. Under the assumption of an elastic mode choice model, the flow constraint can be formulated as follows:

\[ \left[ q^*_b, q^*_r, q^*_m \right] = \phi(S, q^*_b, q^*_r, q^*_m) \]

where \(q^*_b\) presents the equilibrium flow vector of bus system; \(q^*_r\) indicates the equilibrium flow vector of BRT(Bus Rapid Transit); \(q^*_m\) is the equilibrium flow vector of metro system; \(S\) is the vector of decision variables (transit fare structures, transit services), and \(\phi(\cdot)\) is the transit multimodal assignment function.

Namely, the equilibrium flows of transit mode directly depend on operation strategies and service level:

\[ q^*_b = q^*_b(f_b, F_b) \]
\[ q^*_r = q^*_r(f_r, F_r) \]
\[ q^*_m = q^*_m(f_m, F_m) \]

(1) Formulation of transit assignment
It is assumed that passengers do not have perfect knowledge on the timetable of the transit lines and transit mode would be chosen by minimizing their perceived total travel cost. Simultaneously, it is assumed that passengers are expected to board the first coming vehicle at each station if there is spare capacity. Therefore, the result of mode choice leads to a Stochastic User Equilibrium (SUE) assignment on transit network.

The logit-based model is used to formulate the flow constrain. Hence, the generalized utility function is adopted to indicate the preference on ridership mode as below:

\[ U_i = U - C_i + \xi_i, \quad i = 1,2,\cdots,n \]

where \(U_i\) is the utility on transit mode \(i\); \(U\) is a constant term representing the utility received through transit travelling; \(C_i\) is the generalized cost of choosing transit service \(i\), which can be described by transit fare and service level of the mode; \(\xi_i\) is the uncertainty in specifying utility of selecting mode \(i\), and
supposing the random \( \xi_i \) be identically and independently distributed Gumbel variables with mean zero. Hence, based on the constant transit demand, the transit assignment problem can be represented as a logit-based formula of the modal split at aggregated demand level (Anderson et al., 1992; Oppenheim, 1995).

\[
q_i = Q \exp(\theta C_i) / \sum_{i=1}^{n} \exp(\theta C_i)
\]

where \( q_i \) is the demand on mode \( i \); \( Q \) is the total demand in transit market; \( \theta \) is a positive parameter related to the standard deviation of random variables and its value can be estimated from survey data.

(2) Formulation of cost

The generalized cost of ridership \( C_i \) can be expressed as a function of fare and travel time. Simultaneously, the transit mode choice depends on these factors. In addition, the passengers frequently have no enough estimation on congestion within transit vehicles (the reasons for congestion occurring in experience of transit service). Therefore, the total generalized cost can be calculated as follow:

\[
C(f, L, V, g) = \sum_{i} f_i + \left[ \beta_{bd} \left( \sum_{a,d} L_{ba} / V_{ba} \right) + \beta_{bw} \left( \sum_{x} W_{bx} \right) \right] + \left[ \beta_{rg} \left( \sum_{e,g} L_{re} / V_{re} \right) + \beta_{rw} \left( \sum_{y} W_{ry} \right) \right] + \left[ \beta_{mw} \left( \sum_{h,i} L_{mh} / V_{mh} \right) + \beta_{mw} \left( \sum_{z} W_{mz} \right) \right]
\]

where

- \( C \) is the generalized cost of transit travelers;
- \( f_i \) is the fare of transit mode \( i \);
- \( \beta_{bd} \) is the perceived value of time spent on-board a bus service (RMB ¥/h);
- \( \beta_{bw} \) is the perceived value of time spent waiting for a bus service (RMB ¥/h);
- \( \beta_{rg} \) is the perceived value of time spent on-board a BRT service (RMB ¥/h);
- \( \beta_{rw} \) is the perceived value of time spent waiting for a BRT service (RMB ¥/h);
- \( \beta_{mw} \) is the perceived value of time spent on-board a metro service (RMB ¥/h);
- \( \beta_{mw} \) is the perceived value of time spent waiting for a metro service (RMB ¥/h);
- \( L_{ba} \) is the length of bus link \( a \) (km);
- \( V_{ba} \) is the average speed on bus link \( a \) (km/h);
- \( W_{bx} \) represents the user waiting time for a bus line at bus station \( x \) (h);
- \( L_{re} \) is the length of BRT link \( e \) (km);
- \( V_{re} \) is the average speed on BRT link \( e \) (km/h);
- \( W_{ry} \) represents the user waiting time for a BRT line at BRT station \( y \) (h);
- \( L_{mh} \) is the length of metro link \( h \) (km);
- \( V_{mh} \) is the average speed on metro link \( h \) (km/h);
- \( W_{mz} \) represents the user waiting time for a metro line at metro station \( z \) (h);

The waiting time (service interval) \( W_{ik} \) is related to the service frequency \( F_i \), so that we stick to the “average waiting time is a half the headway” rule. Assuming travelers cannot receive information about transit arrival time, then the waiting time is calculated as below:

\[
W_{ik} = \frac{1}{2} F_i \quad \text{if} \quad i=b, r, m; \quad k=x, y, z
\]

where \( F_i \) is the frequency on line \( k \) of mode \( i \).

2.3 Objective Function

In this paper, the objective function employs a weighted sum of user costs and operator costs. User costs correspond to the generalized utility, including fare and travel time (on-vehicle-time and waiting time). Additionally, the congestion within transit vehicles is also taken into account in function; operator costs depend on ticket revenue, variable costs (veh-km produced) and fixed costs (subsidy, labor cost, vehicles maintains etc.). However ticket is a cost for users but revenue for operators, which can annul each other.
Since one aim of this paper is to measure the effects on the choice of transit modes by operation strategies, we only consider ticket as a cost for users in objective function. In this paper, the subsidy to each transit mode from local government is not considered in calculation.

Hence, the objective function include the transit operator costs, \( OC(\cdot) \) and the transit user costs, \( UC(\cdot) \). Their detailed formulation is explained below.

(1) Transit operator costs are presented as:
\[
OC(S) = \sum_{i} (VC_{i} + FC_{i}),
\]
where \( VC_{i} = c_{\text{vm}} \left( \sum_{k} F_{i,k} \cdot L_{i,k} \right) \) is variable costs of transit mode \( i = h, r, m \); \( FC_{i} \) is the fixed system cost which has no influence on optimization and are hence ignored in this paper, and \( L_{i,k} \) is the length of route \( k \) of mode \( i \); \( c_{\text{vm}} \) is the cost per vehicle-km \((\text{\textdollar}/\text{v-km})\) for mode \( i \), which can be assumed as a constant.

(2) Transit user costs includes not only the generalized cost of ridership \( C_{i} \), but also the cost of the congestion within vehicles. The number of transit users on line determines the experience of travelling with the congestion within vehicles. Therefore, the congestion within vehicles is an essential factor in calculation of transit user costs besides fare and travel time, since it can lead to the loss of independence and privacy (Horowitz and Sheth, 1977). Thereby, the cost of the congestion within vehicles of transit user can be defined as passenger perceptions for degree of crowdedness in transit vehicle.
\[
BC_{i,k} = A \left( Tcap_{i,k} - q_{i,k} \right)
\]
where \( BC_{i,k} \) presents the cost of the congestion within vehicles of transit user on line \( k \) of mode \( i \); \( q_{i,k} \) indicates the number of passengers on line \( k \) of mode \( i \); \( A \) is a constant. The closer between the values of \( q \) and \( Tcap \) is, the larger the cost of the congestion within vehicles becomes. Namely, the crowded transit line will increase the cost of the congestion within vehicles. The transit user cost can be calculated by:
\[
UC(S) = \sum_{i} q_{i} C_{i} + BC_{i,k}, \quad i = h, r, m
\]
Therefore, the objective function \( Z \) can be expressed as:
\[
Z(S, q_{h}, q_{r}, q_{m}) = OC(S) + UC(S).
\]

2.4 Optimization Formulations

With two decision variables \( f \) and \( F \), the formulations of optimization of transit operation strategies can be expressed as follows:
\[
\begin{align*}
\text{min } Z(S, q_{h}^{*}, q_{r}^{*}, q_{m}^{*}) & \quad (1) \\
\text{s.t. } \quad & f_{i}^{\text{min}} \leq f_{i} \leq f_{i}^{\text{max}} & (2) \\
& F_{i,k} \leq NT_{i,k} / (2 \cdot rt_{i,k}) & (3) \\
& q_{i,k} \leq TCap_{i,k} & (4) \\
& q_{m}^{*} = q_{m}^{*}(f_{m}, F_{m}) & (5) \\
& q_{h}^{*} = q_{h}^{*}(f_{h}, F_{h}) & (6) \\
& q_{r}^{*} = q_{r}^{*}(f_{r}, F_{r}) & (7)
\end{align*}
\]

Additionally, this model is a problem of nonlinear programming with four constraints on fare control (2), budget (3), capacity (4) and flow (5)-(7). Simultaneously, route choice problem may be included in general situation, as will result in NP-hard. Hence, we can fix route to simplify the complexity of solution in advance. Moreover, the single fare factor which can partly impact on demand cannot absolutely achieve the supply-demand equilibrium. Therefore, the exact optimal solution to this problem is hard to obtain. A remedy to this issue is to use heuristic approaches to find the (near) optimal solution.
3 Solution Algorithms

The heuristic procedure, Genetic Algorithm (GA), is proposed to solve the model (1)-(8). In addition, the Simulated Annealing (SA) algorithm is developed to compare with the result from GA in terms of their efficiency and accuracy in providing the minimum transit cost solution.

3.1 Genetic Algorithm (GA)

The genetic algorithm is a meta-heuristic evolutionary technique which founded on the applications of concepts of natural selection and natural genetics (Goldberg, 1989). The main phases of the algorithm are: initialization, selection, reproduction and termination. In this paper, each solution is represented by a group transit operation strategy, imposed by a gene and penalized by a modification of objective function.

In initialization phase, the feasible solutions, transit operation strategies, is founded to generate the population. Phase 2, selection, aims to compare the chromosomes with a given probability by the fitness function, which can identify good or bad solutions. In this paper, the objective function will be used as fitness function, and the size of population is assumed as a constant. The method of penalty function is adopted to take the constraints (7)-(8) into the objective function (1), and constraints (4), (5), (6) are directly submitted into the objective function. Therefore, the optimization of transit price structure is simplified into a general linear constraint programming model. The fitness function is defined as:

$$\varphi(S,q^*,q_k,q_m) = Z(S,q_k,q_m) + N\left(\max(q_{i,\lambda} - TCap_{i,\lambda}, 0)\right)$$  \hspace{1cm} (9)

where $\varphi$ is the fitness function for selection; $N$ represents penalty factors.

Phase 4 gives the fixed number of iterations as the stop condition of GA. When the algorithm terminates, the feasible solution with low value of objective function is the best solution developed by GA.

3.2 Simulated Annealing (SA)

Simulated annealing (SA) (Kirkpatrick, Gelatt. and Vecchi, 1983) is an enhanced version of local search that has been applied widely to solve many combinatorial optimization problems. SA allows search to proceed to a neighboring state even if the moves cause the value of objective function to become worse in order to escape from the local solution. Namely, a new state (solution) is completely accepted when a move to a neighbor $S'$ in neighborhood $N(S)$ decreases the objective value or leaves it unchanged, otherwise accepted with a probability of $e^{\frac{-\Delta}{T}}$ if $\Delta \geq 0$, where $\Delta = Z(S') - Z(S)$ and $T$ is a parameter called the “temperature”, which depends on the temperature-parameter of the algorithm. Therefore, the value of $T$ varies from a relatively large value to a small value close to zero. These values are periodically reduced by a cooling schedule every $NT$ iteration, where $NT$ is a preset parameter called the epoch length.

The SA algorithm steps for optimization of transit operation strategies as follow.

**Step 1:** initialization. Give the initial transit operation strategies $s^0$, and let $s = s^0$. Set the inner iteration number $M$, the initial temperature $T$ and the final temperature $T_K$, and set $F_{count}=0$, $Rejected=0$.

With the initial solution $s^0$, calculate the $Z(s^0)$.

**Step 2:** iterative computation. While $T > T_{final}$ or $F_{count} \leq 10$, do the following loop ITER times:

1. apply 1-interchange move, randomly select neighbor $s' = s$;
2. calculate $\Delta = Z(s') - Z(s)$;
3. judgment. If $\Delta < 0$, then $s' = s$ , $F_{count}=0$ else set $Rejected=Rejected+1$;
4. temperature decreasing. If $Rejected=ITER$, then set $F_{count}=F_{count}+1$; otherwise set $Rejected=0$;
5. update the temperature $T$ according to equation (10).

**Step 3:** return $S$.

4 Numerical Analysis

Performing the test with the proposed optimization model and algorithms on a small-scale network of Guangzhou is to verify the applicability of the model, the accuracy and efficiency of the algorithms.
Simultaneously, the best solutions will be provided through comparison of the solution found by proposed algorithms and the global optimum. We assume the bus, BRT and metro vehicles travel at a constant speed separately on links. Moreover, the values of parameters in optimization formulas and algorithms, which are calibrated by transit operation data from Guangzhou Bus Company, are listed in Table 1.

### 4.1 Test Transit Network

We conduct a numerical case study on a small-scale network of Guangzhou city, China. Guangzhou urban area had 7.7 million populations and was served by three modes of public transportation (except taxi and ferry modes): bus (includes conventional bus and trolley bus), bus rapid transit (BRT) and rail transit mode (includes metro and Automated People Mover System, APM) in 2010. The development and problems existing in Guangzhou transit system are presented as following.

(1) Transit line system

There are 743 transit lines comprised of 694 bus lines, 41 BRT lines, 7 metro lines, 1 APM line in Guangzhou transit system. Besides, 4,227 stop stations including 4 comprehensive transfer stations are covering whole public transport network. More than 9,100 transit vehicles are operated to provide transportation service of bus trip. In addition, the BRT system on Zhongshan Road was granted “the 2011 Sustainable Transport Award” by the Institute for Transportation Development Policy. In this numerical analysis, three transit lines, including a bus line 261, a BRT line 2 and a metro line 5 between Tianhe overpass (origin) and Majun workroom (destination), are used for calculation and shown in Figure 1. The characters of transit system used are listed in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>GA</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCap</td>
<td>45</td>
<td>NTr</td>
</tr>
<tr>
<td>TCap</td>
<td>70</td>
<td>NTr</td>
</tr>
<tr>
<td>TCap</td>
<td>100</td>
<td>Vb</td>
</tr>
<tr>
<td>NTb</td>
<td>30</td>
<td>Vr</td>
</tr>
<tr>
<td>NT</td>
<td>10</td>
<td>Vm</td>
</tr>
</tbody>
</table>

Note: MG is the maximum number of generations; NN is the number of individuals; NV is dimension of variables; GG is generation gap.

The relatively shorter pedestrian link which has no effects on the nature of optimization of transit operation strategies but increases the complexity of calculation can be omitted.

(2) Transit fare policy

The most common fare systems of conventional bus, BRT and APM in Guangzhou are the flat fare within urban area, which are RMB ¥2 regardless of the length of trip or time period of travel. Especially, when entering the BRT system, every line and transfer can be chosen freely with unlimited conditions. The partial fare system is graduated fare carried by Guangzhou metro where the metro fare is increasing roughly on a per mile basis and transfer is free within all metro system. Therefore, the service fare of the tested metro line is RMB ¥4 according to the scenario of metro fare.

(3) Transit demand

More than 6 million passengers use the integrated services provided throughout the city on working days in Guangzhou. For the bus and BRT networks, the everyday ridership exceeds 5 million passengers. While for the metro network, the everyday ridership is more than 1 million passengers in 2011. In our numerical calculations, the three levels of transit demand (\(Q\)) between origin (Tianhe overpass) and destination (Majun workroom), are considered three levels: \(Q=1900\), \(Q=1500\), and \(Q=1000\).

(4) Problem definition

There are some problems existing in transit operation strategies of Guangzhou transit systems.
according to the survey of transit activity of Guangzhou, 2008.

1) lack of transit fare-demand elasticity;
2) lack of competitiveness between transit modes in the same direction;
3) disequilibrium in demand on transit modes.

Figure 1 Tested transit network: lines and stations

Table 2 The length of links of three kinds of transit line

<table>
<thead>
<tr>
<th>Type of Links</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus links ($L_b$, km)</td>
<td>1.2</td>
<td>1.0</td>
<td>0.6</td>
<td>1.0</td>
<td>0.4</td>
<td>0.6</td>
<td>0.4</td>
<td>0.4</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>BRT links ($L_r$, km)</td>
<td>1.5</td>
<td>1.5</td>
<td>2.3</td>
<td>2.6</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>9.4</td>
<td></td>
</tr>
<tr>
<td>Metro links ($L_m$, km)</td>
<td>1.4</td>
<td>1.8</td>
<td>1.4</td>
<td>2.0</td>
<td>2.5</td>
<td>2.0</td>
<td></td>
<td></td>
<td>11.1</td>
<td></td>
</tr>
<tr>
<td>Pedestrian links ($L_p$, km)</td>
<td>0.3</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Results and Discussion

With the demands with three levels ($Q$=1900, 1500, 1000), the operation strategies of bus, BRT and metro system, including fare structure and service frequency, are optimized, and the results are reported in Table 3-4 and Figure 2-4. In addition, the occupancy rate (rate=ridership/capacity) is defined to further identify the characteristics of distribution of transit demands on each mode in Figure 4.

By the parameter values of optimization formulas and algorithms, the problem of transit operation strategy optimization admits analyzed solutions and global optimum may be found by GA and SA in Figure 2. In these cases, however, the objective values with GA are all less than the values with SA. Therefore, the GA leads to the global optimum and the optimal operation strategies of bus, BRT and metro system gained by GA, are shown in Figure 2, a-2. Namely, the optimal solutions of transit operation strategies of GA are used in results discussion later.

As regards the optimal solutions with GA, the Table 4 gives some information on the variations of transit operation strategies comparing with initial operation strategies. When the transit demand is high ($Q$=1900), the fares of bus and BRT are both down 12% and 17.5%, and service frequencies are simultaneously both rising 13.43% and 63.6%. However, all of the fare and frequency of metro service are to decline (-40.71%) for carrying balance between transit modes. This optimal operation strategies lead to shift the demands from metro mode to others, which achieves a new demand-supply equilibrium and avoids excess passengers taking metro at one time. Clearly, the optimal occupancy rate (bus: 100%, BRT: 97.7%, metro: 96.1%) are more reasonable than the initial occupancy rate (bus: 96.9%, BRT: 71.9%, metro: 104.7%), and as can be read from figure 4, a-2. Therefore, it is to alleviate effectively the over-occupancy of metro mode, meanwhile BRT capacity is used fully under the optimal operation strategies.

For the medium demand ($Q$=1500), the trend of transit operation strategy change is in accordance with the case for high demand except the further reduction of bus fare (RMB ¥0.8, -50.5%). Also the effects of optimal transit operation strategies on demand distribution are similar to the case for high demand.

Considering the lowest demand ($Q$=1000), the bus mode is chosen by mostly transit users because of its flexibility and great coverage, but too less transit demand to supply the mass transit systems including
Table 3 Computational results for the numerical analysis

<table>
<thead>
<tr>
<th>demand</th>
<th>objective value</th>
<th>optimal fare</th>
<th>optimal service frequency</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>GA</td>
<td>SA</td>
<td>GA</td>
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<tr>
<td>Q=1900</td>
<td>Z</td>
<td>-5.26e10</td>
<td>-3.05e6</td>
</tr>
<tr>
<td></td>
<td>f_1</td>
<td>1.65</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>f_2</td>
<td>2.53</td>
<td>4.17</td>
</tr>
<tr>
<td>Q=1500</td>
<td>Z</td>
<td>-1.54e11</td>
<td>-8.99e8</td>
</tr>
<tr>
<td></td>
<td>f_1</td>
<td>1.75</td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td>f_2</td>
<td>2.50</td>
<td>3.63</td>
</tr>
<tr>
<td>Q=1000</td>
<td>Z</td>
<td>1.96e4</td>
<td>2.43e4</td>
</tr>
<tr>
<td></td>
<td>f_1</td>
<td>3.50</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>f_2</td>
<td>2.50</td>
<td>2.33</td>
</tr>
</tbody>
</table>

Table 4 The variations of transit operation strategies comparing with initial operation strategies

<table>
<thead>
<tr>
<th></th>
<th>Q=1900</th>
<th>Q=1500</th>
<th>Q=1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_1</td>
<td>-12.00%</td>
<td>-17.50%</td>
<td>-36.75%</td>
</tr>
<tr>
<td>f_2</td>
<td>-50.50%</td>
<td>-12.50%</td>
<td>-37.50%</td>
</tr>
<tr>
<td>F_b</td>
<td>-60.00%</td>
<td>75.00%</td>
<td>-37.50%</td>
</tr>
<tr>
<td>F_m</td>
<td>+13.43%</td>
<td>+63.60%</td>
<td>-40.71%</td>
</tr>
</tbody>
</table>

Figure 2 Objective values with GA and SA with different levels of demand

BRT and metro. In addition, it allows lower service fare (RMB ¥ 0.8, -60%) and high service frequency (15, +114.29%) to attract the passengers, which leads more than 30% passengers to choose bus. Moreover, the changes of fare and service frequency of metro mode are almost identical to the condition for high and medium demand. On the contrary, the service fare of BRT is forced to increase to upper limit (RMB ¥ 3.5),
and the service frequency are to simultaneously decrease (-40%) in order to cover the operation cost of BRT. This case yields the passengers to change their travel mode choice for BRT. Hence, the role of BRT mode may be lost, and transforming it into bus transit is better choice.

Figure 3 Optimal solutions with GA and SA with different levels of demand

Figure 4 Initial and optimal occupancy rate

5 Conclusions

This paper proposes mathematical models and two algorithms to solve the optimization problem of transit operation strategies under three levels of demand. The models efficiently incorporate the demand of bus, BRT, and metro transit systems to achieve demand-supply equilibrium by the optimal solutions of transit fare and service frequency. We define an objective function that considers not only the utilities of operator and transit user to evaluate the direct cost of transit operation, but also the indirect cost of users’ crowd congestion. Furthermore, the models and algorithms were tested on a real-scale network of Guangzhou. By investigating three levels of transit demand, we find that the optimal transit operation strategies can decrease the operation cost of transit systems and at the same time mitigate users’ crowd congestion during travel. In particular, decreasing the fare and increasing service frequency are the main strategies that we recommend to transit operators, and surplus capacity shall help attract more passengers under high transit demand condition. However, the fare of BRT is too high and the service frequency is too low for covering the operation cost for the lower demand level. Therefore, the BRT service may be altered into regular bus system to achieve flexibility and higher coverage, or it can be terminated directly.

The optimal transit operation strategies from our models, including fare structure and service frequency, are likely to be practically implementable because (i) they were obtained based on an existing
transit network, and (ii) the logit-based mode choice model are calibrated on real data in the same test area. The results might be social acceptable as well: the decrease in fare is beneficial to the passengers; the rising of service frequency will attract more passengers and improve ridership and profit of transit operators, leading to a win-win situation between transit operators and riders. Finally, the numerical case study illustrated the applicability of our models to real systems in terms of yielding optimal strategic planning within acceptable computing times.

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References


