AN APPLICATION-ORIENTED MODEL OF PASSENGER WAITING TIME
BASED ON BUS DEPARTURE TIME INTERVALS

by

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ABSTRACT
Developing a reliable and practical method to estimate passenger waiting time becomes a key issue in the evaluation of service quality for public transit systems. However, existing methods lack of applicability in practices due to the need of the costly data collection. This paper develops an application-oriented model to estimate the waiting time as a function of bus departure time intervals. First, distributions of passenger arrival rates for two types of bus stops are analyzed based on field data collected in Beijing. Bus stops are classified into Type A and B, depending on whether they are connected with urban rail transit systems. The results show that the lognormal distribution has the best fit for Type A bus stop, and gamma distribution provides the best fit for Type B bus stop. Second, considering the convenience to extract the data of bus departure times from existing intelligent transit systems, the relationship between passenger arrival rates and bus departure time intervals is analyzed. It is demonstrated that parameters of the passenger arrival rate distribution for both two types of stops can be expressed by the average and $CV$ (Coefficient of Variation) of bus departure time intervals in functional relationships. Then, an application-oriented waiting time model is proposed. Finally, a model validation is conducted, resulting in the $NMSE$ (Normalized Mean Square Error) of 0.0854 and 0.0126 for Type A and B stops, respectively. Thus, the proposed model is shown to provide a reliable estimation of the average passenger waiting time based on only readily available bus departure time intervals.

Key words: Average Passenger Waiting Time; Bus Departure Time Interval; Passenger Arrival Rate Distribution; $NMSE$
INTRODUCTION

An efficient public transit system with a high quality of service is a crucial determinant to attracting more passengers. For transit trips, passenger waiting time at bus stops is one of the most important performance measures to reflect the quality of the service level for public transit systems (1). Moreover, modal choice decisions are more sensitive to the amount of waiting times than the time on board (2). The waiting time can be a factor that hinders the usage of the bus transit. Thus, developing a reliable and practical estimation method of passenger waiting time is necessary, which is helpful for the transit agency to identify underlying elements for the long waiting time delay, and further implement improvement strategies and control measures.

In this paper, the public transit system in Beijing is chosen for the case study. The public transit system in Beijing has two distinguishing characteristics in comparison with that of other cities. One is that there are a great number of bus stops due to an extensive public transit network of buses and a rapidly expanding subway system. Currently, there are a total of 17 subway lines and 763 bus lines with more than 11,000 bus stops (3). Furthermore, waiting behaviors at various bus stops is different, depending on whether they are connected with urban rail transit systems. For the bus stops connected with the urban rail transit system, most of the passengers disembarks from the rail system and the waiting behaviors of them depend on the transfer time from rail to bus system. Thus, the bus stops connected with the urban rail transit system may be more likely to have passengers arrive in groups, which is different from the randomly arrival of the bus stops not connected with the urban rail transit system. Here in this research, bus stops are classified into two types. One is connected with the urban rail transit system (Type A), and the other one is not connected with the urban rail transit system (Type B). The other characteristic is that new technologies, such as the automatic vehicle locating by GPS (Global Position System) and the automatic fare collection by IC card (i.e., Smart Card), have been widely used in the public transit system of Beijing. Both GPS and IC card systems can provide transit agencies with enormous amounts of data, such as the bus arrival and departure times at each bus stop. Thus, the GPS and IC card systems in Beijing can provide on-line data, which support the transit agency to estimate passenger waiting time and further evaluate the public transit service in a timely manner. In addition, the urban bus transit system in Beijing has short headways to satisfy the high transit demand, usually less than 10 minutes intervals in peak hours. Further, the bus arrival and departure times at each stop are irregular everyday especially during peak hours, due to the traffic congestion. The passenger arrival distribution is affected more or less by the short and irregular bus departure time intervals. Accordingly, the proposed model should incorporate the impact from such short and irregular bus departure time intervals. Hence, the objective of this paper is to develop a reliable passenger waiting model, which can be applied to typical bus stops in Beijing with bus departure time intervals extracted from the GPS and IC card data.

In the following sections, existing studies on passenger waiting time models are first reviewed. Then, the paper analyzes the distribution of passenger arrival rates for two types of bus stops based on field data collected in Beijing. Subsequently, the relationship between the
distribution of passenger arrival rates and bus departure time intervals is analyzed. Further, an
application-oriented waiting time model for two types of bus stops is proposed by the method
of integral calculus. Finally, this study conducts a validating analysis of the proposed model by
comparing the results from the model with the field data. Some conclusions are summarized at
the end of the paper.

LITERATURE REVIEW

Passenger waiting time models have been studied since 1970s. In traditional passenger waiting
time models from early studies, the average passenger waiting time was derived as the sum of
one half of the average headway of buses and the ratio of the headway variance to twice the
average headway, by assuming a random passenger arrival (4). Such a model indicated that the
mean waiting time of passengers decreases as the reliability of the bus service increases (5).
Similarly, the relationship between the average passenger waiting time and bus headway was
widely examined later in passenger waiting time studies.

The work of Seddon and Day (1974) was among the early studies on passenger waiting
time models (6). They obtained a function for the bus passenger waiting time by analyzing
buses in Leeds, UK (6), which was formulated in Equation (1):

\[ w = 1.79 + 0.14\mu \]  

(1)

where \( w \) is the average waiting time (minute); and \( \mu \) is the headway of buses studied
(minute).

A similar result was found by O’Flaherty and Mangan (1990) in Manchester, UK (7), as
shown in Equation (2):

\[ w = 2.34 + 0.26\mu \]  

(2)

Some studies also indicated that waiting behaviors are different depending on different
bus headways (8, 9). For the bus transit service with small headways, passengers rarely need to
consult schedules since vehicles arrive frequently. Therefore, these passengers arrive at the stop
at a random rate. In contrast, for longer headways, passengers generally learn schedules in
advance in order to reduce their waiting times. These passengers arrive at the stop near the
departure time.

Compared with previous literatures, Knoppers and Muller (1995) were the first who
proposed a precise calculation method for passenger waiting time with integral calculus (10),
as shown in Equation (3).

\[ E(w) = \int_{-\infty}^{+\infty} w(p) Pr(p) dp \]  

(3)

Equation (3) shows that the expected mean passenger waiting time \( E(w) \) can be
obtained by integrating passengers’ actual waiting time \( w(p) \) with the corresponding probability
function \( Pr(p) \), where the variable \( p \) represents the actual punctuality deviation of feeder
vehicle arrivals.

Since then, there were also some studies that developed passenger waiting time models
based on distribution curves of passenger arrival rates and the integral calculus method.

Luethi et al. (2007) proposed a model for passenger arrival rates that combine a uniform
distribution with a Johnson SB distribution (11). Normal, lognormal and triangular distributions
had all been used to calculate passenger waiting time \((12, 13)\). Guo et al. (2011) compared normal, exponential, lognormal and gamma distributions while fitting arrival rates of passengers transferring from the rail to buses. The results showed that lognormal and gamma distributions have the best fit for direct transfer and non-direct transfer passengers, respectively \((2)\).

From the above literature review, it can be found that almost all of existing studies showed that headways of buses significantly affect the average passenger waiting time \((14)\). Most of existing waiting time models were based on bus headways and the distribution of passenger arrival rates. However, in order to determine the distribution of passenger arrival rates, it is difficult and also costly to collect field data of passenger arrival times for different types of bus stops. Therefore, this research is aimed at developing an application-oriented passenger waiting time model. This model is intended to overcome shortcomings of the traditional waiting time model by avoiding manually collecting a large number of field data on passenger arrival rates. This model can also estimate passenger waiting time for different types of stops.

**DATA COLLECTION AND PREPARATION**

In order to collect valid data, a preliminary investigation for selecting survey locations is conducted and survey locations are selected on the basis of three criteria. The first criterion is that the bus stop should have an independent waiting platform serving only one bus line, which can help the surveyor identify the passenger who plans to go aboard the bus of the surveyed bus line. The second criterion is that there should be a relatively high demand at the bus stop for the surveyed bus line to ensure an adequate sample size. Finally, for Type A bus stop, the surveyed bus stop should not be far away from the urban subway station. For Type B bus stop, there should be no urban subway station connecting with the surveyed bus stop within an acceptable walking distance.

Based on the above criteria, a total of four bus stops, including two Type A bus stops and two Type B bus stops on four bus lines were selected (see Table 1). The survey was conducted on weekdays during evening peak hours from 5:00 to 7:00 p.m.. Passenger arrival times at bus stops and the bus arrival/departure time were collected. All the time data were recorded in the format of \(hh:mm:ss\).

**TABLE 1 Date Collection**

<table>
<thead>
<tr>
<th>Stop type</th>
<th>Bus stop</th>
<th>(on) Bus line</th>
<th>Survey date</th>
<th>Frequency (trips/peak hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>Xizhimen</td>
<td>Line #105</td>
<td>16(^{th}),17(^{th}),18(^{th}) April 2013</td>
<td>13,13,11</td>
</tr>
<tr>
<td></td>
<td>Dongwuyuan</td>
<td>Line #107</td>
<td>26(^{th}),27(^{th}),28(^{th}) June 2013</td>
<td>8,7,8</td>
</tr>
<tr>
<td>Type B</td>
<td>Baishiqiao</td>
<td>Line #27</td>
<td>16(^{th}),17(^{th}),18(^{th}) April 2013</td>
<td>15,13,14</td>
</tr>
<tr>
<td></td>
<td>South Liuliqiao</td>
<td>Line #349</td>
<td>26(^{th}),27(^{th}),28(^{th}) June 2013</td>
<td>6,7,6</td>
</tr>
</tbody>
</table>

Based on the collected data, passenger arrival time, passenger waiting time, and bus departure time intervals can be calculated. The relationship among these parameters is shown
as Figure 1.

**FIGURE 1** Illustration of passenger waiting time.

In Figure 1, the passenger arrival time is calculated according to the previous bus departure time. The waiting time can be calculated by subtracting the passenger arrival time from the bus departure time. The bus departure time interval is equal to the departure time difference between two consecutive buses, as shown in Equations (4) to (6):

\[ T_{arrival}(i,k) = t_{arrival}(i,k) - t_{departure}(i-1) \]  
\[ T_{waiting}(i,k) = t_{departure}(i) - t_{arrival}(i,k) \]  
\[ T_{interval}(i-1,i) = t_{departure}(i) - t_{arrival}(i) \]

where \( T_{arrival}(i,k) \) is the arrival time of \( k^{th} \) passenger waiting for the \( i^{th} \) bus; \( T_{waiting}(i,k) \) is the waiting time of \( k^{th} \) passenger waiting for the \( i^{th} \) bus; and \( T_{interval}(i-1,i) \) is the bus departure time interval between the \( (i-1)^{th} \) and \( i^{th} \) buses.

In this paper, the data including 1,422 and 1,014 valid records of passengers for Xizhimen bus stop on bus line #105 and Baishiqiao bus stop on bus line #27, which represent the Type A bus stop and Type B bus stop in Beijing respectively and meet the sample size, is used for the model development. And the rest of data is used for the validation of the proposed model. In the following analysis, the data on passenger arrival times are mainly used to determine the passenger arrival rate distributions, and the data on bus departure times are used to identify the relationship between the passenger arrival distribution and the bus departure time interval.

**METHODOLOGY**

**Determination of Passenger Arrival Rate Distributions for Different Types of Bus Stops**

In order to determine the passenger arrival rate distribution, extreme value, exponential, lognormal, gamma and normal distributions are chosen to fit the passenger arrival rate. R-Square and the Sum of Squared Errors (SSE) are used to compare the goodness-of-fit of each distribution. The distribution fitting analyses for both Type A bus stop and Type B stop are completed by using the DFITTOOL (Distribution Fitting Tool) of MATLAB.

**Passenger Arrival Rate Distribution for Type A Bus Stop**

For Xizhimen bus stop, 1,422 records of passengers of line #105 are valid. The records without a bus arrival time before the passenger arrival time or without a bus departure time
after the passenger arrival time during the period of survey are not valid and eliminated. For Line #105, the maximum bus departure time interval at Xizhimen bus stop is 1,159 seconds, while the average is 265 seconds. More than 97.8% of passengers arrived at 600 seconds or less, as shown in Figure 2.

**FIGURE 2** Probability density distribution of passenger arrival time during evening peak hours for Type A bus stop.

Figure 2 illustrates passenger arrival rates during evening peak hours (5:00 p.m.-7:00 p.m.) using extreme value, exponential, lognormal, gamma and normal distribution fits. The results for both R-Square and \( \text{SSE} \) indicate that the lognormal distribution provides the best quality of fit for Type A bus stop with a R-Square of 0.9400 and a \( \text{SSE} \) of 0.0064, which is similar to the results of arrival rate distributions of direct transfer passengers conducted by Guo et al.\(^2\). The probability density distribution of passenger arrival times for Type A bus stop can be expressed by Equation (7) as

\[
f_i(t) = \frac{1}{\sigma t \sqrt{2\pi}} \exp\left( -\frac{1}{2} \left( \frac{\ln t - \mu}{\sigma} \right)^2 \right)
\]

where \( f_i(t) \) is the probability density distribution of passenger arrival times for Type A bus stop; \( t \) is the passenger arrival time variable; and \( \mu \) and \( \sigma \) are the mean and standard deviation of the lognormal probability distribution of the passenger arrival time, in Figure 2 the value of \( \mu \) and \( \sigma \) are 5.0668 and 0.9367 respectively.

**Passenger Arrival Rate Distribution for Type B Bus Stop**

For Baishiqiao bus stop, a total of 1,014 records of passengers are valid. The maximum bus departure time interval at Baishiqiao bus stop is 718 seconds and the average is 250 seconds. More than 98.8% of passengers arrived at 600 seconds or less, as shown in Figure 3.
Figure 3 shows passenger arrival rates during evening peak hours (17:00-19:00) by using extreme value, exponential, lognormal, gamma and normal distribution fits. As shown in Figure 3, the Gamma distribution has the best fit for Type B bus stop with a $R^2$ of 0.9774 and a $SSE$ of 0.0078, which is similar to the results of arrival rate distributions of non-direct transfer passengers conducted by Guo et al.\(^2\). The probability density distribution of passenger arrival times for Type B bus stop can be written by Equation (8) as:

$$f_2(t) = \frac{1}{\beta^\alpha \Gamma(\alpha)} t^{\alpha-1} \exp\left(-\frac{t}{\beta}\right)$$

where $f_2(t)$ is the probability density distribution of the passenger arrival time for Type B bus stop; $t$ is the passenger arrival time variable; and $\alpha$ and $\beta$ are parameters of the probability distribution. The mean and standard deviation of the passenger arrival time for Type B bus stop can be calculated by $\alpha\beta$ and $\sqrt{\alpha\beta}$. In Figure 3 the value of $\alpha$ and $\beta$ are 2.4406 and 72.8073 respectively.

Relationship between Passenger Arrival Rate Distributions and Bus Departure Time Intervals

In existing waiting time models, a large number of field data on passenger arrival times need to be collected manually in order to determine the distribution of passenger arrival rates. Such data collection is difficult and also costly, especially for different bus lines at different types of bus stops. Consequently, the relationship between passenger arrival rate distributions and bus departure time intervals is analyzed because bus departure time intervals can be extracted from existing intelligent transit systems and have an impact on passenger arrival rates according to Equation (4). The relationship between passenger arrival rate distributions and bus departure time intervals might be expressed by certain analytical functions.

The bus departure time interval represents the primary characteristic for the public
transit scheduling and is also used to estimate the median passenger waiting time in transportation models (15). Results of previous studies identified that the bus departure time interval is the most important influencing factor of passenger arrival rates in microscopic distributions analysis (11). Thus, a sensitivity analysis of the bus departure time interval on the distribution parameters is conducted to examine the relationship between the passenger arrival rate distribution and the bus departure time interval. In the sensitivity analysis, two indicators, the average bus departure time interval and the coefficient of variation (CV) of bus departure time intervals at bus stops, are chosen, which reflect the service level and the service reliability (16, 17) of different bus lines. Based on the results of determining the passenger arrival distribution, parameters $\mu$, $\sigma$ and parameters $\alpha$, $\beta$ are chosen to represent characteristics of the passenger arrival rate distribution of different types of bus stops.

In analyzing the impact of different bus departure time intervals on distribution parameters, all of the surveyed data including 11 hours and 12 hours for two types of bus stops are selected. Line #107’s survey on 28th June 2013 is less than two hours, thus only an hour’s data is selected. Then an hour’s data was used as a group for the analysis, thus 11 groups of data sets for Type A bus stop and 12 groups of data sets for Type B bus stop are used. The average and $CV$ of bus departure time intervals in each data group are calculated, and parameters of the passenger arrival rate distribution in each data group are calculated by the DFITTOOL of MATLAB.

**Relationship Analysis for Type A Bus Stop**

Because parameters $\mu$ and $\sigma$ of the lognormal distribution are the mean and standard deviation of the lognormal probability distribution of passenger arrival rates, which are determined by the mean and $CV$ of the passenger arrival time, we assume that parameters $\mu$ and $\sigma$ are also influenced by the average and $CV$ of bus departure time intervals. The regression analysis between the distribution parameters $\mu$, $\sigma$ and the average and $CV$ of bus departure time intervals is conducted as shown in Figures 4 and 5, respectively.

Figure 4 shows that the relationship between the distribution parameters $\mu$ and the average of bus departure time intervals can be represented by a quadratic regression function with a R-Square of 0.9329 as shown in Equation (9)

$$
\mu = -0.00001\mu_b^2 + 0.0098\mu_b + 3.3246
$$

(9)

where $\mu$ is the distribution parameter of passenger arrival rates for Type A bus stop; and $\mu_b$ is the average of bus departure time intervals.
FIGURE 4 Relationship between average of bus departure time intervals and parameter $\mu$. Figure 5 shows that the underlying relationship between the distribution parameters $\sigma$ and the $CV$ of bus departure time intervals can be characterized by a linear function with a R-Square of 0.9387 as shown in Equation (10)

$$\sigma = 0.7207cv_b + 0.3519 \quad (10)$$

where $\sigma$ is the distribution parameter of passenger arrival rates for Type A bus stop; and $cv_b$ is the $CV$ of bus departure time intervals.

FIGURE 5 Relationship between $CV$ of bus departure time intervals and parameter $\sigma$.

Relationship Analysis for Type B Bus Stop

Before analyzing the impact of different bus departure time intervals on distribution parameters $\alpha$ and $\beta$, we derive mathematical relationships between the mean, standard deviation of passenger arrival times and the parameters $\alpha$ and $\beta$ according to the existing relationship in the Gamma distribution. This existing relationship is as follows.

$$\mu_p = \alpha \beta \quad (11)$$
\[ \sigma_p = \sqrt{\alpha \beta} \]  

where \( \mu_p \) is the mean of the passenger arrival time; and \( \sigma_p \) is the standard deviation of the passenger arrival time.

Then, parameters \( \alpha \) and \( \beta \) can be represented by the mean and standard deviation of the passenger arrival time as

\[ \alpha = \left( \frac{1}{cv_p} \right)^2 \]  

\[ \beta = \mu_p cv^2_p \]

where \( cv_p = \frac{\sigma_p}{\mu_p} \) is the coefficient of variation of the passenger arrival time.

Based on the relationship between parameters \( \alpha \) and \( \beta \) and the mean and \( CV \) of the passenger arrival time in Equations (13) and (14), we assume that the parameters \( \alpha \) and \( \beta \) have a similar underlying relationship with the average and \( CV \) of bus departure time intervals. The regression analysis between the distribution parameters \( \alpha \) and \( \beta \) and the average and \( CV \) of bus departure time intervals is conducted as shown in Figures 6 and 7, respectively. Factors of \( 1/cv_b \) and \( \mu_b cv^2_b \) are considered as two new indicators in the following analysis.

Figure 6 illustrates that the relationship between the distribution parameter \( \alpha \) and the indicator \( 1/cv^2_b \) can be represented by a quadratic regression function with a R-Square of 0.9582 as shown in Equation (15)

\[ \alpha = -0.0086 \left( \frac{1}{cv^2_b} \right)^2 + 0.2774 \left( \frac{1}{cv^2_b} \right) + 0.9413 \]

**FIGURE 6 Relationship between \( 1/CV^2 \) of bus departure time intervals and parameter \( \alpha \).**
Figure 7 shows that the relationship between the distribution parameter $\beta$ and the indicator index $\mu_b \cdot cv^2_b$ can be characterized by a linear function with a R-Square of 0.9739 as shown in Equation (16)

$$\beta = 0.538 \mu_b \cdot cv^2_b + 99.047$$

(16)

where $\alpha$ and $\beta$ are the parameters of the passenger arrival rate distribution for Type B bus stop; $cv_b = \sigma_b / \mu_b$ is the coefficient of variation of bus departure time intervals; $\mu_b$ is the average deviation of bus departure time intervals; and $\sigma_b$ is the standard deviation of bus departure time intervals.

![Graph showing the relationship between $u^* CV^2$ of bus departure time intervals and parameter $\beta$.](image)

FIGURE 7 Relationship between $u^* CV^2$ of bus departure time intervals and parameter $\beta$.

Passenger Waiting Time Model for Different Types of Bus Stops

**Passenger Waiting Time Model for Type A Bus Stop**

For a given bus departure time interval, the standard probability density function of the passenger arrival time for Type A bus stop can be expressed by Equation (17) as

$$g_h(t) = \frac{f_i(t)}{\int_0^h f_i(\tau)d\tau}$$

(17)

where $g_h(t)$ is the standard probability density function of the passenger arrival time for Type A bus stop within a bus departure time interval $h_i$; $h_i$ is the departure time interval of the $i^{th}$ bus; and $t$ is the arrival time variable for passengers varied from 0 to $h_i$.

Thus, the average waiting time for a given bus departure time interval can be derived using Equation (18) as

$$\overline{W_i}(h_i) = \int_0^{h_i} (h_i - t) g_h(t) dt$$

(18)

where $\overline{W}(h_i)$ is the average waiting time within a bus departure time interval $h_i$. 

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Finally, the average waiting time of all passengers can be obtained as shown in Equation (19).

\[
\bar{W}_1 = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\int_{0}^{h} (h_i - t) f_1(t) \, dt}{\int_{0}^{h} f_1(t) \, dt} \right)
\]

where \( \bar{W}_1 \) is the average waiting time of all passengers; and \( n \) is the number of bus departure time intervals for Type A bus stop.

**Passenger Waiting Time Model for Type B Bus Stop**

By using the same method, we can obtain the average waiting time model of all passengers for Type B bus stop as shown in Equation (20).

\[
\bar{W}_2 = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\int_{0}^{h} (h_i - t) f_2(t) \, dt}{\int_{0}^{h} f_2(t) \, dt} \right)
\]

**VALIDATION**

In order to validate the applicability of the proposed models, this section analyzes and compares results from the case study using field data and results from the proposed waiting time model. The proposed model is applied to Dongwuyuan bus stop on line #107 (Type A), and South Liuliqiao bus stop on line #349 (Type B), respectively. First, we obtain bus departure time intervals and passenger waiting times during each bus departure time interval in each surveyed period (evening peak hours on 26th, 27th, 28th June 2013). Then, the average and CV of bus departure time intervals at each period can be calculated. Thus, we can get the average passenger waiting times at Dongwuyuan stop and South Liuliqiao stop in each bus departure time interval by the proposed model, respectively, as shown in Figure 8. A total of 43 and 38 data sets for Type A and Type B bus stops are used for validation, respectively.
To evaluate the proposed model, $NMSE$ (Normalized Mean Square Error) is used in this paper, which can evaluate the average relative discrete degree between the field and estimated values \((18)\), as shown in Equation \((21)\).

\[
NMSE = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{w_{\text{real}} - w_{\text{model}}}{w_{\text{real}} w_{\text{model}}} \right)^2
\]  

where \(w_{\text{real}}\) is passenger waiting time in the field, and \(w_{\text{model}}\) is passenger waiting time.
estimated by the model, and $\bar{w}_{real}$ and $\bar{w}_{model}$ are the mean of the field and estimated passenger waiting time, respectively.

In an accurate model, the NMSE should be close to 0. Based on existing studies, NMSE $<0.5$ is an acceptable limit (19). For the calculation results in this paper, the NMSEs in three days’ evening peak hours at Dongwuyuan stop and South Liuliqiao stop are 0.0854 and 0.0126, respectively.

The average passenger waiting time in the field and that estimated by the proposed model at Dongwuyuan stop and South Liuliqiao stop are very close with relative errors of -6.79% and 7.40%, 1.88% and 6.00%, -2.11% and 1.92% in each evening peak hours, which are considered to be reasonable, as shown in Table 2.

Table 2 Field vs. Estimated Values of Average Passenger Waiting Time in Each Period for Two Bus Stops

<table>
<thead>
<tr>
<th>Bus stop</th>
<th>Time</th>
<th>Field value(s)</th>
<th>Estimated value(s)</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dongwuyuan stop</td>
<td>26th June_p.m. peak</td>
<td>230.3</td>
<td>214.7</td>
<td>-6.79%</td>
</tr>
<tr>
<td></td>
<td>27th June_p.m. peak</td>
<td>249.8</td>
<td>254.5</td>
<td>1.88%</td>
</tr>
<tr>
<td></td>
<td>28th June_p.m. peak</td>
<td>302.6</td>
<td>296.2</td>
<td>-2.11%</td>
</tr>
<tr>
<td>South Liuliqiao stop</td>
<td>26th June_p.m. peak</td>
<td>342.1</td>
<td>367.4</td>
<td>7.40%</td>
</tr>
<tr>
<td></td>
<td>27th June_p.m. peak</td>
<td>302.2</td>
<td>320.3</td>
<td>6.00%</td>
</tr>
<tr>
<td></td>
<td>28th June_p.m. peak</td>
<td>329.4</td>
<td>335.7</td>
<td>1.92%</td>
</tr>
</tbody>
</table>

From the above analysis, it can be shown that passenger waiting time model for both Type A and Type B bus stops can be used to calculate passenger waiting time with a high accuracy. Moreover, in practice, the bus departure time at each bus stop can be extracted from the GPS and IC card systems, which are installed in buses, so the bus departure time intervals are available inputs to the proposed passenger waiting time model. Therefore, the proposed passenger waiting time model in this paper is more targeted on applications, which has a good potential to be applied in estimating average passenger waiting times for different types of bus stops in Beijing.

**CONCLUSIONS**

Based on field data in Beijing, an application-oriented model of the average passenger waiting time for different types of bus stops is proposed, which is a function of bus departure time intervals. As the only input of the proposed model, bus departure time intervals can be easily extracted from the GPS and IC card data. Therefore, the proposed model has a good applicability in estimating average passenger waiting times for different types of bus stops in Beijing. Main findings in this study can be summarized as follows:

1. Distributions of passenger arrival rates for both Type A and Type B bus stops were analyzed based on field data collected in Beijing, and it was found that lognormal distribution had the best fit for Type A bus stop, and the gamma distribution provided the best fit for Type B bus stop.
2. The relationship between the passenger arrival rate distribution and the bus departure time interval was analyzed. It was shown that distribution parameters of the passenger arrival rate for both Type A and Type B bus stops could be expressed by the average and CV of bus departure time intervals in functional relationships.

3. An application-oriented model of average passenger waiting time for both Type A and Type B bus stops was proposed by the method of integral calculus. The model was validated by comparing the field value and estimated value of passenger waiting time. The validation results showed that the proposed model had a high accuracy with a relative error no more than 8%.

In future studies, the influence of the next bus arrival time information service on the passenger waiting behavior at bus stops need to be considered. The classification of bus stops can be further expanded to refine the proposed model. A study on waiting time models for suburban bus lines with longer bus departure time intervals (often more than 15 minutes) can be conducted. In addition, for the model application in other cities, a calibration and validation study of the proposed passenger waiting time model is needed.

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