UE-based Location Model of Rapid Charging Stations for EVs with Batteries that Have Different States-of-charge

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ABSTRACT

The aim of this research is to develop a location model of rapid charging stations for electric vehicles (EV) in urban areas considering the batteries’ state-of-charge (SOC) and the users’ charging and traveling behaviors. EVs are developed to prepare for the energy crisis and reduce greenhouse gas emissions. In order to help relieve range anxiety, an adequate number of EV charging stations must be constructed. In urban areas, the construction of rapid charging stations is needed because there is inadequate space for slow-charging equipment. The objective function of the model is to minimize EVs’ travel fail distance and total travel time of the entire network when the link flow is determined by user equilibrium (UE) assignment. The remaining fuel range (RFR) at the origin node is assumed to follow a probabilistic distribution in order to reflect users’ charging behavior or technical development. The results indicate that the location model described in this paper can identify locations for charging stations by using a probabilistic distribution function for the RFR. And the location model, which is developed based on UE assignment, is likely to consider the congested traffic conditions of urban areas in order to avoid locating charging stations where they could cause further traffic congestion. The proposed model can assist decision makers in developing policies that encourage the use of EVs, and it will be useful in developing an appropriate budget for implementing the plan.
INTRODUCTION

The electric vehicle (EV) is one of the most popular alternative-fuel vehicles (1). However, range anxiety has restricted the pace at which EVs have penetrated the market. The construction of an adequate number of EV charging stations can help relieve this range anxiety (2). Considering the budget constraints, choosing where such stations should be located is an important issue. Access to EV charging stations will impact the use rates of EVs, decisions concerning their use, the percentage of miles attained with electricity, the demand for petroleum, and power consumption at various times during the day (3, 4). So the problem of properly locating EV charging stations is an essential topic, and some important studies have been conducted in the past few years (5-10).

To formulate a practical model for determining the appropriate locations for EV charging stations, several variables must be considered, including the vehicle range (VR), batteries’ state-of-charge (SOC), users’ charging behavior, and travel preferences. In the early stages of EVs, the targeted consumers were people who traveled almost exclusively within the urban area (11). In existing models for locating EV charging stations in urban areas, slow-charging equipment was targeted, and the objectives of the existing studies were to optimize the total usage of electrical power, maximize profit, and minimize costs. The location of rapid charging stations in urban areas also is very important because adequate space cannot be made available to accommodate the larger numbers of slow-charging equipment that would be necessary. However, most studies are based on parking behavior, and there is a lack of research on charging on route. Rapid charging stations in urban area can help increase accessibility to charging to a greater extent than such stations could in rural areas. Therefore, current planning involves establishing charging stations first in urban areas and then expanding their availability to intercity roads (12). However, rapid charging stations, at which EV users can recharge during their trips, have not been considered in the most of the studies.

Flow-refueling location models (FRLMs) have been developed to find adequate location of gas station for vehicles that need refuel during their trip. FRLMs for alternative-fuel vehicles are extended models of flow-capturing location models (FCLMs) that were developed for convenience stores by Hodgson (13, 14). An FCLM is a maximum covering model, and it cannot handle the multiple refueling stations needed for paths longer than the VR. Vehicle range (VR) is the distance that a vehicle can travel when it is fully charged, and FRLMs can be extended by adding the vehicle-range constraint.

However, it is difficult to apply FRLMs to urban areas for two reasons. First, the travel paths used in these models were determined exogenously. In the existing models, all of the vehicles from the same origin-destination (OD) pair must be assigned to one path. This is reasonable for inter-city trips, but it would rarely occur in urban areas, because numerous alternative paths are available. In practice, drivers can detour to charge their vehicles they so desire. Some studies have considered such detours, but only from the standpoint of the probability of trip availability (15, 16). Second, the SOC level was assumed to be 0.5 because only the marginal case of alternative fuel vehicles was considered in the studies of FRLMs.
If each vehicle has a different remaining fuel range (RFR), the path used by each vehicle may be different. RFR can be calculated by multiplying VR by the SOC level of the battery. When a constant RFR is used, the number of stations that should be constructed can be underestimated in urban areas. Summarizing the literature, two variations are required to develop the location model for EV charging stations in an urban area. One is determining the travel path endogenously, and the other is assuming probabilistic RFR. In this way, the model can simulate EV users’ behaviors reasonably and extend the results by enhancing batteries’ volume or charging performances. The summary of the literature review and the contribution of this study are presented in Table 1.

TABLE 1 Literature Review and the difference of this study (5-10, 14-24)

<table>
<thead>
<tr>
<th>Previous studies</th>
<th>Objective function</th>
<th>Station</th>
<th>Spatial scope</th>
<th>RFR</th>
<th>Travel path</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuby and Lim (2005)</td>
<td>Minimum failure</td>
<td>AFV</td>
<td>INT</td>
<td>D</td>
<td>EXO</td>
</tr>
<tr>
<td>Wang (2009)</td>
<td>Minimum number of stations</td>
<td>AFV</td>
<td>-</td>
<td>D</td>
<td>EXO</td>
</tr>
<tr>
<td>Upchuch et al. (2009)</td>
<td>Minimum failure cost</td>
<td>AFV</td>
<td>INT</td>
<td>D</td>
<td>EXO</td>
</tr>
<tr>
<td>Ip et al. (2010)</td>
<td>Minimum operational cost</td>
<td>EV</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Hanabusa and Horighuchi (2011)</td>
<td>Entropy maximization</td>
<td>EV</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ge et al. (2011)</td>
<td>Minimum users’ loss</td>
<td>EV</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wang et al. (2011)</td>
<td>Maximum net income</td>
<td>BS</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Frade et al. (2011)</td>
<td>Maximum covering</td>
<td>EV-S</td>
<td>INN</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Chen et al. (2013)</td>
<td>Minimum access cost</td>
<td>EV-S</td>
<td>INN</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Proposed Model</strong></td>
<td><strong>Minimum failure cost</strong></td>
<td>EV-R</td>
<td>INN</td>
<td>P</td>
<td>ENDO</td>
</tr>
</tbody>
</table>

*note: *= not considered, AFV = station for alternative fuel vehicle, EV = station for electric vehicle, BS=battery switch station for EV, EV-S=slow-charging station for EV, EV-R=rapid charging station for EV, INT = intercity, INN = inner-city, D = deterministic, P = probabilistic, EXO = exogenously determined (all-or-nothing assignment), ENDO = endogenously determined (user equilibrium assignment)*
The aim of the research reported in this paper is to develop a location model of rapid charging stations considering vehicles’ ranges, batteries’ SOC, and users’ charging and travel behaviors. The model was formed as a bi-level optimization model in which the main problem was formulated to determine the locations of the stations and the patterns of use by EVs. The sub-problem was formulated to determine link flow based on the user-equilibrium principle. To solve the problem in reasonable time, a modified, simulated annealing algorithm was proposed. The applicability of the model was tested in a network in the example networks.

The structure of this paper is as follows. In section 2, the location model of rapid charging stations based on the user-equilibrium principle is formulated, and a heuristic algorithm is proposed that assists in efficiently determining the approximate solution of the problem. In section 3, potential applications of the proposed model on a simulated network are performed, and the results are compared with those provided by existing methods. Our conclusions and recommendations for future research are presented in section 4.

MODEL FORMULATION

The proposed model is an uncapacitated facility location problem to minimize travelers’ costs. The model is a modification of a $P$-median problem combined with a user-equilibrium problem. The proposed model was based on the following considerations. First, the vehicle range is assumed to be longer than the distance between the origin and destination for all OD pairs. This means that the EV’s battery would not have to be recharged more than once during the trip. Thus, we can eliminate the vehicle-range constraint from the FRLM.

Second, it was assumed that the remaining fuel range at the origin node followed a probabilistic distribution. The state of technical development or the supply of slow-recharging equipment at the origin node could affect the SOC. An EV that has a long range must be charged longer than an EV that has a shorter range. If charging time becomes shorter or larger numbers of slow-recharging equipment are offered, the SOC level can be higher. If the remaining fuel range is assumed to be constant, as Capar and Kuby did (18), the RFR function is a unit impulse function. The cumulative distribution function of RFR, which is used in the literature, can be written as shown in equation 1. In this study, the RFR function was assumed to be a probabilistic distribution function, such as a uniform distribution, an increasing distribution, and a triangular distribution. The distribution functions that were assumed in this study are shown in equations 2-4.

Constant function: $G(r) = \begin{cases} 0 & \text{if } 0 \leq r < 0.5r_V \\ 1 & \text{if } 0.5r_V \leq r \leq r_V \end{cases}$

Uniform function: $G(r) = \frac{1}{r_V} \times r$
Increasing function : 
\[ G(r) = \frac{1}{rV^2} \times r^2 \]  

Triangular function : 
\[ G(r) = \begin{cases} 
\frac{2}{rV^2} \times r^2 & \text{if } 0 \leq r < 0.5r_V \\
\frac{4}{rV} \times r - \frac{2}{rV^2} \times r^2 - 1 & \text{if } 0.5r_V \leq r \leq r_V 
\end{cases} \]  

where:

1. \( r \) = remaining fuel range in distance
2. \( r_V \) = vehicle range in distance
3. \( G(r) \) = cumulative distribution function of \( r \)

The probability of how many EVs can travel with or without charging can be calculated by the RFR distribution function. Figure 1 shows the relationship between the type of RFR distribution function and trip ratio. An increasing distribution may be found when people can easily charge their parked EVs. However, a uniform or a triangular distribution may be found when sufficient slow-charging equipment is not installed to accommodate the requirements of the EVs.

Flow from \( i \) to \( j \) : 100
Path used for trip without recharging : \( i \rightarrow j \)
Path used for trip with recharging : \( i \rightarrow k \) (station) \( \rightarrow j \)

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Constant distribution</th>
<th>Uniform distribution</th>
<th>Triangular distribution</th>
<th>Increasing distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_V = 20 )</td>
<td>Trip failure</td>
<td>0</td>
<td>35</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Trip success</td>
<td>100</td>
<td>65</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>without recharging</td>
<td>0</td>
<td>25</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>with recharging</td>
<td>100</td>
<td>40</td>
<td>63</td>
</tr>
<tr>
<td>( r_V = 40 )</td>
<td>Trip failure</td>
<td>0</td>
<td>18</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Trip success</td>
<td>100</td>
<td>82</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>without recharging</td>
<td>100</td>
<td>62</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>with recharging</td>
<td>0</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

**Figure 1** RFR distribution functions and travel path by RFR.

Third, trips are classified by users’ recharging behaviors. An existing fossil-fuel vehicle can be refueled easily, because there are many gas stations available. But there are far fewer charging stations for EVs than there are gas stations for fossil-fuel vehicles, so it was assumed that, before departure, the drivers of
EVs chose where they would recharge their EVs. A user’s behavior is determined based on the remaining fuel range displayed in the vehicle’s instrument panel. If the RFR is greater than the distance between the origin of the trip and its destination, the travelers can go to their destination without recharging, while they must locate a charging station or travel using gasoline when the RFR is less than the distance to the destination. Here, we propose two new decision variables, $\hat{y}_{ijk}$ and $\hat{y}_{ij}$, to divide the entire trip into three groups. The decision variable $\hat{y}_{ijk}$ is trip ratio travels via station $k$ among whole trip from origin $i$ to destination $j$. The decision variable $\hat{y}_{ij}$ is the ratio who cannot travel with EV. Travelers who can no longer travel in their EVs may travel by public transportation, taxi, or incur the cost of an emergency service to recharge their EV. In the flow refueling location model, travel paths are given as input data. As shown in Figure 2, three paths are available with the same distance from node 1 to node 8, i.e., 1-2-4-8, 1-5-4-8, and 1-5-7-8. In the existing model, just one path is available among the three paths. Here, we assume that the consumption of the battery’s charge is proportional to the distance traveled because the RFR is displayed in kilometers, and users usually use this information to decide whether their EVs must be charged on their routes or not.

<table>
<thead>
<tr>
<th>Travel path</th>
<th>FRLM (17-19)</th>
<th>Proposed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path used when the station is located at node 2</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>Path used when the station is located at node 5</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>

*note: The RFR distribution function is assumed to be a uniform function. The link travel time is assumed to be constant.*

**FIGURE 2 Travel paths by location of charging station.**

Finally, a traveler’s trip follows the user-equilibrium principle in terms of mean travel time, while the purpose of locating rapid charging stations is to minimize social costs, including travel time cost and the penalty associated with EV travel failure. The notation for formulating the model is as follows:

(a) Sets

\[ N = \text{node set, indexed by } n \ (N \ni n) \]
(b) Number of location

\[ P \] = number of charging stations

(c) Weights

\[ \gamma \] = additional penalty of failed travel
\[ \omega = \frac{\gamma}{1 + \gamma} \]

(d) Variables about remaining fuel range

\[ r \] = remaining fuel range
\[ g_i(r) \] = probability distribution function about remaining fuel range on origin node \( i \)
\[ G_i(r) \] = cumulative distribution function of \( g_i(r) \)
\[ \tilde{G}_{ij} \] = failure ratio travel between O-D pair \( i-j \);
\[ \tilde{G}_{ij} = G_i(c_{ij}) \]

(e) Node-based variables

\[ Q_{ij} \] = total demand between O-D pair \( i-j \)
\[ \bar{y}_{ijk} \] = charging ratio at the station \( k \) of travel between O-D pair \( i-j \) \( (0 \leq \bar{y}_{ijk} \leq 1) \)
\[ \bar{y}_{ij} \] = failure ratio of travel between O-D pair \( i-j \)
\[ z_k = \begin{cases} 1 & \text{if we locate at candidate node } k \\ 0 & \text{otherwise} \end{cases} \]
\[ v_{ik} = \begin{cases} 1 & \text{if station } k \text{ is the nearest from origin node } i \\ 0 & \text{otherwise} \end{cases} \]
\[ c_{ij} \] = minimum fuel consumption between O-D pair \( i-j \)
\[ \xi_{ikk'} = \begin{cases} 1 & \text{if } c_{ik} \leq c_{ik'} (\forall i,k,k') \\ 0 & \text{otherwise} \end{cases} \]

(f) Link-based variables

\[ x_a \] = flow on link \( a \)
\[ t_a \] = travel time on link \( a \)
\[ c_a \] = fuel consumption on link \( a \)
**Node/link/path-based variables**

- $f_{ij}^h$ = flow on path $h$ between O-D pair $i-j$
- $\delta_{ah}^{ij}$ = indicator variable; $\delta_{ah}^{ij} = \begin{cases} 1 & \text{if link } a \text{ is on path } h \text{ between } O - D \text{ pair } i - j \\ 0 & \text{otherwise} \end{cases}$

The proposed model is a bi-level optimization model. The main problem is a location-allocation problem to minimize the social cost, while the sub-problem is the trip assignment problem based on the user-equilibrium principle. In the main problem, the EV trip failure ratio, the trip ratio via each station for each origin-destination pair, and the location of charging station are determined. The equations were formulated based on the general facility-location problem (25). In the sub-problem, link flow was determined. The equations in the sub-problem are the modification of Beckmann’s mathematical programming (26). The mathematical model is as follows.

**Main Problem:**

\[
\min \left[ \omega \sum_i \sum_j Q_{ij} c_{ij} \bar{y}_{ij} + (1 - \omega) \sum_a x_a t_a(x_a) \right] 
\]

**Subject to:**

\[
\sum_k z_k = P
\]

\[
\bar{y}_{ijk} \leq z_k \quad \forall i, j, k
\]

\[
v_{ik} \geq \sum_{k'} \xi_{ikk'} z_{k'} + z_k - P \quad \forall i, k
\]

\[
\bar{y}_{ij} \geq v_{ik} \left[ \bar{G}_{ik} \xi_{ikj} + \bar{G}_{ij} (1 - \xi_{ijk}) \right] \quad \forall i, j, k
\]

\[
\bar{y}_{ij} + \sum_k \bar{y}_{ijk} = \bar{G}_{ij} \quad \forall i, j
\]

\[
\bar{y}_{ijk} \xi_{ik} (1 - \xi_{ikk'}) \geq \bar{G}_{ik} \xi_{ikj} + \bar{G}_{ij} \xi_{ijk} (1 - \xi_{ijk}) \quad \forall i, j, k
\]

\[
\bar{y}_{ijk} \geq 0 \quad \forall i, j, k
\]

\[
z_k, v_{ik} = 0, 1 \quad \forall i, k
\]

$x_a$ satisfies the sub-problem. \( \forall a \)

**Sub-Problem:**
\[
\min \sum_a \int_0^{x_a} t_a(\chi) d\chi
\]

Subject to:

\[
\sum_h f_h^{ij} = \bar{Q}_{ij} + \tilde{Q}_{ij} \quad \forall i, j
\]

\[
\sum_h f_h^{ik} = \sum_j Q_{ij} \tilde{y}_{ijk} \quad \forall i, k
\]

\[
\sum_h f_h^{kj} = \sum_i Q_{ij} \tilde{y}_{ijk} \quad \forall k, j
\]

\[
x_a = \sum_i \sum_j \sum_h f_h^{ij} \cdot \delta_{ah}^{ij} + \sum_i \sum_k \sum_h f_h^{ik} \cdot \delta_{ah}^{ik} + \sum_k \sum_j \sum_h f_h^{kj} \cdot \delta_{ah}^{kj} \quad \forall a
\]

\[
t_a(\chi) = t_0 \cdot \left(1 + \alpha \left(\frac{\chi}{\text{cap}}\right)^\beta\right)
\]

The objective function (eq. 5) of the main problem is to minimize the weighted average of the network travel time and EV trip failure penalty. The constraint (eq. 6) stipulates that \( P \) stations should be located. The constraints (eq. 7) state that any user traveling from origin \( i \) to destination \( j \) cannot be charged at station \( k \) unless a charging station is located at node \( k \). Constraints (eq. 8) are established for finding the nearest station \( k \) from origin node \( i \). If station \( k \) is the nearest station from origin \( i \), the number of stations that are at the same distance from node \( i \) or farther is \( P \). Thus, if the value of equation \( \sum_k \xi_{ik} z_k + z_k = P + 1 \), station \( k \) is the nearest station from origin \( i \), and \( v_{ik} \) becomes 1. Constraints (eq. 9) define the travel failure ratio \( \tilde{y}_{ij} \). If the nearest station from the origin is at the same distance or closer than the destination, the travel failure ratio is the probability that RFR is shorter than the distance between the origin and the nearest station. If the nearest station from the origin is farther than the destination, the travel failure ratio is the probability that RFR is shorter than the distance between the origin and the destination. Constraints (eq. 10) state that the sum of the travel failure ratio and the travel success ratio with charging at any of the stations is the same as \( \bar{G}_{ij} \), which is the ratio that EVs cannot travel without charging from the origin to the destination. Constraints (eq. 11) state that the probability of being unable to go from origin \( i \) to station \( k \) is equal to or smaller than the sum of the travel failure ratio and the travel success ratio with charging at any closer stations. Constraints (eq. 12) state that the travel ratio has a non-negative value. Constraints (eq. 13) are the integrality constraints. Link flows are determined at the sub-problem.
The sub-problem is the traffic assignment model with user equilibrium (eq. 15-20). To simulate the charging behavior, OD flow is split into two groups; one group consists of those who travel from origin to destination directly (eq. 16), and the other group consists of those who travel and use charging stations (eqs. 17-18). A travel with charging can be divided into two individual trips; one from origin to station (eq. 17), and another from station to destination (eq. 18). As a result, OD flow with charging is double counted. The split ratio is determined at the main problem. It is related to the user’s charging behavior. If link travel time is not a function of link flow, but a constant, all the decision variables can be determined without sub-problem.

The objective function is formulated as a multi-objective function based on EV trip failure penalty and total travel time. Two objective functions are combined with weighting factor \( \omega \). An EV traveler tends to give up her or his trip with the EV if the detour cost is greater than the trip failure penalty. When \( \omega \) approaches 0, EV travelers decide their trip depending on travel cost. However, when \( \omega \) approaches 1, more travelers travel with their EV. If \( \omega \) is 1, the objective functions become the same as the performance indices of Upchurch et al. Figure 3 shows the assignment results by varying \( \omega \).

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2~0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assignment result</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Failure (( \hat{y}_{18} ))</td>
<td>0.65</td>
<td>0.40</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Charging at node 3 (( \hat{y}_{183} ))</td>
<td>0.00</td>
<td>0.00</td>
<td>0.05</td>
<td>0.30</td>
</tr>
<tr>
<td>Charging at node 4 (( \hat{y}_{184} ))</td>
<td>0.00</td>
<td>0.20</td>
<td>0.25</td>
<td>0.00</td>
</tr>
<tr>
<td>Travel without charging (( \hat{y}_{18} ))</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Travel failure cost</td>
<td>549</td>
<td>338</td>
<td>296</td>
<td>296</td>
</tr>
<tr>
<td>Total travel cost</td>
<td>845</td>
<td>845</td>
<td>855</td>
<td>904</td>
</tr>
</tbody>
</table>

**FIGURE 3** Assignment results by weight \( \omega \).
Since it is a combinatorial, optimization problem, the location problem can be solved only through an enumeration technique. The computation complexity of an enumeration technique tends to increase exponentially as the feasible regions become larger. Therefore, many heuristic methods have been developed for solving location problems effectively, but they cannot be applied directly to the proposed model. The reason is that the gradient function or descent direction of the objective function cannot be calculated easily because the proposed model contains a sub-problem that has an objective function that is different from that of the main problem.

The objective function for each solution of location can be evaluated in polynomial time by the following steps. At the first time, the nearest station is calculated using the given \( z_k \) and \( P \) by equation 18.

\[
v_{ik} = \begin{cases} 
1 & \text{if } z_k - P + \sum_{k'} \xi_{ikk'}z_{k'} = 1 \\
0 & \text{else} 
\end{cases} \quad (21)
\]

Then, trip ratio (\( \hat{y}_{ij}, \hat{y}_{ijk} \)) can be determined by solving a simple linear programming problem, as shown in equations 22-27. The link flows, which are used for evaluating the objective function, can be derived from the sub-problem (equations 15-20). The user equilibrium assignment problem is a convex, non-linear programming problem, and it can be solved by using the Frank-Wolfe algorithm (26).

\[
\min \left[ \omega \sum_i \sum_j Q_{ij}c_{ij} \hat{y}_{ij} + (1 - \omega) \sum_i \sum_j \sum_k Q_{ij} \pi_{ijk} \hat{y}_{ijk} \right] \quad (22)
\]

Subject to:

\[
\begin{align*}
\hat{y}_{ijk} & \leq z_k^* & \forall i, j, k \\
\hat{y}_{ij} & \geq v_{ik} \left[ g_{ik} \xi_{ikk} + \bar{g}_{ij} (1 - \xi_{ikk}) \right] & \forall i, j, k \\
\hat{y}_{ij} + \sum_k \hat{y}_{ijk} & = \bar{g}_{ij} & \forall i, j \\
\hat{y}_{ij} + \sum_k \hat{y}_{ijk} \xi_{ikk'}(1 - \xi_{ikk'}) & \geq \bar{g}_{ik} \xi_{ikk} + \bar{g}_{ij} \xi_{ijk}(1 - \xi_{ikk}) & \forall i, j, k \\
\hat{y}_{ijk} & \geq 0 & \forall i, j, k
\end{align*}
\]

where:

\[
\pi_{ijk} = \begin{cases} 
c_{ik} + c_{kj} - c_{ij} & \text{if } c_{ik} + c_{kj} > c_{ij} \\
0 & \text{else} 
\end{cases}
\]

\( z_k^* \) is given

\[
v_{ik} = \begin{cases} 
1 & \text{if } z_k - P + \sum_{k'} \xi_{ikk'}z_{k'} = 1 \\
0 & \text{else} 
\end{cases} \]
To find a suitable solution, the objective function should be evaluated for every solution set. To do this efficiently, the modified, simulated annealing algorithm was used to solve the proposed model. The solution algorithm is shown in Figure 4. In order to make the algorithm function more efficient, the cooling and annealing schedules were controlled based on repetitive works.

**FIGURE 4 Flowchart of the solution algorithm.**
APPLICATION AND RESULTS

We checked the validity and applicability of the proposed model on the modified Sioux-Falls 24-node network, which was first used by LeBlanc (27). This network is not considered to be realistic, but it has been used in many publications to debug code or examine the formats of data (28). The maximum distance between the origin and the destination was 12.1 km in the original network data, so the test network was enlarged to five times its original distance. Figure 5 summarizes the information on the base condition.

(a) Network and OD

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Number of nodes</td>
<td>24</td>
</tr>
<tr>
<td>Demand on peak (trip/hr)</td>
<td>360,600</td>
</tr>
<tr>
<td>Number of candidates</td>
<td>24</td>
</tr>
<tr>
<td>Maximum distance (km)</td>
<td>60</td>
</tr>
</tbody>
</table>

(b) Input parameter

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Weight ((\omega))</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(c) Travel time function

<p>| | |</p>
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<tr>
<th></th>
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<tbody>
<tr>
<td>BPR function</td>
<td>(\alpha = 0.15), (\beta = 4.0)</td>
</tr>
</tbody>
</table>

(d) RFR assumption

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<table>
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</thead>
<tbody>
<tr>
<td>Vehicle Range (km)</td>
<td>60</td>
</tr>
<tr>
<td>Distribution function</td>
<td>Probabilistic</td>
</tr>
</tbody>
</table>

*note: The sizes of the circles indicate the departure volume from each node. The thicknesses of the lines indicate the capacity of each link.*

FIGURE 5 Parameters and input data.

The main differences between the existing model and the proposed model were the RFR assumption and the way in which the recharging station was chosen. In the existing model, every vehicle had the same SOC and can be recharged only at the station on the shortest path. Only a few studies have considered the possibility of taking a detour for recharging. The proposed model chooses the detour path for travel via the recharging station endogenously.

First, we compared the solution of the existing model and the proposed model. Because the former’s objective function was to minimize trip failure and the total travel cost was included in the proposed model, the solutions can be different. However, it is difficult to determine whether one solution is significantly better than the other.

When the RFR was assumed to be half of the vehicle range, RFR was 50 km in this application, and the distances of most trips were shorter than the RFR (Figure 6). Thus, only one station should be
constructed. Therefore, such an assumption cannot simulate the travel pattern of daily trips in an urban area. This is the reason that we should use the probabilistic RFR distribution. We compared the proposed model with the existing model under the assumption that the RFR distribution follows the uniform distribution function.

![Figure 6: EV travel failure by the assumption of RFR distribution function](image)

We solved the problem using the proposed model and the existing model based on the probabilistic RFR distribution function and compared their results. Model I is the existing model, in which only the shortest paths were available. By adding constraints \( \pi_{ijk} \cdot \hat{y}_{ijk} = 0 \) for all \( i, j, k \) to the main problem, paths longer than the shortest paths must be used. If the reduced cost from origin node \( i \) to destination node \( j \) via station \( k \) is greater than 0 (\( \pi_{ijk} > 0 \)), station \( k \) is not used (\( \hat{y}_{ijk} = 0 \)). If station \( k \) is chosen (\( \hat{y}_{ijk} > 0 \)), it means that station \( k \) is on the shortest path from \( i \) to \( j \). Model II has a detour available for recharging.

The number of trip failures and the cost of trip failures were decreased by increasing the number of charging stations in both models. But no significant pattern of decreasing or increasing network costs was found. The comparison of the two models indicated that, for the model that had a detour available, trip failure and trip failure cost were worse than they were for the other model, but the total travel time costs were better. This means that the proposed model can reflect travelers’ behaviors of making detours to recharge so they can finish their trips in EV mode.

Finally, we analyzed the solution by the objective function. When the objective function is to minimize the number of trip failures or the trip-failure costs, the total travel time of the network tended to be larger. If the facilities were located based on the objective function, including only the trip-failure term, they were likely to make the traffic condition worse.
FIGURE 7 Comparisons of performance indices by detour possibility.

(a) EV trip failure (veh)

(b) EV trip failure cost (veh·km)

(c) Total travel cost (veh·hr)
CONCLUSIONS AND FUTURE RESEARCH
In this paper, we reported the development of a model for locating rapid charging stations in urban areas. The assumption made for RFR distribution helped to reflect the technology development or demand variations. By using an assignment model that was user-based, charging and traveling behaviors could be considered reasonably. The modified, simulated annealing algorithm was proposed to solve the problem in polynomial time. According to the comparison with existing methods, the proposed model produced more stable solutions for various inputs, while the other methods produced different solutions when the number of stations or the vehicles’ ranges were changed.

From the results, we identified three significant implications. First, the present models used to locate charging stations are only for slow-charging equipment, which can be installed in parking lots. But to enhance the penetration of EVs, rapid charging stations are needed so they can be used during trips. Therefore, we developed a location model for rapid-charging stations. Second, the location model with constant RFR can produce different solutions when the vehicles’ ranges are changed. This makes it difficult to determine definitively the best locations for EV charging stations when the technology is advancing rapidly. Therefore, the location model that we developed can contribute to determining stable locations for charging stations by using a probabilistic RFR distribution function, even though the technology is being developed continuously. Third, the location model without the UE-based assignment problem is not likely to consider the congested traffic condition of urban areas. When only a few charging stations were installed during the EVs’ introduction stage, many detours occurred for charging during trips, which can make traffic worse. This implies that the UE-based assignment should be included in the charging station location problem.

The model developed in this study provides a suitable solution for the location of rapid charging stations in an urban area because it reflects the probabilistic RFR distribution and the users’ practical charging and traveling behaviors. To consider the viewpoint of decision makers, we can include construction costs or operating costs in the objective function. Therefore, the model provides a theoretical basis for determining suitable locations for rapid charging stations, and it can be applied directly to a real network by using the modified algorithm developed in this study.

The proposed model can be improved in further research and assessment work. If the RFR distribution function or battery consumption profile is surveyed in practice or predicted more reasonably, the proposed model can provide a more accurate solution for the targeted area without any modifications of the model. The optimal number of stations can be evaluated when the model is extended to include the capacity of each station, charging speed, and the construction cost and operating cost of each station. By combining this model with the location of slow-charging stations, or considering vehicle mixing, various types of stations can be optimized, which is a subject for future research.
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REFERENCES


