Optimal Allocation of Life Cycle Cost, System Reliability, and Service Reliability in Passenger Rail System Design

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Yung-Cheng (Rex) Lai*, Chia-Tsong Lu, and Ya-Wen Hsu

Railway Technology Research Center
Department of Civil Engineering
National Taiwan University
Room 313, Civil Engineering Building
No 1, Roosevelt Road, Sec 4, Taipei, Taiwan, 10617

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Yung-Cheng (Rex) Lai
+866-2-3366-4243
yclai@ntu.edu.tw

Chia-Tsong Lu
+866-2-3366-4243
r02521523@ntu.edu.tw

Ya-Wen Hsu
+866-2-3366-4243
sandy870210@gmail.com

*Corresponding Author
ABSTRACT

A number of products with specific cost and reliability for each subsystem can be selected from several equipment suppliers to design a new rail system or upgrade an existing system. Decision makers have to balance the trade-off among life cycle cost (LCC), system reliability (e.g., mean time between failures), and service reliability (e.g., delay) to optimally allocate resources. This research develops an optimization framework with an alternative evaluator and an investment selector to determine an optimal investment plan with a specific allocation on cost, system reliability, and service reliability. Empirical case studies indicate that this optimization process can efficiently and successfully evaluate all of the possible alternatives and determine an optimal investment set according to design service reliability or LCC. This tool can help railway agencies and companies maximize their return on investment and provide reliable service to their customers.

Keywords
Rail Transportation, System Design, Reliability Allocation, Life Cycle Cost
INTRODUCTION

Reliability is one of the most important performance indicators for rail systems (1, 2). These systems consist of different subsystems, and each has its own reliability and life cycle cost (LCC). To design or maintain a rail system with a specific system reliability, planners have to carefully allocate the reliability and budget by examining the trade-off between cost and reliability.

Reliability allocation is a popular topic in designing hardware and/or software systems. Various reliability allocation methods have been developed to allocate goal system reliability into subsystem reliability under cost-effective conditions. These methods are generally categorized into two types, namely, (1) weighting method and (2) optimization method (3, 4). Weighting method is adopted if the quantitative relationship between system reliability and LCC is unavailable. Experts typically determine the weights for cost and reliability of each subsystem from past experiences in similar systems (5–9). However, weight determination is subjective, and thus, the resulting weights are usually case-specific without general applicability.

Optimization method (e.g., mathematical programming) is used to allocate reliability upon establishing a quantitative relationship between subsystem cost and reliability (3). Optimal reliability allocation involves minimizing cost under reliability constraints or maximizing reliability under budget constraints (10, 11). This optimization process requires a clear relationship between cost and reliability for each subsystem, which is best captured by exponential functions. However, adopting exponential functions in the formulation results in nonlinear optimization models that cannot guarantee global optimal solutions (3, 12, 13).

Existing studies on reliability allocation define reliability as the mean time between failures (MTBF) and discuss its relationship with cost (13, 14). For rail systems, MTBF represents system (or subsystem) reliability but does not consider its effect on passengers (15). Service reliability (e.g., delay or on-time arrival percentage) is more favorable than system reliability because it considers customer satisfaction. Therefore, the relationship between system reliability and service reliability should also be established in decision making. Planners have to balance the trade-off among service reliability, system reliability, and LCC to optimally allocate reliability and cost (Figure 1).
A few studies have applied reliability allocation to capital investment for rail systems. Anderson (16) proposed an allocation method with consideration of delay and utilized Lagrangian multipliers to combine LCC and service reliability to identify optimal solutions. This method requires continuous and linear relationships between cost and service reliability, which is not always available or reasonable. Moreover, practical constraints during actual applications cannot be easily accommodated without mathematical programming formulation (e.g., linear or integer programming models). Consequently, we develop an optimization framework with alternative evaluators (AE) and investment selectors (IS) to assist decision makers in optimally allocating service reliability, system reliability, and LCC when designing or upgrading rail systems. This tool can help railway agencies and companies maximize their return on investment (ROI) and provide reliable service to their customers.

**OPTIMIZATION FRAMEWORK**

A number of products for each subsystem can be selected from several equipment suppliers to design a new rail system or upgrade an existing system. Each product has LCC and reliability in terms of MTBF or mean distance between failures (MDBF); this information is obtained from suppliers. However, planners initially have to establish a relationship between system reliability and service reliability for each alternative. This relationship is then incorporated with investment selector to determine the optimal allocation of reliability and LCC. The following paragraphs define the three important elements in decision making, followed by the optimization framework:

- **LCC**: LCC for railway systems typically includes capital investment, operating cost, and maintenance cost within the planning period. Some products have low capital investment but high operating and maintenance costs, whereas other products have high capital
investment but low operating and maintenance costs. Therefore, employing LCC is more appropriate in decision making than solely employing capital investment.

- **System reliability**: System reliability is defined as the MTBF or MDBF. System reliability is used to estimate the failure frequency based on particular exposure rates (in train-hour or train-km). A high MTBF or MDBF results in superior system reliability. The relationship between system reliability and LCC is obtained from suppliers.

- **Service reliability**: Service reliability identifies the effect on passengers. This study defines service reliability as on-time arrival percentage (without any buffer), which is the proportion of on-time operations in terms of total system operating time (in train-hour) [Equation (1)].

$$r_{sys} = \left( \frac{P - \sum_{i=1}^{l} d_i}{P} \right) \times 100\%,$$

where:

- $r_{sys}$ = On-time arrival percentage
- $P$ = Total system operational time (in train-hour) in a defined period
- $d_i$ = Delay (in train-hour) of subsystem $i$

Conventional on-time percentage is computed with a buffer (e.g., 5 min). If the scheduled and actual arrival times of a train service are within the buffer, then delay is not computed against on-time percentage. However, for our purpose, we take into account all delay in this decision making process without adopting any buffer in order to reveal actual service reliability.

Figure 2 shows the optimization framework for reliability and LCC allocation. The proposed framework includes two important modules, namely, AE and IS. Based on investment alternatives and demand, AE initially evaluates all of the possible alternatives and generates an investment alternative table with their associated LCC, system reliability, and effect on service reliability. With complete information gathered on all of the alternatives, IS subsequently determines the optimal set of investment alternatives according to design reliability or LCC.
FIGURE 2  Optimization framework for reliability and LCC allocation.

ALTERNATIVE EVALUATOR (AE)

AE is used to evaluate LCC, system reliability, and service reliability for every possible alternative. LCC and system reliability (in terms of MTBF or MDBF) are obtained from suppliers, but this process requires an established relationship between service reliability and system reliability, which is computed using Equation (2). The equation computes the delay for subsystem $i$ with alternative $k$, which is the product of the number of failures ($T_i/M_{ik}$), average number of online trains ($N$), and average delay time ($Q_{ik}$).

$$D_{ik} = \frac{T_i}{M_{ik}} N Q_{ik} \quad \forall i \in I, k \in K,$$

where:

- $D_{ik}$ = Delay (in train-hour) of subsystem $i$ with alternative $k$
- $M_{ik}$ = MTBF or MDBF of subsystem $i$ with alternative $k$
- $T_i$ = Operational time or distance of subsystem $i$ in a defined period
- $N$ = Average number of online trains
- $Q_{ik}$ = Average delay (in hours) from a failure of subsystem $i$ with alternative $k$

$M_{ik}$ is provided by the supplier, and $T_i$ and $N$ are computed by demand, cycle time, train capacity, and design (or average) loading factor. For example, $N = (\text{Demand in Maximum Loading Section} \times \text{Cycle Time}) / (\text{Train Capacity} \times \text{Loading Factor})$. The average delay from a failure of subsystem $i$ with alternative $k$, denoted by $Q_{ik}$, is key to evaluating the service reliability of an alternative. This parameter is estimated using historical data from similar systems; otherwise, this parameter is determined by simulating the service effect from possible types of failures. After computing $D_{ik}$, AE consolidates LCC, system reliability (in MTBF or
MDBF), and service reliability of all of the alternatives and creates an investment alternative table for IS (Table 1).

### TABLE 1 Investment Alternative Table

<table>
<thead>
<tr>
<th>Subsystem (i)</th>
<th>Alternatives (k)</th>
<th>MDBF ($M_{ik}$) (train-km)</th>
<th>LCC ($C_{ik}$) (billion dollars)</th>
<th>Delay ($D_{ik}$) (train-hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train</td>
<td>1</td>
<td>29,274</td>
<td>16.63</td>
<td>393</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>39,799</td>
<td>16.76</td>
<td>289</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>62,192</td>
<td>17.05</td>
<td>185</td>
</tr>
<tr>
<td></td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Electricity</td>
<td>1</td>
<td>29,274</td>
<td>20.86</td>
<td>2,144</td>
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<tr>
<td></td>
<td>2</td>
<td>38,521</td>
<td>22.48</td>
<td>1,629</td>
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<tr>
<td></td>
<td>3</td>
<td>48,133</td>
<td>24.32</td>
<td>1,304</td>
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</tr>
<tr>
<td>Track</td>
<td>1</td>
<td>36,716</td>
<td>14.81</td>
<td>1,363</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>56,073</td>
<td>15.31</td>
<td>892</td>
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<tr>
<td></td>
<td>3</td>
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<td>Signal</td>
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<td>30.03</td>
<td>1,110</td>
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<td></td>
<td>2</td>
<td>31,039</td>
<td>31.01</td>
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<td></td>
<td>3</td>
<td>39,799</td>
<td>36.45</td>
<td>817</td>
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<tr>
<td>Communication</td>
<td>1</td>
<td>36,716</td>
<td>8.56</td>
<td>3,395</td>
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<td>2</td>
<td>53,899</td>
<td>9.75</td>
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<td>3</td>
<td>78,465</td>
<td>11.77</td>
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<tr>
<td>Station</td>
<td>1</td>
<td>67,941</td>
<td>3.91</td>
<td>277</td>
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<td></td>
<td>2</td>
<td>84,846</td>
<td>1.95</td>
<td>222</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>123,063</td>
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<td>153</td>
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</tbody>
</table>

### INVESTMENT SELECTOR (IS)

IS determines an optimal investment plan for rail systems based on available alternatives. The optimal investment plan identifies the best alternative for every subsystem according to acceptable LCC or service reliability. Two optimization models are developed, namely, (1) the minimum cost model (MCM), which minimizes total LCC according to acceptable service reliability, and (2) the maximum reliability model (MRM), which maximizes service reliability according to available LCC.
The optimization models employ the following notations. $i$ and $k$ are indices that represent the subsystem type and product alternatives. $I$ and $K$ are the sets of $i$ and $k$, respectively. $C_{ik}$ is LCC of alternative $k$ of subsystem $i$. $D_{ik}$ is delay of alternative $k$ of subsystem $i$. $P$ is total operational time per year. $R$ is design service reliability. $B$ is design LCC. Two sets of decision variables are used: (1) $\delta_{ik}$ is a binary variable that represents whether alternative $k$ of subsystem $i$ is selected; and (2) $d_i$ is the delay of subsystem $i$.

**Minimize Cost Model (MCM)**

MCM minimizes LCC while performing service reliability constraint. The mixed-integer programming (MIP) model for MCM is as follows:

\[
\begin{align*}
\min & \quad \sum_{i \in I} \sum_{k \in K} C_{ik} \delta_{ik} \\
\text{s.t.} & \quad \sum_{k \in K} \delta_{ik} = 1 \quad \forall i \in I \tag{4} \\
& \quad d_i = \sum_{k \in K} D_{ik} \delta_{ik} \quad \forall i \in I \tag{5} \\
& \quad \left( \frac{P - \sum_{i \in I} d_i}{P} \right) \times 100\% \geq R \tag{6} \\
\text{and} & \quad \delta_{ik} \in \{0,1\} \quad \forall i \in I, k \in K \tag{7} \\
& \quad d_i \geq 0 \quad \forall i \in I \tag{8}
\end{align*}
\]

Equation (3) minimizes total LCC over the planning horizon. Total cost is the summation of costs of all subsystems required in the design process. Equation (4) ensures that each subsystem selects only one alternative. Equation (5) computes subsystem delay according to investment selection. Equation (6) guarantees that system service reliability is greater than or equal to design service reliability.

**Maximize Reliability Model (MRM)**

MRM maximizes reliability with constraints on the design LCC. The MIP model for MRM is as follows:
Equation (9) maximizes service reliability in terms of on-time arrival percentage. Equations (10) and (11) are the same as Equations (4) and (5); the former is the constraint on the number of alternatives per subsystem, whereas the latter is the computation of subsystem delay. Equation (12) ensures that LCC of the optimal investment plan is not greater than the design LCC.

**CASE STUDY**
We implement the optimization process in two case studies utilizing empirical data obtained from a metro system in Taiwan to demonstrate their potential use in rail system design and upgrade. Case I aims to design a new metro system by selecting appropriate alternatives for subsystems according to design service reliability. MCM is employed in Case I to determine an optimal investment plan with minimal LCC. Case II aims to improve the reliability of an existing rail system subject to constraint on available increment in LCC (i.e., budget constraint). MRM is employed in Case II to determine an optimal investment plan with maximum increase in service reliability.

**Case I: New System Design**
A system should be carefully designed at the design stage according to the desired level of service reliability. We consider the design of a 25-km passenger rail system, which comprises six subsystems, namely, train, signal, communication, electricity, station, and track. Each subsystem has a number of alternatives. Estimated demand is 140,000 passengers per day. Figure 3 shows...
LCC (in New Taiwan Dollars) and MDBF of possible alternatives for each subsystem. This information is obtained from the suppliers. To examine the possible scenarios, we assume that the design service reliability (on-time arrival percentage) is from 95% to 99%, with 1% increments.

![Figure 3: Subsystem alternatives for the new system design.](image)

AE initially evaluates all of the alternatives and subsequently establishes the relationship between system reliability and service reliability. In Equation (2), $M_{ik}$ is obtained from the supplier (Figure 3). $T_i$ and $N$ are computed based on the required train service according to demand, train capacity, cycle time, and design loading factor, which results in 5,411,347 (train-km/year) and an average of 31 online trains. $Q_{ik}$ is estimated based on historical data from existing systems that have similar characteristics as the new design. Table 1 shows that communication and electricity are the two largest sources of delays. $P$ is 172,539 train-hour per year. After computing $D_{ik}$, AE then consolidates LCC, system reliability (in MDBF), and service reliability of all of the alternatives and creates an investment alternative table (Table 1) for IS.

IS is coded into a General Algebraic Modeling System (GAMS) and is efficiently solved by CPLEX within seconds. Table 2 shows that the total cost range is 92.83 billion to 195.94 billion. High design service reliability results in high MDBF and required LCC. However, every subsystem under different design service reliabilities has its own options for specific MDBF and LCC and, thus, has different increase rates.
Figure 4 shows the MDBF of each subsystem under the desired level of service reliability. MDBF distributions for all of the subsystems at 95% service reliability are similar except for station. The difference in this allocation becomes obvious as service reliability level increases. Station has the largest MDBF in all of the scenarios, and its increase rate is the highest among all of the subsystems, followed by track, train, communication, electricity, and signal. Large increases in MDBF indicate less frequency for service failure. However, the effect on service reliability also depends on the consequence of service failure (in delay per failure). Besides system and service reliability, IS also considers LCC for determining the best combination among all of the alternatives.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Design Service Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>95%</td>
</tr>
<tr>
<td><strong>MDBF</strong></td>
<td></td>
</tr>
<tr>
<td>Train</td>
<td>29,274</td>
</tr>
<tr>
<td>Electricity</td>
<td>29,274</td>
</tr>
<tr>
<td>Track</td>
<td>36,716</td>
</tr>
<tr>
<td>Signal</td>
<td>29,274</td>
</tr>
<tr>
<td>Communication</td>
<td>36,716</td>
</tr>
<tr>
<td>Station</td>
<td>84,846</td>
</tr>
<tr>
<td><strong>LCC</strong></td>
<td></td>
</tr>
<tr>
<td>Train</td>
<td>16.63</td>
</tr>
<tr>
<td>Electricity</td>
<td>20.86</td>
</tr>
<tr>
<td>Track</td>
<td>14.81</td>
</tr>
<tr>
<td>Signal</td>
<td>30.03</td>
</tr>
<tr>
<td>Communication</td>
<td>8.56</td>
</tr>
<tr>
<td>Station</td>
<td>1.95</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>92.83</td>
</tr>
</tbody>
</table>
Figure 5 demonstrates the LCC allocation for each subsystem. LCC generally increases with service reliability. The increase in total LCC from 95% to 97% is modest for design service reliability. However, total LCC increased sharply from 97% to 99% because of the nonlinear relationship between cost and reliability. Allocation proportions to different subsystems are the same for design service reliability from 95% to 97%. Electricity and communication proportions become notably high when the level increases to 98%. LCC proportion allocated to the signaling system also sharply increases at 99% design service reliability to meet the high standard in service reliability.
Case II: Existing System Improvement

Case II utilizes MRM to improve the reliability of an existing rail system according to the design LCC. Cost used in the improvement is usually fixed, and system reliability is maximized according to this cost constraint. Service reliability of the existing system is 97%. To examine the possible scenarios, we assume an increment ranging from 1 billion to 5 billion, with an increment of 1 billion. Figure 6 shows the alternatives for possible improvements on the existing system. Each alternative is associated with an increment in LCC and system reliability (in MDBF). Not all of the subsystems can be easily changed for the existing system. Therefore, we consider alternatives for communication, electricity, and track in this case.
The process is coded in GAMS and efficiently solved by CPLEX within seconds. Figures 7 and 8 demonstrate the increment in both MDBF and LCC for each scenario in Case II. High design LCC generally results in high MDBF (system reliability). Track has the most significant increase in MDBF, followed by communication and electricity. The distinction in the MDBF increment among the three subsystems increases with the design LCC.

**FIGURE 6** Subsystem alternatives for the existing rail system.

**FIGURE 7** MDBF increment for each subsystem in optimal system upgrade.
According to Figure 8, the higher the budget, the higher the increment of LCC. More LCC have been allocated to electricity, and communication for all scenarios because impact on delay from communication and electricity failures is more severe than that from track failures (Table 1). Figure 9 shows the increase in service reliability from possible scenarios of system improvement. Service reliability increases from 0.1% to 0.5% (1 billion to 5 billion). Increasing system reliability at this stage is typically more difficult than increasing it at the design stage because the available improvement alternatives are limited and usually cost more for the existing system.
DISCUSSION

A number of products for each subsystem can be selected from different equipment suppliers to design a new rail system or upgrade an existing system. Decision makers have to balance the trade-off among service reliability, system reliability, and LCC to optimally allocate resources. Case studies indicate that reliability can be increased more effectively during the design stage (for new systems) than during the improvement stage (for existing systems) with the same amount of money. Therefore, carefully designed systems with good resource allocation could avoid costly reworking and improvement.

The accuracy of the reliability data obtained from suppliers is a key to successfully determine the optimal investment plan through this process. We recommend the following three ways to verify the accuracy of MTBF or MDBF: (1) rigorous test process and data; (2) actual performance data; and (3) certificate of conformity (CoC). Test data on the reliability of a particular subsystem should be obtained using a rigorous process and standard approved by rail transit agencies or companies. The actual performance data of a particular subsystem in a rail system represent actual reliability in real-world operations. CoC issued by a third-party examiner can be used to verify the reliability of data. Among these methods, rigorous test data are generally required for a particular product, and their credibility can be further improved with actual performance data and/or CoC. Agencies or companies should specify the required data and documents for subsystems.
CONCLUSIONS

This research develops an optimization framework with AE and IS to assist decision makers in optimally allocating service reliability, system reliability, and LCC when designing or upgrading rail systems. Empirical case studies indicate that the optimization process can efficiently and successfully evaluate all of the possible alternatives and determine optimal investment sets according to design service reliability or LCC. This tool can help railway agencies and companies maximize their ROI and provide reliable service to their customers.

REFERENCES


