Optimizing Passenger Transfer Coordination in a Large Scale Rapid Rail Network

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ABSTRACT
Urban rapid rail transit, or metro, is an essential travel mode for daily commuters in many metropolitan areas. Transfer stations, as hubs connecting various transit lines spatially, used to be the bottlenecks of the network due to its high volume of transfer passengers during peak periods. Coordinating train arrivals at the hubs by adjusting train departing times from the terminal may significantly reduce passenger transfer time. A mathematic model is developed and the optimal justification of train departure times which minimizes the total transfer time was found by a simulated annealing algorithm. A real-world metro network with five lines intersecting at thirteen stations is applied to demonstrate the effectiveness of the proposed methodology. The passenger origin-destination demand of the study network was estimated based on data provided by an automatic fare collection system.

KEYWORDS
Optimization; Transfer time; System coordination; Metro network; Timetable; Simulated annealing
INTRODUCTION
As a major transportation tool in metropolitan area, urban rapid rail transit with exclusive right-of-way, or called Metro, is an essential travel mode for daily commuters, which offers frequent services to accommodate high demand and mitigates the escalation of traffic congestion. Transfer stations, as hubs connecting various transit lines spatially, used to be the bottlenecks of the network due to its high volume of transfer passengers during peak periods. The optimal timetables of different lines in a Metro network are desirable, so that convenient and smooth transfer may be expected and the system productivity can be improved.

Transfer delay has been deemed as an important service quality indicator for evaluating the efficiency of public transit systems. Coordinated transfers for passengers from one line to another can reduce the total travel time. Transfer optimization has been concerned by many previous studies. Rapp and Gehner (1) presented a four-stage interactive graphic process, which minimizes transfer delays by modifying dispatching offsets. Bookbinder and Désilets (2) formulated a transfer optimization problem as a quadratic function, in which the overall passenger disutility for bus transfer trips was minimized. Adamski (3) developed a dynamic model for optimizing dispatching control based on a probability model for transfer waiting times.

The timetabling and scheduling optimization of a transit system considering transfer coordination was investigated in past decades. Klemt and Stemme (4) developed a computer aided scheduling algorithm through synchronizing the transit dispatching to minimize total transfer times. Voß (5) formulated a schedule synchronization approach to minimize passengers’ waiting time at the transfer nodes. Later, Daduna and VoB (6) developed a Tabu search algorithm for the same optimization objective. Goverde (7) formulated a mathematical model to minimize all affected waiting times associated with initial train departure delays, including the waiting times of originating and through passengers on the current transfer station.

Scheduling optimization has been applied on the bus system as well. Deb and Chakroborty (8) formulated a transit scheduling problem to minimize overall waiting time subject to resource and service related constraints. Ceder et al. (9, 10) optimized a synchronized timetable for a given network by maximizing the number of simultaneous bus arrivals at transfer nodes. The problem was formulated with a mixed-integer linear programming approach, in which a heuristic algorithm was used to search for the optimal solution. Cevallos and Zhao (11) developed a genetic algorithm to search for the optimal timetable for a large scale bus network which yielded the minimum transfer times considering the ridership data at all transfer locations and the randomness of bus arrivals. Shafahi and Khani (12) aimed to minimizing the waiting time at transfer stations base on a departure time setting model and a headway setting model solved by genetic algorithm. Later on, Khani and Shafahi (13) took both headway and departure time into account for transfer optimization in transit networks.

Furthermore, many studies have focused on optimizing intermodal coordination. Lee and Schonfeld (14) optimized slack times of coordinated transit routes, which minimized the total transfer cost, considering stochastic vehicle arrivals at a single transfer station. Chien and Schonfeld (15) optimize headways, station/stop locations, and route spacing for an integrated bus and rail network, which minimized the total cost, consisting of user and supplier costs. Chowdhury and Chien (16) optimized the dispatching times of vehicles of connected routes at a transfer station, which minimized the total cost including connection delay and missed-connection costs incurred by transfer passengers and vehicle holding. Chowdhury and Chien (17) formulated
a model to optimize a coordinated service provided by multiple transit modes including a train line and its feeder bus system. The minimized objective total cost function, including supplier and user costs, was yielded by the optimized headways and slack times. As an extension of their previous studies (16, 17), Chowdhury and Chien (18) developed a four-stage procedure to optimize coordination among routes at multiple transfer stations in an intermodal transit network, which minimized the total cost. Concerning about coordination between trains and buses, Shrivastava and Dhingra (19, 20) studied a schedule optimization model to minimize transfer time of passengers between trains and buses.

Up to now, few studies concentrated on transfer coordination via optimizing dispatching time and timetable for a large scale metro network. Ting and Schonfeld (21) used a heuristic algorithm to jointly optimize the headways and slack times by minimizing the total costs of operating a multiple-hub transit network. Wong et al. (22) presented a mix-integer programming model to optimize non-periodic timetable synchronization, which minimizes the transfer waiting times by adjusting trains’ run times, station dwell times, dispatch times, turnaround times at the terminals with a heuristic approach. Fang et al. (23) developed an optimization model to minimize total waiting time of transfer passengers and inboard passengers. A multi-layers coordination policy was proposed to find small time shifts of the proposed single-line timetables to optimize the urban mass transit system. Aksu and Akyol (24) proposed a genetic algorithm that creates routes coordination to reduce the transfer time for connecting passengers. The objective is to minimize the total system cost, including the in-vehicle, waiting, and transfer costs for all the passengers served by the transit system and the operating cost of all transit vehicles.

Traditionally transfer coordination for various transit systems is conducted by minimizing the passenger waiting time, where adjustments such as dispatching time, headway, running time, dwell time, etc. were utilized to optimize transfer coordination. However, previously studies were usually based on a simplified network with one or two lines. It appears that previous research on minimizing total transfer time in a large scale metro network is still minimal.

This study aims at developing a mathematic model to optimize coordination among lines in a real-world large scale metro network, in which transfer walking time and choice of transfer locations are considered. A simulated annealing algorithm is developed to search for an optimal solution. We intent to justify train dispatching time but not the headway of existing timetable; therefore, there would be no addition cost and safety concerns to operation. A real-world metro network with five lines intersecting at thirteen stations is applied to demonstrate the effectiveness of the proposed method. The passenger origin-destination demand of the study network was estimated based on data provided by an automatic fare collection system.

METHODOLOGY
A simplified, general metro network, shown in Figure 1, is used to define the variables used in the problem formulation. Intersecting metro lines operate on fixed headways and meet at hubs. Each line’s headway is derived to take into account passengers load, capacity, fleet size, etc. Trains running on each line depart from the starting terminal at discrete points in time and reach subsequent stations downstream according to the schedule some time later. While the line headways are fixed, our intent was to optimize train dispatching times from the terminal in order to minimize the total transfer passenger waiting time at the hubs.
A mathematic model is developed to minimize the total transfer waiting time incurred by passengers at transfer stations. The decision variable in the model is the time adjustment of discrete train dispatching time for each train line.

**A General Study Metro Network**

The network consists of a set of lines denoted as $L$, and a set of stations defined as $N$. A metro line $l (l \in L)$ provides a directional service. Each line consists of a set of stations $N_l (N_l \in N)$ as shown in Figure 1. Each station is given a unique station ID. For example, IDs of stations of line 1 begin with 1 and end at $N_1$. The IDs of stations on line 2 start from $N_1+1$ and end at $N_2$. Thus, stations of line $l$ are labeled from $N_{l,1}+1$ through $N_l$. Each line is split into 2 lines – outbound and inbound– connecting a pair of end terminals. For line $l$, the hourly service frequency is denoted as $F_l$, and the index of trains is denoted as $m (m=1, 2, \ldots, F_l)$. A set of transfer stations is denoted as $S$, in which a transfer station $s (s \in S)$ is also referred to as $u_l$ of line $l$ as well as $u_{l'}$ of line $l'$ ($u_l \in N$).

**FIGURE 1 A simplified metro network with a transfer station.**

The variables used in model formulation are defined in Table 1, and the assumptions for formulating the proposed model are:

1. Train running times between stations and dwell times at stations are deterministic within a time period (i.e. one hour). However, they may vary over different periods.
2. Passengers arrive at any origin station are random. However, transfer passengers arriving at transfer stations are concentrated and dependent on the arrivals of trains which deliver them.
3. The OD passenger demand matrix is known, which is estimated based on data provided by an automatic fare collection system (AFCS).
4. The train service capacity is sufficient to meet demand and the dispatching headways at the terminals are identical of each line within a time period, but may vary over different lines.
5. The average walking time between any two lines at a transfer station is known.
6. The choice of transfer location is determined by means of the shortest travel time, the sum of in-vehicle and transfer waiting time.
7. Most passengers will make one transfer to reach the final destination.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Units</th>
<th>Description</th>
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<tbody>
<tr>
<td>( A_{mli} )</td>
<td>-</td>
<td>Arrival time of train ( m ), station ( i ), line ( l )</td>
</tr>
<tr>
<td>( D_{mi} )</td>
<td>-</td>
<td>Departure time of train ( m ), station ( i ), line ( l )</td>
</tr>
<tr>
<td>( ij )</td>
<td>-</td>
<td>Indices of stations on metro network ((i, j \in N, N ) is the set of stations for metro network)</td>
</tr>
<tr>
<td>( l/l' )</td>
<td>-</td>
<td>Index of lines ((l, l' \in L, L ) is the set of lines)</td>
</tr>
<tr>
<td>( F_l )</td>
<td>trains/hr</td>
<td>Hourly train service frequency on line ( l )</td>
</tr>
<tr>
<td>( h_l )</td>
<td>hr</td>
<td>Train headway of line ( l )</td>
</tr>
<tr>
<td>( m/m' )</td>
<td>-</td>
<td>Index of pick-up/delivery trains on line ( l/l'(m=1,2,\ldots, F_l; m'=1,2,\ldots, F_l') )</td>
</tr>
<tr>
<td>( R_{ij} )</td>
<td>pass/hr</td>
<td>Hourly passengers from station ( i ) to ( j ) who transfer at station ( s )</td>
</tr>
<tr>
<td>( r_{ij} )</td>
<td>pass/train</td>
<td>Average number of passengers per train of line ( l' ) from station ( i ) to ( j ) who transfer at station ( s )</td>
</tr>
<tr>
<td>( s )</td>
<td>-</td>
<td>Index of transfer stations ((s \in S, S ) is the set of transfer stations)</td>
</tr>
<tr>
<td>( t_{rs} )</td>
<td>hr</td>
<td>Transfer walking time from line ( l' ) to ( l ) at transfer station ( s )</td>
</tr>
<tr>
<td>( t_{ij} )</td>
<td>hr</td>
<td>Passenger travel time from station ( i ) to ( j ) via transfer station ( s )</td>
</tr>
<tr>
<td>( t_{Ron,l',ml,s} )</td>
<td>hr</td>
<td>Transfer waiting time from train ( m' ) of line ( l' ) to train ( m ) of line ( l ) at transfer station ( s )</td>
</tr>
<tr>
<td>( t_{Wi} )</td>
<td>hr</td>
<td>Average wait time at station ( i )</td>
</tr>
<tr>
<td>( t_{Vij} )</td>
<td>hr</td>
<td>In-vehicle time from station ( i ) to ( j ) through transfer station ( s )</td>
</tr>
<tr>
<td>( \alpha_W )</td>
<td>-</td>
<td>Ratio of the average wait time to service headway</td>
</tr>
<tr>
<td>( \beta_{ij} )</td>
<td>-</td>
<td>A binary variable. ( \beta_{ij} = 1 ) as transfer is needed from station ( i ) to ( j ); otherwise, 0</td>
</tr>
<tr>
<td>( \gamma_{ij} )</td>
<td>-</td>
<td>A binary variable. ( \gamma_{ij} = 1 ) when passenger travel from station ( i ) to ( j ) via transfer station ( s ); otherwise, 0</td>
</tr>
<tr>
<td>( \lambda_{m; l',ml,s} )</td>
<td>-</td>
<td>A binary variable, ( \lambda_{m; l',ml,s} = 1 ) when passengers travel with train ( m' ) of line ( l' ) to train ( m ) of line ( l ) via transfer station ( s ); otherwise, 0</td>
</tr>
</tbody>
</table>

**Demand Characteristics**

*Transfer passenger*

The number of hourly passengers traveling from origin station \( i \) to destination \( j \) is denoted as \( Q_{ij} \). It is estimated using the data collected by AFCS. The passengers are classified into transfer and non-transfer passengers. Passengers accessing the origin stations are random, but transfer passengers arrivals follow the time of a delivery train arrival, we focus on minimizing total
transfer waiting time incurred by transfer passengers. A binary variable $\beta_{ij}$ is set to 0 if no transfer is needed for passenger from station $i$ to $j$; otherwise, is equal to 1.

$$\beta_{ij} = \begin{cases} 1; & \text{Transfer is needed} \\ 0; & \text{Transfer is not needed} \end{cases}$$ (1)

The hourly demand from station $i$ to $j$ with transfers at station $s$ is denoted as $R_{ijs}$. It is derived by

$$R_{ijs} = Q_{ij} \beta_{ij}$$ (2)

where $s$ can be determined by passenger travel choice. This will be described in the next section. Since the passengers’ arrivals at any origin station $i$ are assumed random, the average passengers per train from station $i$ to $j$ with transfer at station $s$, denoted as $r_{ijs}$, can be calculated as

$$r_{ijs} = \frac{R_{ijs}}{F_l}$$ (3)

The transfer occurs when a passenger takes train $m'$ of line $l'$ from origin station $i$ through transfer station $s$, to get on board of train $m$ of line $l$ to destination station $j$. Therefore, the origin station $i$ and destination station $j$ belong to line $l'$ and $l$, respectively. $F_l$ represents the hourly number of dispatched trains on line $l$. Hence, the number of average passengers transferring from line $l'$ to $l$ at transfer station $s$ per train, denoted as $r_{ijs}$, can be calculated by summing up average passengers from origin station $i$ of line $l'$ to destination station $j$ of line $l$ through transfer station $s$. The flow of transfer passengers at transfer station $s$ is shown in Figure 2. Thus,

$$r_{ijs} = \sum_{i=N_{i-1}+1}^{N_i} \sum_{j=N_{j}+1}^{N_j} r_{ijs}$$ (4)

Note that, the transfer station $s$ is the station $u_{l'}$ of line $l'$, meanwhile it is also the station $u_l$ of line $l$. If passengers cannot transfer from line $l'$ to $l$ at transfer station $s$, then $r_{ijs}$ is equal to 0.

![FIGURE 2 Flows of transfer passengers at a transfer station.](image-url)
Choice of a Transfer Station

Passengers are assumed to choose the shortest path to reach the destination. Thus, the travel path is determined by

$$\min \left\{ \gamma_{ijs} t_{ijs} \right\} \text{ for all } i, j \text{ and } s$$  \hspace{1cm} (5)

For instance, if a transfer passenger is able to board train $m'$ of line $l'$ at origin station $i$, arrive at station $s$ and then take train $m$ of line $l$ to destination station $j$, the transfer station choice, denoted as $\gamma_{ijs}$, is equal to 1. $\gamma_{ijs}$ is 0 if $s$ is not the transfer station which yielded the shortest travel time for this passenger. Thus,

$$\gamma_{ijs} = \begin{cases} 1; & \text{Choose transfer station } s \\ 0; & \text{Not choose transfer station } s \end{cases}$$  \hspace{1cm} (6)

The travel time $(t_{ijs})$ consists of wait, in-vehicle and transfer time. It is defined as

$$t_{ijs} = t_{wi} + t_{vjs} + t_{rmls}$$  \hspace{1cm} (7)

where the average wait time at origin station $i$, denoted as $t_{wi}$, is the average initial wait time, a fraction $\alpha_w$ of headway on line $l'$. The average initial wait time is assumed a fraction of the headway. Thus,

$$t_{wi} = \alpha_w h_l$$  \hspace{1cm} (8)

The in-vehicle time, denoted as $t_{vjs}$, includes the in-vehicle time from origin station $i$ of line $l'$ to transfer station $s$ and from transfer station $s$ to destination station $j$ of line $l$, which can be formulated as

$$t_{vjs} = (A_{m'l's} - D_{m'l'i}) + (A_{m'd} - D_{m'ds})$$  \hspace{1cm} (9)

where $A_{m,l,i}$ and $D_{m,l,i}$ represent the arrival and departure times of train $m$ on line $l$ at station $i$. The transfer time $t_{rmls}$ is determined by the departure time of pickup train $m$, the arrival time of delivery train $m'$ transfer walking time from line $l'$ to $l$ and transfer connection availability between train $m'$ of line $l'$ and train $m$ of line $l$. Thus,

$$t_{rmls} = (D_{mls} - A_{m'l's} - t_{rs})\lambda_{m'l'mls}$$  \hspace{1cm} (10)

where transfer walking time $(t_{rs})$ includes the time of a passenger getting off a delivery train $m'$ of line $l'$ and walking to the boarding area of pickup train $m$ of line $l$. The average transfer walking time at transfer station $s$ between any two lines is obtained from a survey, it is fixed assuming walking speed of all passengers are the same. For the binary variables $\lambda_{m'l'lmls}$, the value of 1 means that train $m'$ of line $l'$ arrives early enough that passenger can transfer to train $m$ of line $l$ at station $s$; otherwise, it is 0. Thus,

$$\lambda_{m'l'lmls} = \begin{cases} 1; & D_{mls} \geq A_{m'l's} + t_{rs} \text{ Train m' and m are connected} \\ 0; & D_{mls} < A_{m'l's} + t_{rs} \text{ Train m' and m are not connected} \end{cases}$$  \hspace{1cm} (11)

Note that, there are only one train $m$ of line $l$ that has connectivity with train $m'$ of line $l'$. The travel path for any passengers can be determined by Eq.5.
**Objective Function**

The objective is to minimize the total waiting time of transfer passengers, which is denoted as $T_R$. It is a sum of the waiting time at transfer stations for all transfer passengers. It is equal to the number of passengers from train $m'$ of line $l'$ to train $m$ of line $l$ via transfer station $s$ ($r_{l's}$) multiplied by the average transfer waiting time denoted as $t_{Rm'l'm,s}$.

$$T_R = \sum_{x=1}^{X} \sum_{l=1}^{L} \sum_{m=1}^{M} \sum_{l'=1}^{L} \sum_{m'=1}^{M} r_{l's} t_{Rm'l'm,s}$$  \hspace{1cm} (12)

where $r_{l's}$ and $t_{Rm'l'm,s}$ can be determined by Eqs. 4 and 10.

**SOLUTION METHOD**

In addition to developing a model, the key component of this paper is to develop an effective procedure to reach an optimal solution, namely the minimum total transfer waiting time. For a metro network consists of multiple lines intersecting at many stations, the hourly frequency of line $l$ is denoted as $F_l$. Therefore, the optimization of timetable for a large scale metro network contains large solution spaces, becoming a large combinatorial optimization problem.

In previous studies, the Heuristic Algorithm (HA) and Genetic Algorithm (GA) had been widely applied to search for an optimal solution. Compared with HA and GA, SA is easier to find the global optimum. Simulated Annealing (SA) method has been regarded as an effective algorithm to produce a global optimal solution within a reasonable computing time (25-31). It has been used in transit route network design (30) and train timetable optimization for a subway system (31).

An efficient solution algorithm based on SA is developed. A step-by-step procedure for optimizing coordinated operation over multiple lines in the metro network system is discussed below. It is as follows:

**Step 1:** Input the original timetable of line $l$ ($l \in L$) and the initial dispatching time of train $m$ on line $l$ at start terminal, which is defined as $D^{(0)}_{m1}$ and collected in the matrix $X^{(0)}$.

$$X^{(0)} = (D_{1,1,1}, D_{1,2,1}, ..., D_{F_1,1,1}, D_{2,2,1}, ..., D_{F_1,2,2}, D_{1,1,2}, D_{2,2,2}, ..., D_{F_1,2,2}, 1, 1, 1)$$  \hspace{1cm} (13)

**Step 2:** Adjusting $D_{m1}$ with $v_i$, so the adjusted dispatching time can be derived as

$$D_{m1} = D^{(0)}_{m1} + v_i$$  \hspace{1cm} (14)

where $v_i$, defined as $\{v_1, v_2, v_3, ..., v_L\}$ representing allowable range for schedule justification, is

$$-\frac{h_j}{2} < v_i < \frac{h_j}{2}$$  \hspace{1cm} (15)

**Step 3:** Apply SA (See Figure 3) to search for the optimal $v^*_i$. The flow chart of SA is illustrated in Figure 3. The steps of SA include:

**Step 3-1:** Initialize SA and input the variables and parameters for the developed model.

**Step 3-2:** Generate the initial solution and calculate the objective function $f_0$. Let $f_i = f_0$.

**Step 3-3:** Generate a new solution by using the solution from previous iteration, and calculate the objective function $f_j$. Let $u = u + 1$.
Step 3-4: Compare object values of last two iterations $f_i$ and $f_j$. If $f_j \geq f_i$, go to step 3-5; Otherwise, let $f_i = f_j$ and denote the current solution is the best solution, then go to step 3-6.

Step 3-5: Verify the Metropolis criterion. Calculate $\Delta f_{ij} = f_j - f_i$. If $\exp(-\Delta f_{ij}/t_i) > e$ ($e$ is a random number, $e \in (0, 1)$), let $f_i = f_j$; Otherwise, go to step 3-6.

Step 3-6: Reduction criteria of temperature. If $u \geq U$, $t_{i+1} = at_i$, $u = 0$, go to step 3-7; Otherwise, go to step 3-3.

Step 3-7: Check if the stop criteria (e.g. current temperature $t_{i+1} \leq t_f$ or maximum iteration $I$) is satisfied. If yes go to Step 4; otherwise, go to Step 3-3.

Step 4: Terminate SA search and output the best solutions $v^*$, $D_{ml}^*$ and $T_{R}^*$ . Update the arrival and departure time at each station of timetable according to the calculation based on Eqs. 16 and 17.

\[
D_{ml}^* = D_{ml}^{(0)} + v_i^* 
\]  
\[
A_{ml}^* = A_{ml}^{(0)} + v_i^* 
\]

FIGURE 3 The Proposed Simulated Annealing Algorithm.

CASE STUDY
The model was applied to a metro network in a large metropolitan area of southern China. The network consists of five lines (designated as lines A, B, C, D, and E, respectively) with 13 transfer stations, as shown in Figure 4. It is worth noting that the five metro lines are operated by three different agencies, and thus the timetables of these lines are designed independently from each other. It is fair to say that the coordination among lines may not have been the primary concern when the overall network is concerned. Potential improvement existed for enhancing traveler experience by minimizing his/her travel time while not impacting negatively the productivity of
each line. Since each line provides bi-directional service (outbound and inbound), the network is represented by 10 directional lines with 262 stations. The total length of lines is 157 track-km for the network. The travel ridership comes from the AFCS. There are more than 2 million passengers in a typical weekday.

![FIGURE 4 The studied metro network.](image)

The distribution of passengers by time of day is shown in Figure 5. The figure shows that the AM and PM peak hours are 8:00 am to 9:00 am and 6:00 pm to 7:00 pm, respectively. The regular operation hours each day is from 6:30 am to 11:00pm. Since the AM peak demand is greater than the PM’s, our focus was on adjusting trains that serve the AM peak. The input parameters needed by this optimization, including OD demand matrix, original dispatching timetables, train timetables, transfer walking time (It has been observed via a survey.), and other operator parameters, are summarized in Table 2. The total ridership for the whole metro network in the AM peak is 165,080 pass/hour; nearly 45% of the riders need a transfer to reach their destination station.
FIGURE 5 The ridership distribution over time on a typical weekday.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Units</th>
<th>Baseline Values</th>
<th>Variables</th>
<th>Units</th>
<th>Baseline Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1 / h_2$</td>
<td>min</td>
<td>3/3</td>
<td>$F_1 / F_2$</td>
<td>Trains/hr</td>
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<tr>
<td>$h_3 / h_4$</td>
<td>min</td>
<td>6/6</td>
<td>$F_3 / F_4$</td>
<td>Trains/hr</td>
<td>10/10</td>
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<tr>
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<td>7.5/6.7</td>
<td>$F_5 / F_6$</td>
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<td>3.53/3.15</td>
<td>$F_7 / F_8$</td>
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<td>$h_9 / h_{10}$</td>
<td>min</td>
<td>6/6</td>
<td>$F_9 / F_{10}$</td>
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<td>4.5</td>
</tr>
<tr>
<td>$t_{r7}$</td>
<td>min</td>
<td>2.5</td>
<td>$a_w$</td>
<td>-</td>
<td>0.5</td>
</tr>
</tbody>
</table>

SA Algorithm Computational Issues
The SA algorithm is programmed within MATLAB. The parameter values of the SA algorithm are shown as follows: initial temperature $t_0=10000$, reduction parameter of temperature $\alpha=0.9$, number of iteration $I=1000$, terminate temperature $t_f=1$, number of iteration at current temperature $U=100$. The computation time of proposed model was around 2 hours to find the optimal solution on a laptop (Intel (R) Core(TM) i5-3360M CPU @2.8GHz with 8 GB RAM and 64-bit Operating System). Figure 6 shows the value of objective function vs. iteration during the optimized process.
Figure 6 Total transfer waiting time vs. iteration.

Optimal Solution
The optimized scenario is compared with the original operation plan, in which the optimized dispatching time and minimized transfer waiting time are illustrated as in Table 3 and 4. The proposed model and simulated annealing are applied to the morning peak hour: 08:00-09:00 am. There are 133 trains dispatched during this period. The transfer waiting time of passengers is reduced by 20,516 minutes on total in morning peak hour. The generated dispatching time and timetables reduce the total transfer waiting time of passengers, although the average transfer waiting times are increased for some transfer stations. The optimized adjustment of dispatching time and the comparison of results with original operation are shown as below.

The optimized adjustment of dispatching time for metro line is shown in Table 3. The dispatching time of line 2, 3, 4, 7, 8 and 10 should be postponed, while line 1, 5, 6 and 9 should be dispatched ahead of schedule.

<table>
<thead>
<tr>
<th>Item</th>
<th>Units</th>
<th>Values</th>
<th>Item</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td>min</td>
<td>-1.4</td>
<td>$v_2$</td>
<td>min</td>
<td>1.4</td>
</tr>
<tr>
<td>$v_3$</td>
<td>min</td>
<td>3.0</td>
<td>$v_4$</td>
<td>min</td>
<td>1.7</td>
</tr>
<tr>
<td>$v_5$</td>
<td>min</td>
<td>-3.6</td>
<td>$v_6$</td>
<td>min</td>
<td>-2.0</td>
</tr>
<tr>
<td>$v_7$</td>
<td>min</td>
<td>0.8</td>
<td>$v_8$</td>
<td>min</td>
<td>1.6</td>
</tr>
<tr>
<td>$v_9$</td>
<td>min</td>
<td>-2.9</td>
<td>$v_{10}$</td>
<td>min</td>
<td>3.0</td>
</tr>
</tbody>
</table>

As indicated in Table 4, the total wait time of transfer passengers reduced 9.57% ($=20,516$ minutes/hr) after adjusting the dispatching times in the AM peak, albeit the transfer waiting times at transfer stations 6, 9, and 10 slightly increase.
TABLE 4 Comparison of transfer waiting time with original and optimal operation

<table>
<thead>
<tr>
<th>Transfer Station</th>
<th>Original Operation (min)</th>
<th>Optimized Solution (min)</th>
<th>Saved Time (min)</th>
<th>Percentage of Saved Time (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_1$</td>
<td>16,165</td>
<td>15,391</td>
<td>774</td>
<td>4.79</td>
</tr>
<tr>
<td>$s_2$</td>
<td>20,696</td>
<td>17,952</td>
<td>2,744</td>
<td>13.26</td>
</tr>
<tr>
<td>$s_3$</td>
<td>24,078</td>
<td>21,875</td>
<td>2,203</td>
<td>9.15</td>
</tr>
<tr>
<td>$s_4$</td>
<td>39,337</td>
<td>33,825</td>
<td>5,512</td>
<td>14.01</td>
</tr>
<tr>
<td>$s_5$</td>
<td>13,511</td>
<td>12,377</td>
<td>1,134</td>
<td>8.39</td>
</tr>
<tr>
<td>$s_6$</td>
<td>7,742</td>
<td>7,983</td>
<td>-241</td>
<td>-3.11</td>
</tr>
<tr>
<td>$s_7$</td>
<td>16,023</td>
<td>14,547</td>
<td>1,476</td>
<td>9.21</td>
</tr>
<tr>
<td>$s_8$</td>
<td>11,375</td>
<td>10,212</td>
<td>1,163</td>
<td>10.22</td>
</tr>
<tr>
<td>$s_9$</td>
<td>12,312</td>
<td>12,509</td>
<td>-197</td>
<td>-1.60</td>
</tr>
<tr>
<td>$s_{10}$</td>
<td>5,755</td>
<td>6,106</td>
<td>-351</td>
<td>-6.10</td>
</tr>
<tr>
<td>$s_{11}$</td>
<td>16,723</td>
<td>14,436</td>
<td>2,287</td>
<td>13.68</td>
</tr>
<tr>
<td>$s_{12}$</td>
<td>12,298</td>
<td>11,367</td>
<td>931</td>
<td>7.57</td>
</tr>
<tr>
<td>$s_{13}$</td>
<td>18,452</td>
<td>15,371</td>
<td>3,081</td>
<td>16.70</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>214,467</strong></td>
<td><strong>193,951</strong></td>
<td><strong>20,516</strong></td>
<td><strong>9.57</strong></td>
</tr>
</tbody>
</table>

Take Line 9 in morning peak hour as an example, the comparison between the original and adjusted dispatching time at the start terminal is shown in Figure 7. The schedule from the model has the same frequency as original operation and also provides passengers with constant and equal headways scheme. Since we only adjust the train dispatching time but not the headway, the safety spacing between trains has been ensured as usual. Hence, the optimized schedule would not cause operating difficulties and impact the operational complexity. There is no change required in headway for each line and no addition cost to operation is needed by adjusting the dispatching time at a start terminal. It should be mentioned that the effectiveness of the model would still depend upon the ability of the train operator to adhere to schedules.

FIGURE 7 Comparison of dispatching time at start terminal for Line 9.

CONCLUSION AND FUTURE RESEARCH

Efficient and safe transfer of passengers in hubs of a metro system is critical for daily functioning of the metro network. The coordination at transfer station can provide immediate and smooth
transfer for passengers. This paper has proposed a mathematic model to coordinate transfer in a metro network. The results show that a substantial reduction in travel time for transferring passengers is possible. The savings can transfer into work and labor productivity as the workers can arrive to work less tired and stressed compared to the original baseline operation. There is no change required in headway for each line and no increase of operator cost is needed by adjusting the dispatching time at the start terminal. To formulate model, original operation including dispatching time and timetables were taken into consideration. The proposed SA-based approach was applied to search the optimal solution and able to solve the transfer coordination problem. The numerical results indicated transfer waiting time saving significantly. Since the optimizing solution will not impact the operation complexity, thus, it is facilitated to be implemented in reality for operation agency. The model and approach can be applied seamlessly for many metro systems around the world.

The future study will involve the following investigations. The assumption of random passenger arrivals at origin stations can be relaxed to follow a more realistic distribution. Also, the passengers who require more than one transfer should be considered in the extended model. Train capacity constraint during peak period with trains arriving at the transfer station with high loads should be further analyzed.
REFERENCES


17. Chowdhury, M. S., and Chien, S. Optimization of transfer coordination for intermodal transit


