MULTI-OBJECTIVE TRANSIT FREQUENCY OPTIMIZATION: SOLUTION METHOD AND ITS APPLICATION TO A MEDIUM-SIZED CITY

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Multi-objective transit frequency optimization: solution method and its application to a medium-sized city

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ABSTRACT

Frequency setting takes place at the strategic and tactical planning stages of public transportation systems. The problem consists in determining the time interval between subsequent vehicles for a given set of lines, taking into account interests of users and operators. The result of this stage is considered as input at the operational level. In general, the problem faced by planners is how to distribute a given fleet of buses among a set of given lines. The corresponding decisions determine the frequency of each line, which impacts directly on the waiting time of the users and operator costs. In this work, we consider frequency setting as the problem of minimizing simultaneously users’ total travel time and fleet size, which represents the interest of operators. There is a trade-off between these two measures; therefore we face a multi-objective problem. We extend an existing single-objective formulation to account explicitly for this trade-off, and propose a Tabu Search solving method to handle efficiently this multi-objective variant of the problem. The proposed methodology is then applied to a real medium-sized problem instance, using data of Puerto-Montt, Chile. We consider two data sets corresponding to morning-peak and off-peak periods. The results obtained show that the proposed methodology is able to improve the current solution in terms of total travel time and fleet size. In addition, the proposed method is able to efficiently suggest (in computational terms) different trade-off solutions regarding the conflicting objectives of users and operators.

KEYWORDS: transit frequency optimization; multi-objective optimization; tabu search; case study
1. INTRODUCTION

Transit service design entails making decisions concerning several aspects of public transportation systems, namely, line design, timetable construction, fleet/crew assignment and fare determination, among others. Ideally, these decisions should be taken optimally, in the sense of the interests of the whole society. In this work we focus on the task of frequency setting, known as the problem of determining the time interval between subsequent buses on each line of a public transportation system. This problem takes part in both strategic and tactical planning; in the former case, as part of the transit network design, while for the latter, as a way to adjust the services to variations in the demand or the route network (2, 11). In these cases, the main decision maker is a centralized planning and regulating entity. The output of the frequency setting is taken as input to create the operational plans for each line, including e.g. the timetable construction. Here, decisions are usually made by individual operators, which are in charge of running a subset of lines from the whole system.

When considered as an optimization problem, frequency setting should take into account the interests of the main actors involved in the system: users and operators. At the strategic and tactical levels, the planner is represented by a transit agency that is in charge of taking care of the interests of both actors. The existing optimization models usually consider as objective function the maximization of the level of service offered to the users; operators’ interest are included as a constraint stating the maximum level of resources, which allow a profitable operation. Problem data is given by the itinerary of each line and origin-destination (OD) demand within a specific time horizon. An important component of the model is the assignment sub-model, which represents the behavior of the users with respect to a set of lines and frequencies. This sub-model is needed to measure the performance of the system with respect to the users, i.e. the level of service.

The literature concerning transit frequency optimization can be classified into (a) analytical models which admit closed form solutions and (b) mathematical programming formulations either explicit or not, with associated solution algorithms. In the first group, there are formulations that characterize the system in terms of few variables and allow getting a full description of the optimal solution. Despite these models make considerable simplifications of the real system, they allow obtaining practical guidelines which are theoretically well founded, for example the well-known rule of the square root (9, 12). The other important stream of work is based on a detailed characterization of the transit system, in terms of the route network and the demand that should be transported over it. These studies formulate the optimization problem in terms of a graph model, where decision variables are the capacities of the arcs (represented by the frequencies) and the flows which represent the OD-demand that is governed by the assignment sub-model. In the most general case, the objective function seeks to minimize the total travel time, while a maximum fleet size constraint is the main factor which bounds the increase of the frequency, which is desirable from the users’ viewpoint since it reduces the waiting time. Some representative works can be found in (1, 8, 13) and recently in (16). The demand is usually given as a fixed OD-matrix, elastic demand is considered in (16). The existing models usually have real variables which represent line frequencies and passenger flows. In most cases, the mathematical formulations are non-linear, due to the relationships between the frequency with the waiting time and the passenger flows (13). Moreover, the representation of the passenger behavior makes difficult to state an explicit formulation, which precludes the identification of the model structure and therefore the determination of an effective solution method (8). In addition, the realistic modeling of bus capacity and congestion introduces even more difficulties due to bilevel and/or equilibrium formulations (1, 5). Therefore, all solution methods rely on heuristic algorithms either driven by mathematical formulations (1) or purely heuristics (17). These methods have been tested with different cases, including real ones comprising up to 100 lines approximately.

In this work, we contribute in the context of transit frequency optimization on the following specific directions: (a) we present an extension and improvement from our previous single-objective
model and Tabu Search solution method \((10)\), conceived to solve efficiently the multi-objective variant of the problem, and (b) an application to a real case concerning a medium-sized city.

Concerning the first contribution, we propose an extension of our previous formulation and solution method to handle a multi-objective variant of the transit frequency optimization problem, considering the conflicting objectives of users and operators. This approach is aimed to guide the decision maker in the exploration of alternatives representing different levels of service and delivered resources.

Concerning the case study, we use data from the city of Puerto Montt, Chile. This is a medium-sized city of 230,000 inhabitants approximately, with a transit system comprising 20 lines. We use data provided by SECTRA (the transportation planning agency of Chile), which allow building very realistic scenarios since the corresponding information is systematically updated and validated. We use morning-peak and off-peak data, which is not usual in studies concerning transit frequency optimization.

The article is structured as follows. Section 2 provides a description of the methodology, including both model and solution algorithm. In Section 3, we describe the main characteristics of the case study, present the experiments performed and discuss their results. Finally, Section 4 draws some conclusions and lines for future research.

2. METHODOLOGY

The setting of transit frequencies is modeled as a combinatorial optimization problem, where we must assign a frequency value (taken from a given discrete set) to every line given as input data. In the single-objective variant of the problem, any setting of frequencies must respect an upper limit on the available fleet size, while the objective is the minimization of the overall travel time of the users. Each solution is evaluated according to an assignment sub-model, in order to compute its objective value. In \((10)\), this problem is formulated as a mixed integer linear programming (MILP) one, which can be solved to optimality for small-sized instances. For larger instances, a Tabu Search metaheuristic was proposed and implemented and it is the base methodology applied in this work. As an extension of the existing methodology, we propose a variation of the heuristic which solves efficiently a multi-objective version of the problem, considering the conflicting objectives of users and operators.

In the following, we explain the representation of the transit system and the relevant aspects of the assignment sub-model. Then, we formally state the transit frequency optimization problem and the main concepts of the metaheuristic approach used to solve it. Sections 2.1 to 2.3 are mainly taken from our previous paper, see \((10)\). Finally, we present a more detailed description of the extension proposed in this work.

2.1 Transit System and Passenger Behavior

We model the underlying structure of the transit system as a directed graph \(G = (N, A)\). The set of nodes \(N\) represents either bus stops, endpoints of street segments or zone centroids (fictitious points which concentrate the demand). The set of arcs \(A\) represents the bus movements (along street segments) and walking paths (between stop nodes or to/from centroids); special types of arcs represent boarding and alighting of passengers to/from the bus. A transit line is composed by its forward and backward routes or by a single route if it is circular. Each route is a sequence of contiguous arcs in \(G\). Figure 1 illustrates this structure. Moreover, the demand is represented as an origin-destination (OD) matrix, where each non-null entry (called OD pair) has origin and destination centroids (nodes of \(G\)) and an amount of trips per time unit within a specific time-horizon.
The assignment sub-model takes as input the graph representing the transit lines with a setting of frequencies and the OD demand. Its output is a distribution of demand flows representing trajectories between origins and destinations, obtained by applying the hypothesis about the passenger behavior with respect to the given lines and frequencies. We adopt the optimal strategies assignment sub-model (14). A strategy is defined as a set of rules that when applied, enables the user to reach his destination. In terms of the graph $G$ and for a given OD pair, a strategy can be seen as a subset of arcs in $A$ which represents all the lines that the user identifies a priori, for travelling from its origin to its destination. The model assumes that a given user selects the strategy that minimizes his total travel time. To do this, he will select a priori (i.e., before leaving the place where the trip is originated) a set of attractive lines among all the possible lines that connect its origin and destination bus stops (even including transfers). While waiting at the bus stop, the user will take the first bus passing by that stop, belonging to the set of attractive lines determined a priori. An application of the abovementioned model for a single OD pair over a graph $G$ computes:

- The distribution of the corresponding demand, as an assignment of flows $\nu_a$ for each arc $a \in A$.
- The waiting time $WT_n$ at each stop node $n$, calculated as $WT_n = 1 / \sum_{l \in L(n)} f_l$ where $L(n)$ is the set of lines passing by $n$, corresponding to the attractive set identified a priori by the user and $f_l$ is the frequency (buses per time unit) of line $l$.

As a consequence, the result of the assignment sub-model enables to compute the total travel time of the system, which is a performance measure of such a system from the point of view of the users. Note that both flows and waiting times depend on the frequencies, which are decision variables of our problem. Thus, the assignment sub-model must be applied to every new setting of frequencies.

### 2.2 Multi-Objective Optimization Problem

We start from the single-objective MILP formulation proposed in (10) and we consider its multi-objective variant, resulting from casting the fleet size constraint into a conflictive objective with respect to the original objective function. According to (3), the solution of (1)-(8) is not a single optimal one, instead, it is a set of non-dominated solutions in the space of objectives (1) and (2). This set is usually referred as Pareto front. There are different alternatives for arriving to a single solution of a multi-objective problem, namely, a priori, a posteriori and interactive methods (4).

The a priori method fixes the parameters that determine a concrete trade-off between the conflictive objectives, and then it solves a single-objective problem. The a posteriori method finds the complete Pareto front, and then selects a single solution from it. The interactive method incorporates into the optimization method, the information which guides the selection of the desired trade-off level. In every case, that information is an additional input, usually provided by the decision maker.

Table 1 summarizes the notation used in formulation (1)-(8), which assumes a given set of lines $L$ and a predetermined set of frequencies $\Theta = \{\theta_1, ..., \theta_m\}$, which is the discrete domain of values that can be assigned to each line. Constraint (3) states that exactly one value of $\theta$ is assigned to each line, while constraint (4) is a conservation flow expression which ensures the demand is routed from origin to destination. Constraint (5) is a flow-splitting expression coming from the optimal strategies assignment sub-model, while constraint (6) states that demand can flow only through...
enabled arcs. Finally, constraint (7) states the non-negative nature of flow values and constraint (8) states the binary nature of variables which indicate the setting of frequencies. For details concerning the reasoning and justification of this formulation, we refer to (10).

\[
\begin{align*}
\min_{y,v,w} & \quad \sum_{k \in K} (\sum_{a \in A} m_{ak} y_{ak} + \sum_{n \in N} w_{nk}) \\
\min_{y,v,w} & \quad \sum_{l \in L} \sum_{f = 1..m} \theta_{lf} y_{lf} \sum_{a \in A} c_a \\
\text{s.t.} & \quad \sum_{f = 1..m} y_{lf} = 1 \quad \forall l \in L, \\
& \quad \sum_{a \in \text{out}(n)} v_{ak} - \sum_{a \in \text{in}(n)} v_{ak} = b_{nk} \quad \forall n \in N, k \in K, \\
& \quad v_{ak} \leq \theta_{lf} w_{nk} \quad \forall n \in N^p, a \in \text{out}(n), k \in K, \\
& \quad v_{ak} \leq \delta_k y_{lf}(a) \quad \forall a \in A^l, k \in K, \\
& \quad v_{ak} \geq 0 \quad \forall a \in A, k \in K, \\
& \quad y_{lf} \in \{0,1\} \quad \forall l \in L, f \in 1..m.
\end{align*}
\]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A (A^p))</td>
<td>Set of arcs (boarding arcs) with generic element (a)</td>
</tr>
<tr>
<td>(N (N^p))</td>
<td>Set of nodes (stop nodes) with generic element (n)</td>
</tr>
<tr>
<td>(K)</td>
<td>Set of OD-pairs with generic element (k)</td>
</tr>
<tr>
<td>(L)</td>
<td>Set of lines with generic element (l)</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>Set of frequencies indexed by (f) in the range (1..m)</td>
</tr>
<tr>
<td>(\text{out}(n), \text{in}(n))</td>
<td>Set of outgoing and incoming arcs of node (n), respectively</td>
</tr>
<tr>
<td>(\delta_k)</td>
<td>Amount of demand corresponding to OD-pair (k)</td>
</tr>
<tr>
<td>(b_{nk})</td>
<td>Constant value equal to 1 (-1) if node (n) is the origin (destination) of OD-pair (k) and 0 otherwise</td>
</tr>
<tr>
<td>(c_a)</td>
<td>Cost of arc (a)</td>
</tr>
<tr>
<td>(v_{ak})</td>
<td>Flow of OD-pair (k) over arc (a)</td>
</tr>
<tr>
<td>(w_{nk})</td>
<td>Waiting time multiplied by flow of OD-pair (k) in node (n)</td>
</tr>
<tr>
<td>(y_{lf})</td>
<td>Equal to 1 if frequency (f) is assigned to line (l)</td>
</tr>
<tr>
<td>(l(a), f(a))</td>
<td>Line and frequency corresponding to arc (a), respectively</td>
</tr>
</tbody>
</table>

### 2.3 Metaheuristic Solution Method

The combinatorial aspect of the problem denoted by (1)-(8) is approached through a metaheuristic solution method based on Tabu Search (6). The original implementation (10) performs a local search on the discrete domain determined by \(\Theta^L\) looking for the best possible values of \(y\) in the context of the single-objective problem. The search advances according to a compound move (which defines a neighborhood structure), that changes the frequencies of two lines in \(L\): one decrease and one increase. A line can change its frequency only to a value which is contiguous in \(\Theta\) to its current frequency. Thus, the compound move can be seen as a redistribution of the available buses among the lines of the system. But this intuitive move has the challenge of avoid getting trapped in local optima. Therefore, we apply tabu concepts to the basic local search performed through compound increase and decrease moves. A tabu list records the last iteration of the search where each line has increased or decreased its frequency. These tabu active statuses expire once a number of iterations is reached or when there is a reason to generate more valid moves (aspiration criteria). Moreover, we should note that the evaluation of each new solution entails an invocation to the assignment sub-model, which may slow down the performance of the overall algorithm when it is invoked repeteadly. Taking into account these observations, we implemented the aspiration plus strategy (7) which explores a reasonable number of neighbors (settings of frequencies) considering that too few may constrain the search and too much may slow down the process.
2.4 Multi-Objective Solution Method

The Tabu Search explained in Section 2.3 uses a short-term memory to record moves which should be avoided in subsequent iterations. In this work, we introduce an extension by introducing a long-term memory (7). This type of memory is expected to bring the opportunity to the algorithm of learning from its achievements in a wider horizon. Thus, the long-term memory can enhance the Tabu Search by exploring solutions not ordinarily found, and by intensifying the search in regions of promising solutions. Let define \(sol_1 (sol_m)\) as the solution in which every line frequency has value \(\theta_1 (\theta_m)\). Note that \(sol_1 (sol_m)\) has the lowest (highest) fleet size and the highest (lowest) travel time. Also, note that we can reach \(sol_1 (sol_m)\) by applying a finite number of decrease (increase) operations to any solution \(s\) and vice-versa.

We consider an oscillation boundary determined by the fleet size expression (2). By using exclusively operations increase and decrease, we may potentially explore the whole solution space between \(sol_1\) and \(sol_m\), which trajectory may cross the oscillation boundary several times. Whenever the boundary is reached, an intensification phase is applied, by staying in the corresponding fleet size level until a given number of iterations has passed without improvement. At each iteration, the short-term memory strategies are applied. We also maintain a critical event memory which records the history of the most recent critical events (crossings of the oscillatory boundary) that happened over the search. The critical events corresponding to the first feasible solution found and every new best solution, are recorded in this memory. Thus, we have a measure of the most recent and frequent values of the frequencies of the critical events. Every time \(sol_1\) or \(sol_m\) has been reached, we bias the search to take the least frequent solutions using this memory, by penalizing the path to the most frequent solutions over a defined number of iterations. We call this variation as long-term memory Tabu Search.

Using the algorithmic component described above, we handle the multi-objective problem by following these steps:

1. Find a set of initial solutions using the long-term memory Tabu Search.
2. Construct some statistic solutions from the initial solutions.
3. Find new solutions by searching the path between the solutions from steps 1 and 2.
4. With all the solutions found in previous steps, select the non dominated ones.

To find the initial solutions we extend the long-term memory Tabu Search to manage multiple critical levels (equal to the number of different frequencies: \(m\)). As initial solution for level \(i\), we select the one having all frequencies equal to \(\theta_i\) (note that the only solution for level 1 is \(sol_1\) and for level \(m\) is \(sol_m\)). After the execution of the long-term memory Tabu Search we obtain a set of \(m\) best solutions, one for each level (including \(sol_1\) and \(sol_m\)). To construct the statistic solutions we use the mean, mode and median of the best solutions of each level found in the previous step. At step 3 we build paths between the solutions of contiguous levels obtained in the steps 1 and 2. Starting from the solution of level 1 we search towards the solution of level 2, then towards the solution of level 3 and so on, until level \(m\) (upward path). Similarly, we search in a downward path (beginning at level \(m\)). At each step of both paths, an exhaustive search of all the neighbors is done to select the best one. Because of the penalty in the objective function and the fleet size upper limit is set to the fleet size of the current solution, both paths differs. The paths are explored for each type of solution one at a time: best, mean, mode and median. A simple filtering process is performed in step 4.

Notice that the algorithm explained above produces an entire set of non-dominated solutions (which is an approximation to the optimal Pareto front of problem (1)-(8)) in a single run. This is particularly relevant from the computational point of view, since it does not imply solving a single-objective problem several times. Thus, it is a multi-objective metaheuristic (4).

3. EXPERIMENTS AND RESULTS

We tested the methodology proposed in this work, using a real case corresponding to the city of Puerto Montt, Chile. The city has a public transportation system comprising 20 bus lines, each one
having forward and backward itineraries, with headways (inverse of frequency) in the range $[1.5, 16.1]$ minutes. Data were provided by SECTRA, the transportation planning agency of Chile. The underlying graph $G$ comprises 733 nodes and 1662 arcs, including 70 zone centroids and corresponding access (walk) links. Figure 2 shows the bus network of the city. We consider two scenarios corresponding to peak and off-peak conditions, where the first one comprises 3780 OD-pairs and 8595 trips/hour while the second one comprises 4335 OD-pairs and 3802 trips/hour. As we can expect, the peak scenario entails more trips, concentrated in fewer OD-pairs.

Figure 2 The bus network of Puerto Montt.

According to the aims of our proposed methodology, we performed a set of validating experiments using the real case study of Puerto Montt. We configured the predetermined set of frequencies as $\Theta = \{1/20, 1/12, 1/6, 1/4, 1/3\}$ (values expressed in 1/minute), which are representative values with respect to the current frequencies of the real system. Notice that there are many different values of frequencies in the current solution, so we restricted the predetermined set to a manageable size, since the execution time of the Tabu Search algorithm grows in direct proportion to this size. Figure 3 plots the objective values (total travel time and fleet size) of the obtained results for the peak and off-peak scenarios, according to the following references:

- **Real**: current solution of the city, evaluated using exactly its frequencies.
- **Case**: current solution, evaluated using the frequencies of $\Theta$ which are more similar to the real ones.
- **SOTS**: solution of the short-term memory Tabu Search for the single-objective problem, proposed and implemented in (10).
- **MOTS**: solutions of the multi-objective Tabu Search proposed and implemented in this work.

Note that solutions of MOTS are intended to provide a picture of the different trade-off levels between objectives of users and operators, while SOTS solutions seek to improve the current system (in terms of total travel time) for a fixed value of fleet size. Concerning the obtained non-dominated sets, we can observe that the slopes of both curves are similar for high values of fleet size. This means that an increase in the fleet size (which represents an increase in the resources delivered to the transit system) does not have a strong impact in the level of service represented by the total travel time. For medium and low values of fleet size, the impact of an increase on this factor has a stronger positive impact on travel time. Note that in this analysis (and in the whole study), we are considering inelastic demand with respect to the frequencies. Moreover, we are not taking into account the effects of the bus capacity neither on the passenger behavior nor in the computation of the system performance.
Concerning the performance of the methodologies, Table 2 reports the percentage of improvement in total travel time for a fixed fleet size, with respect to the current solution (the Case solution, evaluated using the same discrete domain of frequencies used in the optimization methods). Also, we report the number of solutions found by each algorithm as well as their execution times in seconds (in a Core i5 computer). Regarding improvements in total travel time, we observe small values, which in fact are consistent with values reported in the literature for this problem (1, 10).

In terms of computational efficiency, a direct comparison of execution times throws a reduction of 70% for the algorithm proposed in this work, with respect to its original variant. Furthermore, given that a single execution of MOTS produces an entire Pareto front which includes the solution delivered by SOTS (as it can be seen in Figure 3), the relative efficiency of the multi-objective metaheuristic is much greater. Note that if we want to obtain different trade-off solutions...
using the SOTM algorithm, we have to perform different runs, for different values of maximum fleet size. Moreover, given that the SOTM algorithm has proven to be capable of finding solutions very close to the global optimum for the single-objective problem \((10)\), we may expect that solutions found for Puerto Montt are also close to the optimal ones.

TABLE 2 Performance of the Methodologies

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Solution method</th>
<th>% Improve</th>
<th># Solutions</th>
<th>Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>SOTS</td>
<td>1.94</td>
<td>1</td>
<td>4223</td>
</tr>
<tr>
<td></td>
<td>MOTS</td>
<td>na</td>
<td>241</td>
<td>1235</td>
</tr>
<tr>
<td>Off-peak</td>
<td>SOTS</td>
<td>4.07</td>
<td>1</td>
<td>4565</td>
</tr>
<tr>
<td></td>
<td>MOTS</td>
<td>na</td>
<td>266</td>
<td>1227</td>
</tr>
</tbody>
</table>

NOTE: na = not applicable.

Finally, Table 3 shows the frequencies (index in discrete set \(\Theta\)) of the current solution and the optimized one for peak and off-peak scenarios. We reported values for both forward and backward itineraries for each one of the 20 lines, in order to be consistent with the current solution, which has this characteristic. Note that a small index means a low frequency while a large index means a high frequency. We have marked the decreased frequencies (the optimized ones with respect to the current ones) with light grey, and the increased frequencies with dark grey. We can observe that in general terms, the changes suggested by the optimization method are consistent between both scenarios, but the specific suggested (optimized) values are different. These results also contribute to validate the proposed methodology as well it shows the relevance of performing the study for different demand scenarios.

TABLE 3 Current and Optimized Frequencies

<table>
<thead>
<tr>
<th>Line</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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</thead>
<tbody>
<tr>
<td>peak</td>
<td>current</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
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<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>optimized</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>1</td>
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<td>5</td>
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<tr>
<td>off-peak</td>
<td>current</td>
<td>4</td>
<td>3</td>
<td>3</td>
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<td>4</td>
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<tr>
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<td>optimized</td>
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<td>3</td>
<td>2</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>1</td>
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</table>

4. CONCLUSIONS

In this work, we have proposed a new method for optimizing transit frequencies. We considered a multi-objective variant of an existing model and we have extended its solution method. Both model and algorithm proposed in this work, take advantage of the multi-objective nature of the problem, to propose a computationally efficient solving method. Although the proposed algorithm is heuristic, its accuracy is inferred from previous results which show its ability of producing near optimal solutions.

The proposed method solves a case corresponding to a real system in acceptable execution time. Special effort has been made in solving the multi-objective variant, which entails finding an entire set of non-dominated solutions. The computational efficiency is very relevant in a case such as Puerto Montt, in which there are both a dense network and a dense origin-destination matrix. In this case study, the proposed method improves current solutions by 2% to 4% approximately; these values are consistent with improvements found in the literature for transit frequency optimization.
The optimized frequencies are not the same for the peak and off-peak scenarios, which shows the relevance of performing the optimization for each scenario separately.

For further work, several issues concerning the realism of the model could be included. Taking into account congestion and demand elasticity would provide a more realistic modeling framework, in particular for this case study. These characteristics become more relevant in the peak scenario. Another interesting future study would be the application of transit network optimization methods to the case of Puerto Montt. Reported travel time improvements for medium-sized cities reach 30% (15), which is much higher than the improvements obtained in this work. Nevertheless, we have to mind that transit network optimization methods are more complex and their results usually are harder to implement in practice, given the potential disruptions they may cause in users due to system usability concerns.

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