Estimation of Optimal Inventory Levels at Stations of a Bicycle Sharing System

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ABSTRACT

Bicycle sharing systems are getting increasingly popular around the world as a sustainable and eco-friendly transportation mode which can address many urban transportation issues. However, the highly asymmetric usage of such systems leads to an imbalance in the system. The agencies try to balance the system by repositioning bicycles within the system. A very good tool for calculating the desired inventory levels based on the future usage of the system is essential for this activity and ensuring a minimum desired level-of-service to the users. This paper proposes a model for predicting the optimal inventory levels at the stations to take care of the future usage using the historic usage data of the system. The usage at the stations is modeled as a Markov process and the expected penalty values are calculated for the different starting inventory levels. A Mixed Integer Program (MIP) is developed to ensure that the number of bicycles in the system is conserved. The approach is tested for a bicycle sharing system in Antwerp, Velo Antwerpen. The computational times are found to be extremely reasonable. The results show that the levels-of-service can be improved and considerable savings can be obtained for the systems by reducing the resources used in the repositioning operation.
INTRODUCTION
A bicycle sharing system is a type of a bicycle rental system. A typical bicycle sharing system has stations spread throughout a city from where users can either pick up or drop off bicycles. There has been a lot of interest in such systems recently because of their rapid growth around the world. From the first system that opened in Amsterdam, Netherlands in 1965 till today, there have been many changes in the technology, the business model and the safety measures used in bicycle sharing systems (DeMaio (1)).

The increased interest in bicycle sharing systems is due to the numerous benefits that such systems have. The positive impacts of these systems include easing the congestion, reducing the pollution, and increasing the mobility and accessibility for the users (DeMaio, Shaheen et al., Shaheen and Guzman (1, 2, 3)). There are also many challenges that bicycle sharing systems face. One of the major challenges is the system imbalance issue (Raviv et al. (4)). As explained by Vogel and Mattfeld (5), the system gets imbalanced due to many reasons such as the high number of one way trips and a preference for particular stations for picking up or dropping off bicycles, among others. The agencies rebalance the system by operating vehicles to go around the system, pick up bicycles from some stations and drop them off at other stations. This operation is called bicycle repositioning. Castillo-Manzano and Sánchez-Braza (6) show that there is some self-rebalancing in the system by the users. The agencies may also provide incentives to the users to improve the status of the system. However, Fricker and Gast (7) claim that this kind of rebalancing operation is not enough for maintaining a good level-of-service in the system. Agency-controlled repositioning of bicycles in the system is necessary to ensure the satisfaction of the users.

An integral component of the repositioning operation is the process of calculating the desired inventory levels at the stations and hence, the number of bicycles to reposition within the system. This requires a very good approach that combines the usage forecast in the system with inventory management of the stations in the system (Raviv and Kolka (8)). The bicycle sharing systems usually record the data of the usage of the system. This paper proposes an approach of coming up with the desired inventory levels at the stations using the historic usage data in bicycle sharing systems.

In this work, the demand for bicycles or empty slots at a station is modeled as a Markov process. A penalty is associated with each user that does not get a bicycle to pick up or an empty slot to return a bicycle at the station. The expected penalty function is calculated as a function of the different starting inventory levels at the station. To ensure that the total number of bicycles in the system remains conserved, a Mixed Integer Program (MIP) is formulated with a constraint on the total number of bicycles in the system. The inventory levels at the stations that minimize the expected penalty of the system are calculated from this program. The key contribution of this research is a very practical and efficient approach to the inventory management problem using the available data. This approach may be used for static repositioning (SBRP) (Raviv and Kolka (8)) or dynamic repositioning (DBRP) (Raviv and Kolka, Benarbia et al. (8, 9)) depending on the system and the time of the operation.

The paper is organized as follows: Section 3 reviews the current work on forecasting methods and inventory management in bicycle sharing systems and related problems. Section 4 de-
scribes the problem mathematically and proposes the MIP model to solve the problem to optimality. Section 5 presents the numerical experiments and discusses the results. Section 6 concludes the work and discusses the recommended future research directions.

**LITERATURE REVIEW**

This section reviews the literature on the demand forecast in PBSSs and other related studies. The key points of the proposed work are also explained and contrasted with the existing approaches in the literature.

Bicycle repositioning is the operation in which the agency-operated vehicles move bicycles within the system to ensure the desired availability of bicycles and empty slots during the operating hours. This activity is essential in determining the success of the bicycle sharing system as it depends on how well the user demands are met (DeMaio, Hoye (1, 10)). There has been a lot of interest recently in the repositioning activities of bicycle sharing systems. Raviv et al., Benarbia et al., Benchimol et al., Chemla et al., Angeloudis et al., Rainer-Harbach et al., Dell’Amico et al. (4, 9, 11, 12, 13, 14, 15) proposed different models for the Static Bicycle Repositioning Problem. Contardo et al. (16) looked at the Dynamic Bicycle Repositioning Problem.

Barth and Todd (17) developed a simulation model for the demand of cars in a multiple station car sharing system. They calculated the optimal number of cars needed in the system as a function of the total trips made in the system to minimize different measures of performance such as the average customer waiting time and the number of cars repositioned. Caggiani and Ottomanelli (18) proposed a Neural Networks based approach for simulating the demand in a bicycle sharing system. They then used Fuzzy Logic concepts to make decisions regarding the repositioning operations.

Froehlich et al. (19) used the data from Bicing system in Barcelona to identify clusters of stations with similar usage patterns. They also developed 4 different models (Last Value, Historic Mean, Historic Trend and Bayesian Network) to predict the demand at the stations for different prediction windows. Kaltenbrunner et al. (20) used time-series analysis (Auto Regressive Moving Average) on the Bicing data to predict the future state of the stations.

Rudloff and Lackner (21) developed Poisson, Negative Binomial, and Hurdle models to predict the state of the stations considering various factors, including the weather conditions. Regue and Recker (22) proposed separate models using Linear Regression, Neural Networks, and Gradient Boosting Machines. They used the data from the Hubway system in Boston to test the performance of the models and the Gradient Boosting Machine model was found to outperform the other models.

Most of the works in the literature deal with the problem from a purely demand forecasting point-of-view. There have not been many studies which look at the inventory management of the system to come up with the desired inventory levels at the stations after the repositioning operation. Raviv and Kolka (8) modeled the demand as a Markov process. They calculated a convex User Dissatisfaction Function (UDF), and the upper and lower bounds of the penalty function for
each inventory level. They then come up with the optimal inventory levels for all the stations in the system separately. They compare the results with the results from a simulation model and the solution from the system experts of the Tel-O-Fun system in Tel Aviv, Israel. However, their model does not consider the overall inventory level of the system in deciding the optimal inventory levels. This might lead to cases where the total number of bicycles is not conserved in the system.

This research tries to address some of the gaps in the existing literature. The research can be classified into two parts: (1) The calculation of the expected penalties at the stations for different inventory levels. This is done using the historical data of the usage of the systems. The authors propose modeling the usage at the stations as a Markov Process for this purpose. This is a good way of modeling the usage since the change of the state of the station in a given time interval is independent of the state of the station in the previous time intervals. (2) The formulation of the MIP to get the optimal starting inventory levels for all the stations in the system. This takes care of the constraints such as the conservation of the total number of bicycles in the system before and after the repositioning operation and the capacity constraints of the station. There is a huge emphasis on the practicality in this approach. The proposed approach can be easily used by the bicycle sharing systems.

The following section defines the problem mathematically and presents the MIP model to solve it.

**PROBLEM STATEMENT AND METHODOLOGY**

This section presents the problem definition and the methodology adopted in the paper.

**Problem Definition and Notations**

A network of stations of a bicycle sharing system is given. The historical usage data of each station is given. This usage data includes the data about the number of bicycles picked up and returned at the stations at a specific time interval over a long period of time. For the purpose of this work, the data used is the net bicycles returned at the stations. The net returns is the difference between the number of bicycles returned and the number of bicycles withdrawn from the station at the particular time interval. The inventory levels at all the stations at the beginning of the operation are given and the goal is to compute the desired inventory levels by the end of the repositioning operation. All the stations are considered to be independent of each other. In case a user does not find a bicycle or an empty slot at a station, the user is assumed to not go to any other station for the purpose and a penalty is incurred.

Table 1 shows all the notations that are used in the penalty function calculation part of the paper. All the notations are for a specific station of the system.
A Markov process is a stochastic process in which the future state of the system depends only on the current state of the system. Consider a station of a bicycle sharing system with 10 bicycles at the beginning of the time interval \( T_i \). Depending on the time of the day, the state of the station at the end of the time interval \( T_i \) will change with some probability. This probability does not depend on what the inventory level of the station was at the time interval \( T_{i-1} \). For example, the probability that there will be 15 bicycles at the station at the end of the time interval \( T_i \) will be the same irrespective of whether there were 0 bicycles or 5 bicycles at the beginning of the previous time interval \( T_{i-1} \). Since the transition probabilities depend only on the current state of the station, a Markov process is an ideal way of modeling the usage at the station.

The state vector is a vector containing the probabilities of seeing the station in the possible states at the beginning of the time intervals. An example of a state vector is as follows.

\[
X_t = \begin{bmatrix}
0.35 \\
0.25 \\
0.4
\end{bmatrix}
\]

This means that at the beginning of the time interval \( t \), the station is expected to be in state 1 with a probability of 0.35, in state 2 with a probability of 0.25 and in state 3 with a probability of 0.4. The state vectors for all the time intervals are calculated using the initial state vector and the transition probability matrices for the station. The initial state vector is a vector with 1 as one of the elements and all remaining elements as 0. This is done to calculate the expected penalties for all the different starting inventory levels at the station.

The transition probability matrix is a matrix which contains the probabilities of transitioning from one state to all the other possible states in the time interval. An example of a transition probability matrix is as follows.

\[
P_t = \begin{bmatrix}
0.3 & 0.4 & 0.3 \\
0.2 & 0.5 & 0.3 \\
0.4 & 0.4 & 0.2
\end{bmatrix}
\]

The above transition probability matrix denotes that the probability of the station of re-
maining in state 1 after the time interval is 0.3, going from state 1 to state 2 is 0.4, going from state 
1 to state 3 is 0.3 and so on. The transition probability matrices are calculated from the historical 
usage data of the system. There will be different transition probability matrices for different time 
intervals of the day based on the usage data. The amount of data to use for calculating the transition 
probability matrices may be changed depending on the system and the need of the modeler.

The usage distribution vector $U_t$ of the station is a vector showing the usage of the station 
at the time interval $t$. $C$ is the number of lockers at the station and hence, the maximum number of 
net bicycles returned at the station. If the proportion of times that the net bicycles returned in the 
usage data is $-C$, $-(C - 1)$, $\ldots$, $-1$, $0$, $1$, $\ldots$, $C$ is found to be $y_1$, $y_2$, $\ldots$, $y_C$, $y_{C+1}$, $y_{C+2}$, $\ldots$, 
y_{2C+1}$ respectively, the usage distribution vector is constructed as follows.

$$U_t = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_C \\ y_{C+1} \\ y_{C+2} \\ \vdots \\ y_{2C+1} \end{bmatrix}$$

The usage dissatisfaction vector $\bar{U}_t$ of the station is a vector showing the probability of the 
different states of the station not satisfying the usage demand at the time interval $t$ based on $U_t$. If 
the usage distribution vector $U_t$ is given as above, the usage dissatisfaction vector is constructed as 
follows.

$$\bar{U}_t = \begin{bmatrix} y_1 + y_2 + \cdots + y_C \\ y_{2C+1} + y_1 + y_2 + \cdots + y_{C-1} \\ \vdots \\ y_{C+2} + y_{C+3} + \cdots + y_{2C+1} \end{bmatrix}$$

This means that if the station is in the first state (0 bicycles), then the probability of the 
station not satisfying the usage is the sum of the probabilities corresponding to the usage of $-C$, 
$-(C - 1)$, $\ldots$, and $-1$ in $U_t$ since there can not be more pick-ups than drop-offs at the station in 
that state. Similarly, if there is 1 bicycle in the station, the station would not be able to satisfy the 
usage corresponding to $-C$, $-(C - 1)$, $\ldots$, and $C$ in $U_t$ and so on. Thus, $\bar{U}_t$ is a vector with $C + 1$ 
entries, each for a different possible state of the station.

The analysis is done for each station separately. There is assumed to be no interaction 
among the stations. In reality however, there will be some interaction between the stations. For 
example, if a user is unable to pick up or return a bicycle at a station, there is some chance that the 
user will go to a nearby station. This is one limitation of the work which can be addressed in the 
future.

The state vector for the station under consideration for the beginning of the time interval $t$
is calculated from the state vector at the beginning of the time interval \( t - 1 \) using Equation 1.

\[
X_t = P_{t-1} \times X_{t-1}
\]  

(1)

From Equation 1, it can be shown that

\[
X_{t-1} = P_{t-2} \times X_{t-2}
\]

\[
X_{t-2} = P_{t-3} \times X_{t-3}
\]

\[
\vdots
\]

\[
X_1 = P_0 \times X_0
\]

Equation 2 can be derived from Equation 1. It gives the state vector for any general time interval \( t \) as a function of the initial state vector \( X_0 \) and the transition probability matrices \( P_0, P_1, \ldots, P_{t-1} \).

\[
X_t = \left( \prod_{i=0}^{t-1} P_i \right) \times X_0
\]

(2)

The expected penalty vector \( \delta_t \) for the time interval \( t \) is calculated from the state vectors obtained and the usage dissatisfaction vector by using Equation 3. The elements in the penalty vector are the expected penalties for the time interval for all the different starting inventory levels. The \( \odot \) symbol corresponds to the Hadamard product of the vectors, an element by element multiplication of the two vectors.

\[
\delta_t = X_t \odot U_t
\]

(3)

The expected penalty vector \( \Delta \) for the station is the sum of all the individual expected penalty vectors for all the time intervals, as shown in Equation 4.

\[
\Delta = \sum_{i=1}^{T_f} \delta_t
\]

(4)

The expected penalty function \( F \) is then estimated from \( \Delta \) by using a curve fitting procedure. The penalty function for 4 of the stations are shown in Figure 1. The penalty functions for all the stations obtained by this procedure are found to be convex. This confirms the validity of the approach as Raviv and Kolka (8) showed that the User Dissatisfaction Function is convex. \( F \) is used as an input in the MIP model to calculate the expected penalty of the entire system.

Table 2 shows all the notations that are used in the MIP part of the paper.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>The set of stations ( {1, \ldots,</td>
</tr>
<tr>
<td>( C_i )</td>
<td>The number of lockers at the station ( i )</td>
</tr>
<tr>
<td>( \phi_i )</td>
<td>The expected penalty incurred at the station ( i )</td>
</tr>
<tr>
<td>( L_i )</td>
<td>Integer variable denoting the inventory level at the station ( i )</td>
</tr>
<tr>
<td>( F_i(L_i) )</td>
<td>The expected penalty vector of the station ( i ) as a function of the inventory level</td>
</tr>
<tr>
<td>( N )</td>
<td>The total number of bicycles in the system</td>
</tr>
</tbody>
</table>

**TABLE 2** Notations used in the MIP
FIGURE 1 Penalty functions for 4 stations

The MIP model is formulated as follows.

\[
\min \sum_{i \in S} \phi_i \tag{5}
\]

Subject to

\[
\phi_i \geq F_i(L_i), \forall i \in S \tag{6}
\]
\[
\phi_i \leq F_i(L_i), \forall i \in S \tag{7}
\]
\[
L_i \geq 0, \forall i \in S \tag{8}
\]
\[
L_i \leq C_i, \forall i \in S \tag{9}
\]
\[
\sum_{i \in S} L_i \leq N \tag{10}
\]
\[
\sum_{i \in S} L_i \geq N \tag{11}
\]

The objective function (5) is the total expected penalty of the system. Constraints (6) – (7) calculate the penalty for all the stations. Constraints (8) – (9) are the capacity constraints for the stations. The inventory levels at the stations should always be between 0 and the capacity of the stations. Constraints (10) – (11) conserve the total number of bicycles in the system.
The MIP is an essential part of the demand forecast process since the number of bicycles in the system can not change before and after the repositioning operation. The inventory levels $L_i$ obtained from the MIP are the desired starting inventory levels for the stations that minimize the expected penalty for the time duration in consideration.

The MIP is coded in AMPL and solved using KNITRO commercial optimization solver. The computational times are found to be very reasonable even for a large real-world network.

**NUMERICAL EXPERIMENTS**

This section presents the numerical experiments and the results obtained from the proposed MIP model. The MIP model is applied to a bicycle sharing system in Antwerp, Belgium called Velo Antwerpen. The network map of Velo Antwerpen is shown in Figure 2.


**Dataset**

The net returns is the difference between the total number of bicycles returned and the total number of bicycles withdrawn by the users at a particular station for the time interval. In this dataset, the net returns are available for every 30 minute time interval for an entire year for all the 82 stations. As shown in Figure 3, the net returns follow a clear pattern throughout the year. The figure shows the weekday and weekend patterns for a particular station of the system averaged over a 3 month period and over the entire year. The pattern was found to be very similar for all the weekdays. The demand for the returns and withdrawals is very low on the weekends compared to the demand on the weekdays as can be seen from Figure 3. The dotted lines represent the usage
patterns for the weekends and the solid lines represent the patterns for weekdays.

![Weekday and weekend patterns of the net returns of bicycles at a station](image)

**FIGURE 3** Weekday and weekend patterns of the net returns of bicycles at a station

The peak during the weekends is more spread compared to the peaks observed during weekdays. This is also very intuitive because during weekdays, many people have fixed times to get to the workplace, to get lunch and get back home from work. This results in sharp peaks and valleys. Whereas during weekends, people have no such time limitations. Thus, the peak is spread over a larger time period (5:30 am to 16:30 pm). There are also valleys from midnight to 1:00 am, and from 20:00 pm to midnight. But overall, the usage is very less compared to the usage on the weekdays.

Static repositioning is very suitable for a system where there is not much demand for bicycles throughout the day and where clear usage patterns exist for the stations. The optimal inventory levels can be calculated prior to the repositioning operation and the demand can be served for the whole day or a large part of the day.

**Application on Velo Antwerpen**

This subsection presents the results for the application of the proposed approach on the Velo Antwerpen network. It can be seen from Figure 3 that the usage in the early morning period from midnight to 5:00 am is very less. This period can be used for the static repositioning of bicycles in the system. So, the desired inventory levels are calculated for 5:00 am. The length of the time intervals is 30 minutes so the total number of time intervals under consideration are 38. The transition probability matrices are formed using the historic usage data of the 10 past days.
corresponding to the same day. The total inventory level of the system was 1100 bicycles at the
time for which the data is available. The performance of the approach is then evaluated by using
the data for the day that is in consideration. The results for 6 weekends are summarized in Table
3. Only weekends have been tested since the weekends are very suitable for a static repositioning
operation.

<table>
<thead>
<tr>
<th>Date</th>
<th>No. of stations with 38 intervals satisfied</th>
<th>No. of stations with ≥ 35 intervals satisfied</th>
<th>No. of stations with ≥ 30 intervals satisfied</th>
<th>No. of intervals satisfied for the worst station</th>
<th>Average no. of intervals satisfied</th>
<th>Average % of intervals satisfied</th>
<th>% reduction in total dissatisfactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 14</td>
<td>44</td>
<td>67</td>
<td>82</td>
<td>31</td>
<td>36.57</td>
<td>96.24</td>
<td>49.81</td>
</tr>
<tr>
<td>Jan 22</td>
<td>54</td>
<td>73</td>
<td>81</td>
<td>29</td>
<td>36.99</td>
<td>97.34</td>
<td>58.40</td>
</tr>
<tr>
<td>Feb 26</td>
<td>57</td>
<td>78</td>
<td>82</td>
<td>30</td>
<td>37.24</td>
<td>98.01</td>
<td>56.89</td>
</tr>
<tr>
<td>Jun 24</td>
<td>55</td>
<td>75</td>
<td>82</td>
<td>30</td>
<td>37.07</td>
<td>97.56</td>
<td>61.90</td>
</tr>
<tr>
<td>Jul 15</td>
<td>53</td>
<td>68</td>
<td>82</td>
<td>31</td>
<td>36.74</td>
<td>96.69</td>
<td>86.46</td>
</tr>
<tr>
<td>Jul 21</td>
<td>44</td>
<td>68</td>
<td>81</td>
<td>28</td>
<td>36.54</td>
<td>96.16</td>
<td>40.10</td>
</tr>
<tr>
<td>Average</td>
<td>51</td>
<td>72</td>
<td>82</td>
<td>30</td>
<td>36.86</td>
<td>97.00</td>
<td>58.93</td>
</tr>
</tbody>
</table>

### TABLE 3 Numerical results for the Velo Antwerpen network

38 time intervals cover the whole day (till midnight) and the second column in the table
shows the number of stations for which the demand is satisfied for the entire day. It can be seen
that for a large number of stations in the system, the inventory levels obtained from the approach
can take care of the demand for the whole day. These stations do not require any additional rebal-
ancing during the day. The third column in Table 3 denotes the number of stations for which at
least 35 time intervals have been satisfied. This is an important number because it can denote the
number of stations for which additional repositioning may be required during the day. Stations for
which less than 35 time intervals have been satisfied are the stations with a very high usage. These
stations might face a high number of dissatisfactions if repositioning is not done during the day. A
maximum of 15 stations were found to require rebalancing for the January 14\textsuperscript{th} data. This is a very
small number compared to 82 stations and can be taken care of by even a single van during the day.
This leads to very high cost savings for the agencies. On an average, the number of stations that
require repositioning is found to be just 10. The fourth column shows that almost all the stations
have at least 30 time intervals satisfied.

The proposed approach also ensures that there is not a single station for which the desired
inventory levels gives a very poor performance (less than 25 time intervals satisfied). The fifth
column in the table denotes the number of time intervals satisfied for the station with the worst
performance. The sixth and seventh columns show the average number and percentages of time
intervals satisfied for the entire system. The average number of intervals is found to be very close
to 38 and this shows that the approach can be used to satisfy the demand for a large part of the day
for the entire system.

Another interesting performance metric to look at is the percentage reduction in the total
number of dissatisfied users. The original case is when there is no repositioning during the night
and the inventory levels at the stations are kept as they are. The second case is with the reposi-
tioning after calculating the desired inventory levels using the proposed approach. It is assumed
that there is no repositioning during the day. It can be seen that there is a very high reduction
in the number of dissatisfied users if the inventory levels obtained from the approach are used to
rebalance the system. The level-of-service for the system can thus be improved by a great margin just by having the desired inventory levels at the stations at the beginning of the day.

The computational times for the MIP were found to be in the range of 2-3 seconds for all the cases. This shows that the model is not computationally expensive and has a high ease-of-use for the agency.

CONCLUSION
This paper focuses on the pre repositioning activity of calculating the desired inventory levels at the stations of a bicycle sharing system. The first part of the approach is to estimate the penalty functions for the stations as a function of the starting inventory levels and the second part is to calculate the inventory levels for the stations that minimize the total penalty for the entire system. It is also ensured that the total number of bicycles are conserved in the system before and after the repositioning activity. The proposed approach is not computationally expensive even for the real world network of Velo Antwerpen, a bicycle sharing system with 82 stations. The main contribution of the work lies in the practicality of the approach and ease-of-use for the bicycle sharing systems.

The authors also propose some interesting future research directions to address some of the limitations of the work. The assumption of the demand at each station being independent of the demand and the status of other nearby stations is not very close to reality and needs to be addressed. The data used in this paper includes the usage at the stations. This may not necessarily correspond to the actual demand at the stations. Developing models to estimate the demand at the stations and using that data would also be an interesting problem to address. Moreover, using more sophisticated models to calculate the penalty functions at the stations would lead to much better results.

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References


