THE HILLS ARE ALIVE: IDENTIFYING VERTICAL CURVES USING STRATEGIC HIGHWAY RESEARCH PROGRAM ROADWAY DATA

Daniel L. Carter, P.E. (corresponding author)
UNC Highway Safety Research Center
730 Martin Luther King Jr Blvd, CB #3430
Chapel Hill, NC 27599
919-962-8720 (phone), 919-962-8710 (fax)
daniel_carter@unc.edu

Raghavan Srinivasan, Ph.D.
UNC Highway Safety Research Center
730 Martin Luther King Jr Blvd, CB #3430
Chapel Hill, NC 27599
919-962-7418 (phone), 919-962-8710 (fax)
srini@hsrc.unc.edu

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ABSTRACT
The second Strategic Highway Research Program (SHRP2) conducted a massive data compilation consisting of detailed roadway data, driver behavior, and vehicle kinematics. A research study recently funded by the SHRP2 Implementation Assistance Program is using SHRP2 data to provide valuable information about how drivers behave under various combinations of vertical and horizontal alignment. In this study, it was necessary to identify the presence and location of vertical curves. The objective of this paper is to present and compare two methods for identifying vertical curves using the SHRP2 RID grade data. Each method is assessed against a hypothetical road with artificially created vertical curves and a section of an actual road in Washington State which contains a ground truth vertical curve inventory. The best performing method used a linear correlation element to detect when a set of road segments formed a vertical curve. This method accurately identified the presence of vertical curves without identifying curves where none existed, although it generally underplayed the length of the curves. This is a promising result, since the objective for the overall safety study was to identify vertical curve locations. This paper demonstrates that there are good options for interpreting the data to identify vertical curves. The development of the linear correlation method represented an advancement over the simplistic difference of average grades. Comparisons to a hypothetical road and a real world road with ground truth proved useful to identify the best performing method and determine the optimal parameters for the method to produce the most accurate results.
BACKGROUND
The second Strategic Highway Research Program (SHRP2) was developed to advance the knowledge of transportation topics, one of which is highway safety. SHRP2 conducted a massive data compilation consisting of detailed roadway data in six areas around the U.S. and data on driver behavior and vehicle kinematics for trips traveled on those roads. The roadway data are maintained as the Roadway Information Database (RID) and the driver and vehicle data is maintained as the Naturalistic Driving Study (NDS) (1,2). These datasets are now being made available for research on a variety of topics.

The SHRP2 Implementation Assistance Program (IAP) is focused on supporting the deployment of SHRP2 products (3). One of the IAP activities is providing funding for research studies through a series (rounds) of grants to state DOTs. Round 4 of the IAP addressed highway safety topics (“Concept to Countermeasure – Research to Deployment Using the SHRP2 Safety Data”). One of these research projects granted to North Carolina DOT is the “Evaluation of the interaction between horizontal and vertical alignment on rural two lane roads: An investigation using the SHRP2 naturalistic driving study”. This study is focused examining safety issues related to the combination of horizontal and vertical curves on rural two lane roads. Previous work sponsored by the Federal Highway Administration (FHWA) as part of the Highway Safety Information Systems (HSIS) effort investigated the safety effects of five combinations of horizontal and vertical alignment: horizontal curves and tangents on straight grades, horizontal curves and tangents a type 1 crest vertical curves, horizontal curves and tangents at type 1 sag vertical curves, horizontal curves and tangents at type 2 crest vertical curves, and horizontal curves and tangents at type 2 sag vertical curves (4). Although these CMFs provide valuable information about the safety risk associated with different roadway and traffic control, they do not necessarily provide insight into why certain conditions are higher risk. Data from naturalistic driving studies can provide this insight that studies based only on crash data cannot easily provide.

The intent of the current research study is to use SHRP2 data to provide valuable information about how drivers behave under various combinations of vertical and horizontal alignment. The process for this study was to use the SHRP2 RID to identify a set of road segments with vertical and horizontal curves, select SHRP2 trips that occurred on those segments, obtain vehicle and driver data for those trips, and analyze the data.

In order to conduct this study, it was necessary to identify the presence and location of vertical curves. This was not a straightforward task, since the RID does not explicitly define vertical curvature in the data. Rather, it provides grade data (in percent) approximately every 25 feet, collected by an instrumented vehicle which drove all roads in the study area. It was necessary to develop a method for extracting vertical curve information from the grade data.

OBJECTIVE
The objective of this paper is to present and compare two methods for identifying vertical curves using the SHRP2 RID grade data.

SITE SELECTION AND DATA PREPARATION
The overall study focused on two lane rural roads and was intended to serve as a pilot “proof of concept” study to assess if and how the SHRP2 data could be used to accomplish the study objectives. Thus, only a limited number of sites (curved and straight road sections) was assembled for analysis. To select the study sites, the team began with RID data from New York and Pennsylvania, since these were assumed to have the greatest amount of curves on rural roads.

The team selected routes on these networks by prioritizing the following factors:

- Two-lane roads (critical) – The preliminary selection of two-lane roads was done using the “Lanes” layer from the RID. However, it was necessary to further refine that selection from aerial image inspection due to the fact that the RID provides through lanes in both directions for
undivided roads but separately in each direction for divided roads. Thus, a screening for “Number of lanes = 2” would select two-lane undivided roads and four-lane divided roads.

- Rural area type (critical) – This was assessed through a visual assessment of the road network as well as subsequent aerial image inspection.
- Moderately high number of trips (preferred) – To maximize the efficiency of the NDS data collection, the team sought to identify roads that had more than a minimal number of trips. This was accomplished through referencing the trip density maps provided on the NDS InSight website.
- Promising amount of vertical and horizontal curvature (preferred) – Routes that appeared to have a relatively high amount of horizontal curvature (based on the RID “Alignment” layer) and vertical grade variation (based on the RID “Location” layer) were prioritized in the selection. At this stage, it was not yet possible to determine which routes had vertical curves.

After selecting the potential study routes, it was necessary to assess the route data and determine the start and end points of horizontal and vertical curves. This would be needed to dynamically segment the route into unique sections representing combinations such as “Vertical crest curve on horizontal tangent” or “Vertical sag curve on horizontal curve”. These sections would later serve as the unit of analysis when NDS driver behavior data was obtained.

Determining the start and end points of horizontal curves was straightforward since the “Alignment” layer of the RID is segmented into horizontal tangents and curves. Basic curve attributes are also provided, such as curve radius. However, vertical curvature was not provided in the RID and determining the start and end points of vertical curves was challenging. There was a need for a method to assess the grade data to determine the location and extent of vertical curves.

**DESCRIPTION OF TWO METHODS FOR IDENTIFYING VERTICAL CURVES**

The basic approach taken to assessing the grade data for identifying vertical curves was to conduct a “sliding window” analysis. This approach would begin with the first segment of the route and evaluate the group of adjacent grades within a certain designated window distance to determine if the trend of the values indicated that a vertical curve had been encountered. After each evaluation, the window would slide down the road by one grade section (in the direction of increasing mileposts) and the evaluation would be repeated until the end of the route was reached. However, the question of how to evaluate the group of adjacent grades was the exigent factor in developing and testing two methods – a difference in average grades and a linear correlation of grades.

**Method 1: Difference in Average Grades**

The Difference in Average Grades (DAG) method compares the average of all grades in a window distance upstream of the segment of interest to the average of all grades in a window downstream of the segment of interest. If the difference in the two average grades exceeds a designated threshold amount, then the segment of interest is determined to lie within a vertical curve. Adjacent curve-identified sections are later grouped as a distinct curve. TABLE 1 uses an excerpt of actual RID grade values to demonstrate the DAG method. The table shows results using an example threshold of 0.5 percent and a window consisting of the past and future six segments; if the difference in the average grades of the past window and future window exceeds 0.5 percent, the segment is deemed to be in a vertical curve. For example, the calculation for segment #7 in TABLE 1 would be:

\[
\text{Difference in Average Grades (DAG)} = \text{average}(-1.2,-1.4,-1.4,-1.3,-1.0,-1.4) - \text{average}(-1.6,-1.5,-1.7,-1.9,-2.0,-2.4) = 0.57
\]
The DAG method is a simplistic method which can detect changes in grade sufficient to identify vertical curves. However, initial assessments of the method showed some issues when attempting to identify both small (sharper) vertical curves and long (flatter) curves on the same route. Over the same distance in length, long curves have less change in grade than sharp curves, making it difficult to select a grade difference threshold that will evaluate both correctly. Any particular threshold value would underestimate the length of long curves but overestimate the length of sharp curves. More on the assessment of this method is presented later in this paper.

**Method 2: Linear Correlation of Grades**

The Linear Correlation (LC) method was developed out of a desire to address the main drawback with the DAG method. That is, a method was needed that could equally assess and identify both sharp and flat curves. The answer to this need was found in the common design standards used for all vertical curves. The similarity in sharp and flat curves is that they both follow a parabola (or similar curve within some tolerance for deviations in construction and/or older curve designs). Mathematically, the first derivative of a parabola (Equation 1) is a straight line (Equation 2). If the derivative of the roadway curve could be compared to a straight line, it would be possible to determine how similar the curve is to a parabola.

\[
y = ax^2 + bx + c \quad \text{(Equation 1)}
\]

\[
y' = 2ax + b \quad \text{(Equation 2)}
\]

Conveniently, the RID provides the first derivative of the roadway curve by providing the grade at every 25 to 30 foot interval. If the road were exactly constructed as a perfect parabola (and the grade data were perfectly collected), the trend of the grade values would be a straight line. Thus, the LC method examines the trend of the grade values within a window distance of the segment of interest and calculates a correlation value that reflects the degree to which the grades form a linear trend. The higher the correlation value, the more likely that the segment of interest lies within a vertical curve (i.e., a parabolic group). A correlation value of 1.0 would reflect a straight line.

Even for curves that are not parabolic (e.g., a constant radius circular arc), the trend of the grades is fairly close to a straight line. **FIGURE 1** shows the linear correlation values for an artificially generated set of calculated slopes from parabolic and circular curves. Naturally, the linear correlation values for the perfectly parabolic curve are 1.0; however the correlation values for the circular arc are very close to 1.0.
This fact allows the LC method to detect even those curves that were not built consistent with modern highway design standards or were built with deviations in the construction process.

**FIGURE 1** Linear Correlation of Slope Values for Parabolic and Circular Arcs

The LC method calculates a linear correlation value for the grades of all segments within a designated window distance downstream and upstream of the segment of interest. A linear correlation value is calculated for the grade values captured in the windows. If the trend of the grades is linear within a certain tolerance (linear correlation threshold, such as 0.9), the segment is deemed to be within a vertical curve. TABLE 2 uses an excerpt of actual RID grade values to demonstrate the LC method (these are the same segments used in the illustration of the DAG method in TABLE 1). The table shows results using an example threshold of 0.9 linear correlation and a window consisting of six segments downstream and upstream.

**TABLE 2** Example of Linear Correlation Method

<table>
<thead>
<tr>
<th>Road Segment ID</th>
<th>Segment Grade</th>
<th>Linear Correlation of Grades Using Six Segment Window</th>
<th>Segment is in Vertical Curve (Correlation Meets Threshold)?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.2</td>
<td>0.243</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>-1.4</td>
<td>0.280</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>-1.4</td>
<td>0.512</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>-1.3</td>
<td>0.773</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>-1.0</td>
<td>0.888</td>
<td>No</td>
</tr>
<tr>
<td>6</td>
<td>-1.4</td>
<td>0.882</td>
<td>No</td>
</tr>
<tr>
<td>7</td>
<td>-1.6</td>
<td>0.881</td>
<td>No</td>
</tr>
<tr>
<td>8</td>
<td>-1.5</td>
<td>0.906</td>
<td>Yes</td>
</tr>
<tr>
<td>9</td>
<td>-1.7</td>
<td>0.947</td>
<td>Yes</td>
</tr>
<tr>
<td>10</td>
<td>-1.9</td>
<td>0.976</td>
<td>Yes</td>
</tr>
<tr>
<td>11</td>
<td>-2.0</td>
<td>0.986</td>
<td>Yes</td>
</tr>
<tr>
<td>12</td>
<td>-2.4</td>
<td>0.984</td>
<td>Yes</td>
</tr>
</tbody>
</table>
ASSESSMENT OF METHODS

The crux of the matter is how well the two methods perform in identifying vertical curves based on RID grade data. The assessment of these methods was conducted in two ways. First, the methods were compared to each other using a hypothetical route. Second, the methods were tested for true accuracy by implementing them on a real world route and comparing to a known vertical curve inventory.

Compare Methods Using Hypothetical Road

The first assessment consisted of creating a hypothetical road in the same format as the RID grade data format. Using Excel formulas to generate a dataset, an artificially created series of progressively longer curves were connected with straight grades of 2 percent (FIGURE 2). The dataset was presented in 25 foot sections with grade values for each segment, similar to the RID grade data. The curves were calculated as exact parabolas.

Recognizing that the designation of the window distance could have an effect on the performance of the methods, the authors used two variations of each of the methods in the assessment. They included:
- Difference in Average Grades (DAG) using a window distance of 6 roadway segments
- Difference in Average Grades (DAG) using a window distance of 10 roadway segments
- Linear Correlation (LC) using a window distance of 6 roadway segments
- Linear Correlation (LC) using a window distance of 10 roadway segments

FIGURE 3 shows an example of a profile of one of the curves from the hypothetical road. The curved line shows the actual start and end of the 400 foot curve as presented in the artificial data. Below that are the extents of the curve as calculated by each of the variations of the two methods. The DAG method slightly overestimates when using a window of 6 segments and greatly overestimates when using a 10 segment window. The LC method fairly accurately estimates the start and end of the curve using either a 6 or 10 segment window.

In order to determine the optimal parameters for each method, it was necessary to quantitatively assess the performance of the two methods over the entire hypothetical road. This was done by totaling the number of road segments identified as curve segments that were actually true curve segments and subtracting the number of segments identified as curve segments that were not on a curve. TABLE 3 shows the results of each method variation. The higher the total score, the better the method performed in
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accurately identifying curves on the hypothetical road. This score was used in a trial-and-error iterative fashion to hone in on the optimal parameters for each method. It revealed that a linear correlation of 0.9 performed best for the LC method and a grade difference threshold of 0.45 percent performed best for the DAG method.

In comparing the DAG method to the LC method, TABLE 3 shows that the LC method performed slightly better (higher total scores). The DAG method performed better with the smaller window size, but the window size did not appear to affect the performance of the LC method.

<table>
<thead>
<tr>
<th>Method</th>
<th>DAG (6 segment window)</th>
<th>DAG (10 segment window)</th>
<th>LC (6 segment window)</th>
<th>LC (10 segment window)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correctly identified as curve segments (A)</td>
<td>208</td>
<td>215</td>
<td>212</td>
<td>212</td>
</tr>
<tr>
<td>Incorrectly identified as curve segments (B)</td>
<td>12</td>
<td>47</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Total Score = A – B</td>
<td>196</td>
<td>168</td>
<td>204</td>
<td>204</td>
</tr>
</tbody>
</table>

**Compare to Real World Ground Truth**
The comparison using the hypothetical road showed that both methods could successfully identify the presence of vertical curves and proved useful in determining the optimal parameters for each method. It also indicated that the LC method performed slightly better than the DAG method. However, the true motivation for this effort was to determine how the methods would perform using RID data, so a comparison to a real world dataset was critical.

**Finding a Ground Truth Dataset**
The Seattle area in Washington State is one of the six areas included in the RID. Washington State is also a state which maintains a vertical curve inventory. Thus, the Washington State Department of Transportation (WSDOT) curve inventory was used as “ground truth” to check how accurately the methods identified vertical curves on selected routes in the Seattle area.

In keeping with the objective of the overall study, the selected route for ground truth testing was a two-lane rural road with moderately high amount of vertical curvature (based on the WSDOT inventory). The selected route was a 40-mile portion of US 2 containing 37 vertical curves. This number of curves does not include very flat curves which were not used in the analysis.

**Identifying Ground Truth Curves**
The RID provides many supplemental files not collected by the instrumented vehicle but rather obtained directly from the respective state DOT. In the Washington supplemental files, the RID contains vertical curve inventory as a point feature class with attributes for each point indicating the route ID and starting and ending mileposts of the curve. Ideally, this ground truth curve data would have been compared to the DAG- and LC-estimated curves based on the starting and ending mileposts. However, the linear referencing system used in the RID (called FrMeasure and ToMeasure) was not consistent with the mileposting referencing system used by WSDOT. Thus, spatial position had to be used as the common denominator to compare the ground truth curves with the estimated curves.

The WSDOT geospatial vertical curve inventory was obtained from the WSDOT Roadway Datamart website (5). The appropriate line feature class of the Washington roadway network was obtained directly from WSDOT staff, and the curves were plotted as route events on the road network to produce a set of short segments representing the extents of each vertical curve.

The RID “Location” layer was laid onto the same map with the WSDOT curves, and spatial joining was used to identify which “Location” layer segment were associated with each ground truth curve from the WSDOT inventory. This formed the basis for comparison with the DAG and LC methods.
Identifying Curves using the DAG and LC Methods
The DAG and LC methods were implemented on the three test routes using the “Location” layer data from the RID. As mentioned before, this dataset segmented the route into 25-foot segments with a percent grade value for each segment. The DAG and LC methods evaluated every segment and indicated if it was or was not within a vertical curve. Based on the results of the comparison to the hypothetical road, only the 6-segment window variation of the DAG method was used in the ground truth assessment. Both the 6-segment and 10-segment window variations of the LC method were used, since the hypothetical comparison showed that they performed similarly.

Comparing Methods to Ground Truth
The basis for comparing the DAG and LC methods to ground truth was to examine which road segments were indicated as being part of a vertical curve versus what was indicated by ground truth. TABLE 4 shows an example of a set of 12 road segments from a portion of US 2. The third column shows the indication of whether each segment is in a vertical curve based on the comparison to the WSDOT curve inventory (ground truth). The last three columns show the results of the DAG and LC methods (with two variations of the LC method using different window sizes). For each method, if the calculated value exceeded the designated threshold for that method, then the segment was indicated to be in a curve (shown as bolded and shaded).

<table>
<thead>
<tr>
<th>Segment Number</th>
<th>Grade</th>
<th>Ground Truth Indication of Curve</th>
<th>DAG (6 segment window)</th>
<th>LC (6 segment window)</th>
<th>LC (10 segment window)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>-2.1</td>
<td>No</td>
<td>0.150</td>
<td>0.196</td>
<td>0.467</td>
</tr>
<tr>
<td>41</td>
<td>-2.1</td>
<td>No</td>
<td>0.183</td>
<td>0.135</td>
<td>0.562</td>
</tr>
<tr>
<td>42</td>
<td>-2</td>
<td>No</td>
<td>0.167</td>
<td>0.041</td>
<td>0.611</td>
</tr>
<tr>
<td>43</td>
<td>-2</td>
<td>No</td>
<td>0.117</td>
<td>0.070</td>
<td>0.693</td>
</tr>
<tr>
<td>44</td>
<td>-1.8</td>
<td>No</td>
<td>0.050</td>
<td>0.225</td>
<td>0.742</td>
</tr>
<tr>
<td>45</td>
<td>-1.8</td>
<td>Yes</td>
<td>0.183</td>
<td>0.570</td>
<td>0.771</td>
</tr>
<tr>
<td>46</td>
<td>-2.1</td>
<td>Yes</td>
<td>0.450</td>
<td>0.815</td>
<td>0.804</td>
</tr>
<tr>
<td>47</td>
<td>-2.3</td>
<td>Yes</td>
<td>0.567</td>
<td>0.891</td>
<td>0.832</td>
</tr>
<tr>
<td>48</td>
<td>-2.3</td>
<td>Yes</td>
<td>0.667</td>
<td>0.949</td>
<td>0.855</td>
</tr>
<tr>
<td>49</td>
<td>-2.4</td>
<td>Yes</td>
<td>0.750</td>
<td>0.975</td>
<td>0.910</td>
</tr>
<tr>
<td>50</td>
<td>-2.6</td>
<td>Yes</td>
<td>0.833</td>
<td>0.990</td>
<td>0.952</td>
</tr>
<tr>
<td>51</td>
<td>-2.8</td>
<td>Yes</td>
<td>0.817</td>
<td>0.990</td>
<td>0.970</td>
</tr>
<tr>
<td>52</td>
<td>-2.8</td>
<td>Yes</td>
<td>0.783</td>
<td>0.991</td>
<td>0.984</td>
</tr>
</tbody>
</table>

The quantitative assessment of the methods was done using counts of false positives and false negatives. If a method indicated that a segment was not in a curve, when ground truth indicated that it was, this represented a false negative. Conversely, if a method indicated that a segment was in a curve when ground truth indicated that it was not, this represented a false positive. These false positives and negatives were summed across the entire 40 mile test route to determine how well the methods compared to ground truth. Similar to the comparison to the hypothetical road, the parameters for each method were adjusted in a trial-and-error fashion to hone in on the optimal parameters for each method (lowest resulting counts of false positives and negatives). This revealed that a linear correlation of 0.9 performed best for the LC method and a grade difference threshold of 0.5 percent performed best for the DAG method.
method. These values were almost identical to the values determined in the comparison to the hypothetical road.

TABLE 5 shows the results of this assessment using the optimal parameters for each method. The LC method using a 10-segment window performed the best with the lowest total of false positives and negatives. Although the DAG method resulted in a closely similar number of false negatives (meaning that both methods missed around the same number of curve segments), the DAG method had many more false positives. That is to say, the DAG identified many segments as vertical curves when the ground truth did not. This is a result of the fact that the DAG method only looks at a simplistic difference in grades while the LC method requires that the grades form a specific curved shape. It was more likely that random fluctuations in the road grade resulted in a difference in average grades that exceeded the DAG threshold rather than randomly forming a curved shape which would trigger the LC method.

<table>
<thead>
<tr>
<th>Method</th>
<th>False negatives</th>
<th>False positives</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAG (6 segment window)</td>
<td>248</td>
<td>379</td>
<td>627</td>
</tr>
<tr>
<td>LC (6 segment window)</td>
<td>359</td>
<td>200</td>
<td>559</td>
</tr>
<tr>
<td>LC (10 segment window)</td>
<td>297</td>
<td>107</td>
<td>404</td>
</tr>
</tbody>
</table>

**CONCLUSION**

The comparisons of the DAG and LC methods to a hypothetical road and to a real road with ground truth curve inventory showed that the LC method with a 10-segment window performed the best in correctly identifying road segments that lie in vertical curves. This is consistent with the findings of the comparison to a hypothetical road, in which the LC method also performed the best. In the ground truth comparison, this method produced 297 false negatives and 107 false positives. These must be considered in light of the total number of segments used in the assessment. The 37 vertical curves on the tested portion of US 2 comprised a total of 1010 road segments. A perfect method would have identified all 1010 segments as lying within vertical curves. Thus, the 297 false negatives represent a 70% success rate in correctly identifying vertical curve segments. There were also 5048 segments on the tested portion of US 2 that were not in a vertical curve according to ground truth. A perfect method would not have identified any of these segments as vertical curves. Thus, the 107 false positives represent a 98% success rate in correctly identifying non-vertical curve segments.

These results mean that the LC method accurately identified the presence of vertical curves without identifying curves where none existed, although it generally underplayed the length of the curves. This is a promising result, since the objective for the overall safety study was to identify vertical curve locations.

There are certainly challenges in using RID grade data to identify vertical curves, such as variations in the instrumented vehicle grade readings and deviations in roadway construction. However, this paper demonstrates that there are good options for interpreting the data to identify vertical curves. The development of the linear correlation method represented an advancement over the simplistic difference of average grades. Comparisons to a hypothetical road and a real world road with ground truth proved useful to identify the best performing method and determine the optimal parameters for the method to produce the most accurate results.
REFERENCES