Evaluate the Accuracy of the Photogrammetry-Based Deformation Measurement Method

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ABSTRACT

Triaxial test has been widely used to evaluate unsaturated soil behaviors. Volume change measurement on unsaturated soils during triaxial testing remains a challenge. Recently, a photogrammetry-based method has been developed for unsaturated soil deformation measurement during triaxial testing. Preliminary study indicated that the photogrammetry-based method is a simple, cost- and time-effective option for monitoring soil deformation during triaxial testing. However, there are still concerns regarding the accuracy, reliability, and repeatability of the photogrammetry-based method for soil deformation measurement during triaxial testing.

In this study, to evaluate the point measurement accuracy, validation tests were performed on an aluminum cylinder under different chamber pressure and confining medium (i.e. air, water, and silicone oil which are commonly used as the confining medium for triaxial tests) conditions. A sensitivity analysis was also carried out to evaluate the impact of the system parameters (i.e. refractive indices of air and confining fluid, cell wall thickness) on the volume measurement accuracy of the photogrammetry-based method. Beside these, a triaxial test on a saturated sand specimen was carried out to evaluate the volume change measurement accuracy of the photogrammetry-based method. Effects of target number and mesh pattern on soil volume change measurement accuracy were also evaluated. In addition, the repeatability of the photogrammetry-based method was validated through a series of triaxial tests on unsaturated soils. Some suggestions are also presented to improve the accuracy of the photogrammetry-based method. Results from this study indicate that the photogrammetry-based method is an accurate and reliable option for soil deformation measurement during triaxial testing.

Keywords: photogrammetry-based method, optical ray tracing, triaxial test, volume change, unsaturated soil, accuracy evaluation
INTRODUCTION

Triaxial test has been widely used to evaluate unsaturated soil behaviors. Different from saturated soils, volume change of an unsaturated soil is not equal the water volume change. As a result, the conventional volume change measurement method for saturated soils through monitoring water exchange cannot be applied to measure volume change of unsaturated soils. In the past few decades, several methods including double-wall cell, local displacement transducer, and image-based methods (1-8) have been developed to measure the volume change of unsaturated soils during triaxial testing. However, as summarized in Laloui et al. (9), Hoyos et al. (10), and Zhang et al. (11), all these methods have their limitations. Accurate and reliable volume change measurement on unsaturated soils was a great challenge for researches.

Recently, a photogrammetry-based method, which overcame the most limitations of the conventional image-based method, was developed for the total and localized volume changes of unsaturated soils during triaxial testing (11-12). With the photogrammetry-based method, the conventional triaxial cell, designed for triaxial tests on saturated soils, can be used to characterize unsaturated soils without any modification for volume change measurement purpose. The system setup for the photogrammetry-based method is very simple, which only requires posting measurement targets on the load frame, cell surface, and soil surface. Also, the photogrammetry-based method can be used to measure deformation of both saturated and unsaturated soils with the same system setup. Compared with the conventional deformation or volume change measurement methods, the newly developed photogrammetry-based method provides an accurate, cost- and time- effective way to measure the total and localized deformation of unsaturated soils.

To apply the photogrammetry-based method for soil deformation measurement, after system setup, a series of images are captured for the triaxial chamber with a soil specimen inside. Geometric properties of acrylic cell and orientations of the camera stations where images are captured are determined based on principle of photogrammetry. After this, optical rays from each camera station to the chamber surface are reconstructed and traced to the measurement targets on the soil surface with the help of the Snell’s law. Then, the three dimensional (3D) coordinates of the measurement targets on the soil surface are estimated using the least-square technique based on optical rays traced form different camera stations. With the 3D positions of measurement targets on soil surface, deformation characteristics can be extracted for soil behavior characterization. For the photogrammetry-based method, the cell wall reconstruction, optical ray tracing, least-square estimation, triangular mesh generation, volume and strain calculation processes are rather computational intensive. However, with the self-developed MatLab-based software PhotoSoilVolume, these processes can be completed in seconds.

In Li et al. (12), through a series of triaxial tests on unsaturated soils, the photogrammetry-based method proved to be capable of capturing soil deformation characteristics such as volume change, volumetric strain non-uniformity, barreling, shear band evolution, and full-field strain distribution. Also, the photogrammetry-based method was successfully utilized as a replacement of the conventional volume change measurement method for unsaturated soil characterization using the conventional triaxial apparatus as presented in Li and Zhang (13).

In Zhang et al. (11), validation tests on a stainless steel cylinder and a saturated sand specimen were performed to evaluate the accuracy of the photogrammetry-based method in which water was used as the confining fluid for the triaxial tests. In reality, besides water, air and silicone oil are also widely used as the confining medium for triaxial tests. A discussion regarding the photogrammetry-based method is presented in Salazar and Coffman (14). A
question raised is: is the proposed photogrammetry-based method still applicable if air or silicone oil is used as the confining fluid? In Zhang et al. (11), the soil volume calculation method is based on triangular meshes generated using the measurement targets on soil surface. The calculated volume is highly dependent on the number of the element (tetrahedrons in this case) and the pattern of the mesh. The question is: how many surface points are sufficient to represent the volume of a soil specimen? The impact of number of targets on soil surface and the mesh patterns was not addressed in Zhang et al. (11). Using the triaxial cell (101.6 mm in outer diameter and 203.2 mm in height), the point measurement accuracy on the stainless steel cylinder (38 mm in diameter) was determined to be 0.07 mm as presented in Zhang et al. (11). The question is: will this accuracy be influenced by the size of the acrylic cell and imperfection of the cell including thickness uniformity, cleanliness of the cell wall, light reflection on cell wall, and chamber pressure. As a result, there is a need to further validate the accuracy, reliability, and repeatability of the photogrammetry-based method for soil deformation measurements during triaxial testing.

In this study, to address the concerns raised in Salazar and Coffman (14), a new triaxial test was conducted on an aluminum cylinder using the triaxial cell (166 mm in outer diameter and 304 mm in height) for the validation tests on the saturated sand specimen in Zhang et al. (11). The measurements on the aluminum cylinder were conducted under different confining medium (i.e. air, water, and silicone oil) and chamber pressure (i.e. 0 to 600 kPa) conditions. Besides this, a sensitivity analysis was carried out to assess the impact of the system parameters (cell wall thickness, refractive indices of cell wall, air, and water), on the accuracy of the photogrammetry-based methods. To evaluate the overall volume change measurement accuracy, a new validation test on a saturated sand specimen was conducted. The impact of measurement target number and mesh pattern on soil volume change measurement accuracy was also evaluated. In addition, a series of triaxial tests on unsaturated soils was conducted to evaluate the reliability and repeatability of the photogrammetry-based method. Beside these, several suggestions are also provided to improve the accuracy of the photogrammetry-based method.

ACCURACY EVALUATION

Triaxial Tests on an Aluminum Cylinder

An aluminum cylinder (71 mm in diameter and 142 mm in height), as shown in Figure 1a, was fabricated for the point and volume measurement accuracies evaluation. A total number of 260 measurement targets (20 targets/circle × 13 circles) were posted on the steel cylinder surface to facilitate the measurements and analysis. The used acrylic chamber, as shown in Figure 1b or 1c, is 9.70 mm in thickness with a refractive index of 1.491. A total of 324 measurement targets are posted on the outside surface of the acrylic chamber, which include 6 circles (45 targets / circle) and 3 vertical stripes (18 targets/strip).

The experimental program includes reconstruction of 3D models of the aluminum cylinder using the photogrammetry-based method under following conditions: 1. exposed in air as shown in Figure 1a, 2. installed in the triaxial chamber as shown in Figure 1b with 0, 200, 400, and 600 kPa chamber pressures (air pressure in this case), 3. installed in the triaxial test apparatus which was filled with water as shown in Figure 1c with 0, 200, 400, and 600 kPa chamber pressures (water pressure in this case), and 4. installed in the triaxial test apparatus which was filled with silicone oil as shown in Figure 1d with 0, 200, 400, and 600 kPa chamber pressure.
pressures (oil pressure in this case). Since the modulus of elasticity of aluminum is 69 GPa, it is reasonable to assume the aluminum cylinder to be rigid under a confining pressure less than 600 kPa. As a result, the 3D positions of the measurement targets on cylinder surface provides a good reference to evaluate the measurement accuracy of the photogrammetry-based method. The tests were performed in the following way: (1) firmly fix the stainless steel cylinder on the pedestal of the triaxial test apparatus; (2) capture images from different orientations; (3) carefully install the triaxial chamber; (4) capture images from different orientations at different chamber pressures (in this case, air pressures was increased from 0 to 200, 400, and 600 kPa); (5) release the air pressure in the chamber and then slowly fill up the chamber with water; (6) capture images from different orientations at different chamber pressures (in this case, water pressures was increased from 0 to 200, 400, and 600 kPa). For each measurement, approximately 30 pictures were captured; (7) drain the water out and fill the chamber with silicone oil; and (8) capture images from different orientations at different chamber pressures (in this case, oil pressures was increased from 0 to 200, 400, and 600 kPa).

After testing, under the same global coordinate system, the 3D coordinates of the measurement targets on the chamber surface were determined based on photogrammetric analyses. When assuming to be cylindrical, the radius of the triaxial chamber under different chamber pressure and confining medium conditions was obtained through a least-square method and plotted against chamber pressure as shown in Figure 2. With increasing chamber pressure, the radius of the triaxial chamber continually increased from 82.98 to 83.10 mm under air-, water-, and oil-filled chamber conditions. Theoretically, the cell radius should be the same at the same chamber pressure. However, in Figure 2, the maximum difference of chamber radius is found to be 0.03 mm at the chamber pressure of 400 kPa. The reason for this radius variation is the plastic deformation of the acrylic cell and the measurement error of the photogrammetry method. Within the range of the applied chamber pressure (\( \leq 600 \) kPa), the response of chamber radius was nearly linear as shown in Figure 2.
With the 3D coordinates of the targets on cell surface, the barrel-shaped cell surface was then reconstructed. According to the photogrammetry-based method, with the reconstructed cell surfaces (i.e. outer and inner), the optical ray tracing could be performed to extract the 3D positions of the targets on the cylinder surface. For the measurement at a specific chamber pressure and confining medium condition, many optical rays could be built and traced for each image from the corresponding camera station to the cylinder surface as typically shown in Figure 3a when silicone oil was used as the confining fluid. For a single image, part of the view to the aluminum cylinder surface was blocked by the frame rods, targets and possible stain on cell surface. As a result, optical rays for these targets cannot be constructed as typically shown in Figure 3a. However, for these targets, optical rays could still be built using some other images captured at different view angles. Figure 3b shows a typical optical ray tracing process for a single target on the aluminum cylinder surface using images captured at 12 different camera stations. These 12 rays were then used to best estimate the 3D position of the target using the least square method as presented in Zhang et al. (11).
The 3D coordinates of the measurement targets on the aluminum cylinder surface were obtained using the photogrammetric and photogrammetry-based methods for exposed in air and in the triaxial chamber conditions, respectively. For the photogrammetry-based analysis, the used refractive indices of air, cell wall, water, and silicone oil under different pressures (0 to 600 kPa) were 1.000, 1.491, 1.339, and 1.407. The triaxial chamber was assumed to be barrel-shaped with a uniform wall thickness of 9.70 mm under different chamber pressure and confining media conditions. After the photogrammetric and photogrammetry-based analyses, triangular meshes were generated and the corresponding volumes were calculated using the method presented in Zhang et al. (11). Since the measurement targets did not cover the whole aluminum cylinder, it is worth noting that the volumes calculated based on targets on cylinder surface are not necessarily equal to the real volume of the aluminum cylinder. Figure 4 presents the volumes obtained based on the 3D coordinates of those targets on cylinder surface under different chamber pressure and confining medium conditions.

Since the point measurement accuracy is up to 3 microns as presented in Zhang at al. (11), the volume measurement results of the aluminum cylinder when exposed in air was considered as the “true” volume in this study. When compared with this volume, the average volume measurement accuracy of the photogrammetry-based method is 0.092% and 0.03% when water and silicone oil was used as the confining medium, respectively. Also, the volume measurement accuracy did not vary much at different chamber pressures ranging from 0 to 600 kPa as shown in Figure 4 when water or silicone oil was used as the confining liquid. However, when air was used as the confining medium, the volume measurement accuracy of the photogrammetry-based method was lower than that of the water-filled chamber condition. The volume measurement error increased from 0.11% to 0.43% with increasing confining pressure from 0 to 600 kPa. The
reason for this measurement accuracy difference is believed to be attributed to the used refractive indices of air and water. It is well known that refractive index varies with the wavelength of light. In other words, change in light condition, which means a change in wavelength of light, could result in a different water refractive index.

In this study, the refractive indices of water and silicone oil are obtained through a simple test presented in the Appendix. The built-in flash of the camera was used as the light source for all tests in this study. According to the findings presented in Waxler and Weir (15), water refractive index may change at different pressure and temperature conditions. A pressure change from 0 to 25832 kPa resulted in a change of water refractive index from 1.33423 to 1.33805 at a temperature of 7.64 °C. However, in this study, all tests were conducted at a nearly constant temperature of 23 °C. Also, the chamber pressure only varied from 0 to 600 kPa. No research effort has been dedicated to the study the variation of refractive index of silicone oil at different pressures. In this study, the refractive indices of water and silicone oil were assumed to be constants when chamber pressure varied from 0 to 600 kPa. As shown in Figure 4, no evidence shows that the volume measurement accuracy increased with increasing chamber pressure when water or silicone oil was used as the confining liquid. So, the concern on the influence of chamber pressure on the measurement accuracy of the photogrammetry-based method (Salazar and Coffman (14) on longer exist as long as the refractive index of the confining medium is accurate. According to Owens (16), air refractive index varies with the pressure. Since the refractive indices of air under different pressures are unknow and the test presented in the Appendix is not applicable for the air refractive index measurement, the air refractive index was also assumed to be a constant at a pressure range less than 600 kPa for the photogrammetry-based analysis in this study. The assumption of a constant air refractive index resulted in a measurement error increase from 0.11% to 0.43% when chamber pressure increased from 0 to 600 kPa.

FIGURE 4 Volumes Measured under Different Chamber Pressure and Confining Medium Conditions

In this study, the volumes were measured under different chamber pressure and confining medium conditions. The results show that the volume measurement accuracy increased with increasing chamber pressure when water or silicone oil was used as the confining liquid. However, no such increase was observed when air was used. The assumption of a constant air refractive index resulted in a measurement error increase from 0.11% to 0.43% when chamber pressure increased from 0 to 600 kPa.
With the 3D coordinates of targets on the cylinder surface measured under different conditions (i.e. exposed in air and in the triaxial chamber filled with air, water, and silicone oil), a comparison was made between the results obtained from the photogrammetric and photogrammetry-based analyses as shown in Figure 5. The measurement accuracy at different chamber pressure and confining medium conditions is represented by the gray level of the point. As shown in Figure 5, the overall point measurement accuracy of the photogrammetry-based method when water or silicone oil was used as the confining medium is better than that of the air or water confined condition which is consistent with the overall volume change measurement results. Also, it’s worth noting that the error distribution pattern for the measurements under different chamber pressures are similar to each other as shown in Figure 5. At different chamber pressure and confining medium conditions, instead of randomly distributed, the low measurement errors are always found to be at the right side of the cylinder and the high errors are at the left side of the cylinder. The reasons for this repeatable error distributions are attributed to: (1) the assumption on the triaxial chamber which was best-fitted by an equation represents a axisymmetrically barrel-shaped surface. Using the targets at the bottom circle as shown in Figure 1b as an example, Figure 6a presents the distances between those targets and reconstructed cell surface. The pattern of the error distribution indicates that the cross section of the triaxial cell is elliptical rather than circular; and (2) the assumption of a uniform cell wall thickness (i.e. 9.7 mm) is not consistent with the real cell wall thickness which varies from 9.40 to 9.89 mm based on the measurements using a digital caliper which is sensitive to 0.01 mm. Using the targets at the bottom circle as shown in Figure 1b as an example, Figure 6b presents the thickness of the cell wall at different locations which indicates that one side of the cell is slightly thicker than the other side. If the triaxial cell was perfectly axisymmetric with a uniform thickness, the measurement error of the photogrammetry-based method should be equal to that of the photogrammetric method. However, in this study, the assumption of an axisymmetric surface of cell wall with a uniform thickness inevitably resulted in the error in the traced optical rays due to the imperfection of the cell. The repeatable measurement error can potentially be minimized or at least reduced through calibration (e.g. using a rigid cylinder to determine the measurement error distribution) or more delicate fit (e.g. assuming elliptical instead of circular shaped cell cross section) on the chamber surface.
260

0 kPa, air  200 kPa, air  400 kPa, air  600 kPa, air

261

0 kPa, water  200 kPa, water  400 kPa, water  600 kPa, water
The average point measurement accuracy of the photogrammetry-based method with different confining medium conditions are calculated and summarized in Table 1. Due to the imperfection of the assumption on chamber shape, the overall measurement error increased from 0.003 mm (11) to 0.13, 0.15 and 0.21 mm from the photogrammetric analysis to the photogrammetry-based analyses when silicone oil, water, and air were used as the confining medium. Compared with the average point measurement accuracy of 0.07 mm presented in Zhang et al. (11) when water was used as the confining fluid, the accuracy obtained in this study is nearly twiced. The major reason for this is the used triaxial cells are in different sizes (101.6
and 166 mm in diameters for the cell used in Zhang et al. (11) and this study, respectively). The error in traced optical rays due to the assumption on cell wall would be magnified with the increase in the distance that the optical rays travelled. The point measurement accuracy reduction for the photogrammetry-based analyses from the water or silicone oil confined to the air confined condition is mainly due to the inaccuracy of the air refractive index. The overall measurement accuracy of the photogrammetry-based method when silicone oil is used as the confining fluid is slightly better than that when water is used as the confining fluid. Therefore, silicone oil, water, or some other transparent liquid, which refractive index remains to be constant under a relatively low pressure range (typically less than 1000 kPa), is recommended to be used as the confining medium when the photogrammetry-based method is used for soil deformation measurement during triaxial testing. Another benefit of using silicone oil or water as the confining fluid is the obtained images are in better quality due to the magnification effect of refraction as shown in Figure 1c and 1d when compared with those pictures for the cylinder in the chamber filled with air as typically shown in Figure 1b. The high quality images were very beneficial for accurate reconstruction of the incident optical rays for point position measurements.

**TABLE 1 Summarized Point Measurement Accuracy**

<table>
<thead>
<tr>
<th>Chamber pressure</th>
<th>Measurement error using different confining media (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Air</td>
</tr>
<tr>
<td></td>
<td>Average</td>
</tr>
<tr>
<td>0 kPa</td>
<td>0.210</td>
</tr>
<tr>
<td>200 kPa</td>
<td>0.208</td>
</tr>
<tr>
<td>400 kPa</td>
<td>0.197</td>
</tr>
<tr>
<td>600 kPa</td>
<td>0.213</td>
</tr>
</tbody>
</table>

STD: standard deviation
Sensitivity Analysis

For the photogrammetry-based method, an accurate refraction correction is highly dependent on refraction indices of acrylic cell wall, air, and confining fluid. For the acrylic cell wall, a refractive index of 1.491 was used. Since the thickness of the acrylic cell wall is much less than that of the confining fluid, accuracy of the photogrammetry-based measurement is more sensitive to the refractive index of the confining fluid. To use the photogrammetry-based method for soil deformation measurement during triaxial testing, some system information included camera information, acrylic cell wall thickness, refractive indices of confining fluid and acrylic cell wall were required for optical ray tracing. Camera information which included image sensor size, pixel numbers, focal length, and those distortion parameters could be obtained through camera calibration. So, the acrylic cell wall thickness, refractive indices of the confining fluid and acrylic cell wall were three parameters that could influence the measurement accuracy. Using water filled chamber as an example, in terms of the overall volume measurement accuracy, a sensitivity analysis was performed on three parameters for the photogrammetry-based method. Refractive index of the used water, $n_w$, was determined to be 1.339 using the method presented in the Appendix. Refractive index of the acrylic cell wall, $n_c$, is 1.491 provided by its manufacturer. Cell wall thickness was measured to be 9.70 mm. To perform the sensitivity analysis, each time, one of three parameters was set to be varying and the other two parameters were set to be their original values. The ranges for these three parameters were set to be 1.336 to 1.342 for water refractive index “$n_w$”, 1.488 to 1.494 for cell wall refractive index “$n_c$”, and 9.40 to 10.00 mm for cell wall thickness “$t$”. Sensitivity analysis results are presented in Figure 7.

![FIGURE 7 Sensitivity Analysis Results](image)

As shown in Figure 7, the measured volumes decreased with increases of cell wall thickness, water and cell refractive indices. Refractive index of water dominated the overall measurement accuracy. Since the thickness of cell is much less than the “thickness” of water (i.e. distance between the cell inner surface and the cylinder surface), impact of variations on cell wall thickness and refractive index is not significant when compared with that of water. As
addressed before, the refractive index of water can be influenced by wavelength of light. To achieve an accurate measurement, the refractive index of water is strongly recommended to be measured using the test presented in the Appendix.

**Triaxial Test on a Saturated Sand Specimen**

Beside the accuracy evaluation tests on the aluminum cylinder, a drained triaxial shearing test was performed on a saturated sand specimen to validate the accuracy of the photogrammetry-based method for volume change measurement. The same triaxial cell used for the tests on the aluminum cylinder was used for the test on the sand specimen. Oven dried fine sand (particle size: 0.075 to 0.425 mm) was used to fabricate a cylindrical specimen with a diameter and height of 71 mm and 126 mm. After compaction, the specimen was carefully mounted on the pedestal of the triaxial cell. A vacuum pressure of 50 kPa was applied to hold the sand specimen in place during sealing. Then, a total number of 252 measurement targets (21 targets/circle × 12 circles) were posted on the membrane as shown in Figure 8a. To ensure that the volume change of the specimen can be well represented by the movement of those measurement targets, two circles of measurement targets were posted on the top cap and the pedestal as shown in Figure 8a. In this way, the whole specimen was covered by the measurement targets. After this, the triaxial cell was installed and filled with tap water. The saturation process is consistent with that presented in Zhang et al. (11). When a B value of 0.95 was reached, saturation process was considered to be completed. Then, de-aired water was allowed to enter the saturated sand specimen from the bottom to top to replace the original water, which was full of compressed air inside. The use of de-aired water could reduce the possibility of water cavitation during triaxial testing. Drained triaxial shearing test could be performed after this.

![FIGURE 8 Drained Triaxial Shearing Tests on the Saturated Sand Specimen at Different Axial Displacement Levels](image)

A vertical displacement rate of 0.2 mm/min was applied to the saturated sand at a net confining pressure of 50 kPa. During loading, drainage valve was kept open to allow water
exchange. The volume change of the specimen was recorded by monitoring the amount of water flowed into or out of the sand specimen. At different vertical displacement levels (i.e. 0, 2, 4, 6, 8, 10, 13, 16, and 20 mm), load was paused and drainage valve was shut off. Then, the images were captured around the testing system for volume change measurement. In this way, there was no volume change of the specimen during image capturing. For each volume measurement, approximately 26 images were captured and used. The validation test was stopped when a total displacement of 20 mm was reached. Figures 8a, 8b, and 8c show the images of the specimen at axial displacement levels of 0, 10, and 20 mm. During shearing, the cylindrical specimen gradually turned into barrel-shaped with increasing axial displacement. The 3D coordinates of the measurement targets on the soil surface at different axial displacement levels were obtained using the photogrammetry-based method. Figure 9 shows the soil deformations at different axial displacements. The soil specimen was cylindrical at the initial stage. There was no obvious change in shape when the axial displacement was 2 mm. With increasing axial displacement, the soil specimen gradually bulged into a barrel-shape which is consistent with the observations shown in Figure 8. The shapes were reasonable since the friction between the soil and the loading platens restrained soil from deforming along the radial direction at both ends.

The volume changes of the soil at different axial displacements were calculated based on the triangular meshes shown in Figure 9. The volume changes of the soil specimen were also obtained using the conventional method through direct measurement of the volume of water exchange. A comparison between two methods at different axial displacement levels is presented in Figure 10. Both results indicated that the soil volume increased with an increase of the axial displacement ranging from 0 to 20 mm. The measurement results from the two methods matched well with a slight variation at the axial displacements of 10 and 13 mm. However, the overall measurement accuracy of the photogrammetry-based method is 0.092%. In GDS (17), the accuracy of a well calibrated double-wall cell volume change measurement method was claimed to be 0.25%. So, the photogrammetry-based method provided an alternative, which is in higher accuracy, to the conventional double-wall cell method.
In this study, soil volume changes of the sand specimen were calculated based upon the obtained triangular mesh. Different meshes could be generated if the number of the measurement targets for mesh generation were different. As a result, different volume change results were
expected if the meshes for volume calculation are different. With the 3D positions of measurement targets on soil surface, five triangular meshes using all or part of the targets were generated as shown in Figure 11 for volume change calculation. The total numbers of the measurement targets used for triangular mesh generation are 252, 168, 132, 110, and 88, respectively. The volume change results obtained using five meshes at different axial displacement levels are summarized in Table 2.

![Meshes A to E](image)

**FIGURE 11 Volume Calculation Using Different Meshes**

**TABLE 2 Volume Change Measurement Accuracy Using Different Meshes**

<table>
<thead>
<tr>
<th>Displacement (mm)</th>
<th>Water volume method</th>
<th>Photogrammetry-based method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mesh A</td>
<td>Mesh B</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1.27</td>
<td>0.73</td>
</tr>
<tr>
<td>4</td>
<td>4.72</td>
<td>4.31</td>
</tr>
<tr>
<td>6</td>
<td>8.56</td>
<td>8.22</td>
</tr>
<tr>
<td>8</td>
<td>12.34</td>
<td>12.07</td>
</tr>
<tr>
<td>10</td>
<td>17.09</td>
<td>15.97</td>
</tr>
<tr>
<td>13</td>
<td>22.14</td>
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<tr>
<td>16</td>
<td>26.59</td>
<td>26.12</td>
</tr>
<tr>
<td>20</td>
<td>31.24</td>
<td>31.47</td>
</tr>
<tr>
<td>Mean absolute error</td>
<td>---</td>
<td>0.092%</td>
</tr>
</tbody>
</table>
In Table 2, the mean absolute volume change measurement errors of five meshes are within 0.25% which is an acceptable accuracy for unsaturated soil volume change measurement. However, variations of measurement error exist among five different meshes. The volume change results obtained using meshes A and C with measurement numbers of 252 and 138 are more consistent with the conventional water volume measurement result. Theoretically, meshes generated using more targets tend to provide more representative volume change results. However, in Table 2, it could be found that the accuracy of the volume change results did not exactly increased with an increase in measurement target number. For example, the mean volume measurement error of 0.092% for Mesh C using 132 targets is less than 0.226% for Mesh B using 168 targets. The major reason for this is that the volume measurement accuracy can also be influenced by the pattern of the mesh and the deformation non-uniformity of the soil. Theoretically, under isotropic loading condition, less measurement targets could be used for soil deformation measurement during triaxial testing with acceptable volume change measurement accuracy since soil deformation is relatively uniform. However, during triaxial shearing, soil deformation could be significantly non-uniformly distributed as presented in Li et al. (12). Under this situation, if the soil deformation is axesymmetric, to achieve high volume measurement accuracy, more targets are suggested to be used along the axial direction of the soil. If the soil deformation is not uniform or axesymmetric, more targets are suggested to be used along both the axial and radial directions of the soil.

**Triaxial Tests on Unsaturated Soils**

The point and volume measurement accuracy of the photogrammetry-based method were evaluated through the tests on the aluminum cylinder and a saturated sand specimen. Since the photogrammetry-based method was mainly developed for deformation measurement on unsaturated soils, in this section, more triaxial tests were carried out on unsaturated soils under undrained condition. Locally available Fairbanks silt mixed with Kaolin at a ratio of 85:15 was used to fabricate the unsaturated soil specimens for the undrained triaxial tests. The optimal moisture content and maximum dry density are 15% and 1.836 g/cm$^3$ with a specific gravity of the soil is 2.7. Before compaction, oven-dried soil was mixed with water to a moisture content of 16%. In order to ensure the water is uniformly distributed, the mixed soil was stored in a sealed container for two weeks. After that, the soil was taken out, thoroughly mixed again, and then compacted in 10 layers to 71 mm in diameter and 142 mm in height soil cylinders using the under-compaction procedure (18). The soil specimens were then conditioned to different moisture contents. The same equipment for the triaxial test on the aluminum cylinder and saturated sand specimen was also used for the tests on unsaturated soils. After specimen installation, a total number of 108 measurement targets (9 rows by 12 columns) were posted to the membrane surface for volume change measurement. In this study, seven specimens with different moisture contents were tested under different loading conditions. The specimen information and the applied loading pathes for seven specimens are summarized in Table 3.
As shown in Table 3, four of the unsaturated specimens were first isotopically loaded to different net confining pressures (i.e. 5, 100, 200, and 300 kPa) and then sheared to failure or the maximum axial displacement of 20 mm under the same net confining stresses. Three unsaturated specimens were first isotopically loaded a net confining pressure of 300 kPa, unloaded to 150 kPa, and then reloaded to a net confining pressure of 500 kPa. The net confining pressure was applied with an interval of 50 kPa during loading and 75 kPa during unloading and reloading process. In the triaxial shearing stage, the soils were sheared at a constant loading rate of 1 mm/min. After an isotropic load or axial displacement change, load or displacement was paused and maintained to be constant.

Volumes of the specimens at different loading steps were obtained based upon the obtained 3D coordinates of measurement targets on soil surface. With the measured total weight, moisture content, and specific gravity of the soil, the specific volumes of the soil specimen were then calculated. Figure 12 presents the specific volumes of three specimens during isotropic loading. The soil volume responses of three unsaturated specimens with approximately the same moisture content are similar. Generally, the specific volumes decreased with an increase in isotropic load. During unloading, as shown in Figure 12, soil volumes were not fully recovered which is a clear indication of the elasto-plasticity of an unsaturated soil. Also, a clear loading collapse behavior of tested three specimens are observed in Figure 12.

TABLE 3 Specimen Initial Condition and Loading Paths

<table>
<thead>
<tr>
<th>We</th>
<th>Initial v</th>
<th>Loading path</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.56%</td>
<td>1.656</td>
<td>Isotropic load to 5 kPa, then shear to failure</td>
</tr>
<tr>
<td>14.57%</td>
<td>1.657</td>
<td>Isotropic load to 100 kPa, then shear to 20 mm</td>
</tr>
<tr>
<td>13.42%</td>
<td>1.655</td>
<td>Isotropic load to 200 kPa, then shear to 20 mm</td>
</tr>
<tr>
<td>14.66%</td>
<td>1.656</td>
<td>Isotropic load to 300 kPa, then shear to 20 mm</td>
</tr>
<tr>
<td>14.25%</td>
<td>1.581</td>
<td>Isotropic load to 300 kPa, to 150 kPa, then to 500 kPa</td>
</tr>
<tr>
<td>15.17%</td>
<td>1.624</td>
<td>Isotropic load to 300 kPa, to 150 kPa, then to 500 kPa</td>
</tr>
<tr>
<td>14.63%</td>
<td>1.657</td>
<td>Isotropic load to 300 kPa, to 150 kPa, then to 500 kPa</td>
</tr>
</tbody>
</table>
For four specimens sheared under different net confining stresses (i.e. 5, 100, 200, 300 kPa), the corresponding stress-strain curves were obtained and presented in Figure 13a. When sheared under a net confining pressure of 5 kPa, the applied deviatoric stress initially increased with an increase in the axial displacement, reached a peak at an axial strain of approximately 3%, and then decrease with increases in the axial displacement. When the soils were sheared under a net confining pressure of 100 kPa or higher as shown in Figure 13a, deviatoric stresses increased with an increase in the axial displacement.
Figure 13b shows the volume changes of soils during shearing under net confining pressures of 5, 100, 200, and 300 kPa. When shearing under a net confining pressure of 5 kPa, soil experienced volume increases (shear dilations) throughout the test. By contrast, when the soil specimen with nearly the same level of moisture content was sheared under a net confining pressure of 100 kPa as shown in Figure 13b, soil specimen experienced volume decreases at the early stage of shearing and then gradually increased with increasing axial strain. For specimens sheared under net confining pressures of 200 and 300 kPa, soil volumes decreased with increasing axial strain throughout the shearing stage till an axial displacement of 20 mm.

For the photogrammetry-based method, the volume measurement results at different loading steps are independent from each other which is different from the conventional double-wall cell method. For both volume change results shown in Figure 12 and 13b, the smoothness of the volume variation curves during isotropic and deviatoric loadings indicated that the soil volume measurement results obtained using the photogrammetry-based method are in high precision and reliable. Also, three volume change curves for soils with approximately the same moisture content presented in Figure 12 indicated that the results obtained using the photogrammetry-based method are repeatable.

**Suggestions for Accuracy Improvement**

As addressed above, the photogrammetry-based method is accurate in terms of point and volume change measurements on triaxial specimens. However, errors in this method can be introduced throughout the photogrammetric analysis and the optical ray tracing process. Several suggestions are provided for measurement accuracy improvement as follows.

(a) The used camera is clearly very important for any photogrammetric measurement. Whether a camera is suitable for photogrammetry measurement depends on several factors (such
as focal length and image sensor resolution). To reduce the effort from the photogrammetric analysis, a digital single-lens reflex camera with a fixed focal length lens is preferred (in this case, Nikon D7000 with a 50 mm fixed focal length lens) due to its low cost and acceptable resolution. Theoretically, an image sensor with high resolution would lead to more accurate measurement.

(b) Measurement accuracy of the photogrammetry-based method is highly dependent on camera calibration. Image quality, camera position, and the number of images used can influence the calibration result. Different calibration results would definitely produce different 3D measurement results. Image idealization to apply the pinhole camera model for photogrammetric analysis is based on the lens distortion parameters, format size, and perspective center of the image sensor. These parameters are all extracted from camera calibration. Different lens calibration result could introduce different pixel positions of measurement targets based on idealized images. Also, camera orientation results could be affected by camera calibration results due to different focal length and format size. So, the used camera for the photogrammetry-based analysis is suggested to be calibrated before being used for photogrammetric measurement.

(c) An assumption of barrel-shaped cell wall surfaces (i.e. the outer and inner surfaces) was made for the triaxial chamber reconstruction. Since the cell wall surfaces are the refractive interfaces for refraction correction, accurate optical ray-tracing is highly dependent on the representativeness of the barrel-shaped surface. In this case, to reduce the error from the optical ray tracing process, a transparent and cylindrical cell with a uniform thickness is highly recommended.

(d) The refractive index of water is an important factor that can significantly influence the accuracy of measurement. The refractive index of the confining medium is suggested to be measured using the test presented in the Appendix. Also, the same light source should be used for measuring since the refractive index of the confining medium can be affected by wavelength of light.

(e) To minimize the marking error for the photogrammetric analysis and optical ray reconstruction, high-quality images with larger depth of field are preferred. Therefore, during image capturing, the camera was set at low ISO level (less than 400), small aperture size (F-stop number of 9 or higher), and high shutter speed (< 1/160 s), with the built-in flash on. In this way, high-definition images could be captured.

(f) The least-square method is used for the estimation of the 3D coordinates of measurement targets on soil surface. To reach high measurement accuracy, more images from different view angles are suggested to be captured for the photogrammetry-based analysis. For the measurement on a single target, at least five optical rays are suggested for the least-square estimation.

(g) If necessary, a calibration test, similar to the triaxial test on the aluminum cylinder, could be performed with and without the triaxial chamber conditions. If accurate system parameters such as refractive indices of the confining fluid and cell wall and cell wall thickness are not available, a least square method could be applied to find the “best” combination of the system parameters by minimizing the differences between the measurement results obtained with and without the triaxial chamber conditions.

CONCLUSIONS

The point and volume measurement accuracies of the newly developed photogrammetry-based method were evaluated through triaxial tests on an aluminum cylinder and a saturated sand
specimen when different confining media (i.e. air, water, and silicone oil) were used. Based upon the results from these triaxial tests, the point and volume measurement accuracies were evaluated to be sufficient for the volume change measurement of unsaturated soils during triaxial testing. It should be noticed that the accuracy of the photogrammetry-based method are restricted to the triaxial testing system used in this study. Different measurement accuracy is expected when using different triaxial chambers (i.e. triaxial chambers in different diameter, shape, thickness, and refractive index of the cell wall). Through a sensitivity analysis on the system parameters, the refractive index of the confining medium was found to be very critical on the overall measurement accuracy. When using the photogrammetry-based method, an accuracy evaluation test on a rigid specimen is strongly recommended before being used for any soil deformation measurement during triaxial testing. Through a validation test on a saturated sand specimen, it was found that the volume change measurement accuracy of the photogrammetry-based method can be influenced by the number of the measurement targets on soil surface, the pattern of triangular mesh, and deformation characteristic of the soil during testing. In addition, through a series of triaxial tests on unsaturated soils, reasonable and repeatable volume change results were obtained which indicated that the photogrammetry-based method is accurate and reliable for deformation measurement on unsaturated soils during triaxial testing.
REFERENCES


APPENDIX
Confining Fluid Refractive Index Determination

This photogrammetry-based method can be used for measurement of object under multi-media condition. The following experiment was designed and performed to accurately determine the refractive index of the used confining fluid (silicone oil and water) for the triaxial test. As shown in Figure 14a, several measurement targets were attached to a stainless steel container. Targets at the edge of the container were used to build a coordinate system. Four targets at the bottom of the container were used to back-calculate the refractive index of the confining fluid. Images were captured first to determine the 3D coordinates of those targets using the photogrammetric analysis. The coordinate system was then built based on the measurement targets on the edge of the container. Then, as shown in Figure 14b, the container was filled with the confining fluid and some more measurement targets were placed at the water surface which were used to determine the refractive interface (i.e. air-water interface) for refraction correction. After this, image capturing was performed again. Subsequently, a photogrammetric analysis was performed to determine the 3D coordinates of measurement targets on the confining fluid surface and the edge of the container in the same coordinate system.

![Container with and without the Confining Fluid](image)

(a) without the confining fluid (b) with the confining fluid

Figure 14 Container with and without the Confining Fluid

The 3D coordinates of four targets at the bottom of the container were determined before filling with the confining fluid. The 3D coordinates of the targets on the confining fluid surface were used to determine the refractive interface. Then, for four points in the confining fluid, optical rays were constructed from the corresponding camera positions to the image points based on pinhole camera model. With given refractive interface and a initial guess of the refractive index of the confining fluid, optical rays after refraction were determined. Using the least-square method presented in Zhang et al. (11), the 3D coordinates of four targets can be determined. The refractive index of the confining fluid was back-calculated by minimizing the difference between the 3D coordinates obtained using the photogrammtric (no water in the container) and the photogrammetry-based (the confining fluid in the container) analyses. In this case, the refractive index of the used water and silicone oil were determined to be 1.339 and 1.407, respectively.