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Modeling the spread of infection in public transit networks: A decision-support tool for outbreak planning and control

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32

Abstract

33

34 Identifying components of the public transit system most likely to exacerbate disease spread
35 is critical for public health authorities to be able to plan for epidemics and control their
36 spread. In this work we propose a method to detect such components in a transit network
37 using a three-stage approach. We first use results from a transit simulation model to generate
38 a contact network that is representative of the potential physical encounters between
39 passengers. We then design and run a variety of epidemic scenarios atop this network which
40 vary in their set of initially infected individuals, and properties of the disease. The expected
41 spread (and variance) of the outbreak in terms of the infected passenger set are compared
42 across scenarios. The infected passenger sets are then used to identify the vehicle trips most
43 likely to transport infected passengers during an epidemic, which represent optimal locations
44 to implement vehicle surveillance. Results from a case study using the public transit network
45 from Austin, TX are presented. We show that the infection scenarios vary substantially in the
46 pattern of spread at the individual level, but the set of transit vehicle trips at highest risk of
47 infection is robust to the initial conditions of the outbreak.

48

49 1. INTRODUCTION

50 In many parts of the world a large proportion of the population lives and commutes in
51 increasingly dense conditions, ideal for rapid disease transmission. Moreover, a major
52 challenge is faced when planning for and controlling infectious disease outbreaks due to the
53 stochastic nature of the spreading process (i.e., contact between an infectious and susceptible
54 person may or may not result in a new infection) combined with the lack of information
55 traditionally available on individual's daily contact patterns. For these reasons it is difficult to
56 predict the impact that a new disease might have on a region, and similarly, design the best
57 control strategies for public health authorities to implement during an outbreak.

58 Of particular interest in this study is the role of the public transit system in the spread of an
59 infectious disease, which can act as a potential catalyst in the transmission process within
60 metropolitan regions. We explore the risk posed by a regional public transit system in the
61 event of an epidemic, with the goal of identifying potential vulnerabilities or critical
62 components in the transit system (e.g., super spreading vehicle-trips), which can be
63 prioritized for monitoring and control during an emerging outbreak (1, 2).

64 We propose a network structure to represent the passenger contacts and movements within
65 the public transit system. The network generated is used to model outbreak behavior within
66 the transit system. A range of outbreak scenarios are simulated, and the most likely sets of
67 infected passengers and vehicle trips are evaluated. We present results for a case study which
68 uses transit simulation model results from Austin, TX.

69 2. BACKGROUND

70 The spread of contact-based infectious diseases can be modelled as a dynamic process atop
71 social contact networks (3-7). Various agent based simulation models have been developed
72 and implemented to replicate possible spreading scenarios, predict average spreading
73 behavior, and analyze various intervention strategies for a given network and disease (8-22).
74 Similar models have also been used to explain the propagation of information, ideas and
75 opinions between individuals (23, 24).

76 To accurately predict disease spread, models must exploit characteristics of the population
77 and the disease itself; and thus require large amounts of data, and significant computational
78 resources. However, computational capabilities are exponentially increasing, and
79 developments in data collection are increasingly providing a means for accurate mappings
80 between known individuals. In particular, spatial analysis of transport and communication
81 networks which exploit these data sources are a growing area of research (25-38). The
82 ongoing development of activity-based travel models, which examine why, where and when
83 various activities are engaged in by individuals (39-43), as well as innovations in pedestrian
84 modeling (44) also present additional promising alternatives to generate social-contact
85 networks in the future. These efforts have recently been expanded to include social network
86 modeling, specifically the ability to reproduce spatial structure and interaction between
87 individuals for large-scale social networks (42, 45-47).

88 Of particular applicability to this study are the recent advances in public transit modeling,
89 which are able to provide detailed contact patterns, including temporal patterns (e.g. bus
90 travel time) and spatial patterns (a function of the vehicle size and passenger volume) (48,
91 49). While these methods can potentially allow accurate mappings between known
92 individuals, the data collection and processing required to recreate a real-world contact

93 network is expensive, computationally costly, and time intensive, among other challenges
94 faced such as privacy issues (50-55). As such, it is critical to develop methods which can
95 exploit this data as it becomes available, which is the contribution of this study.

96 Examples of such approaches have been recently addressed in the literature. Wesolowski et
97 al. (56) highlighted the value of mobile network data for modelling human mobility patterns
98 in real time, the results of which can be used in designing surveillance and containment
99 strategies during an outbreak. The authors constructed intra and international mobility
100 patterns in the set of African countries affected by and surrounding the Ebola outbreak. Their
101 work exemplifies the role of increasingly available real-time connectivity data in public
102 health response. Similarly, Sun et al (57) generated a physical encounter network using daily
103 transit data from Singapore smart cards, and then used the contact network generated to
104 compare different individual-based sensor schemes for early detection of simulated outbreaks
105 (58). Their work utilized a “friends as sensors” approach, similar to that in (59-63), applied
106 atop a more complete physical contact network.

107 Our study takes a step further in exploiting a public transport data set for planning city level
108 disease mitigation and control efforts. Whereas Sun et al. (58) attempted to identify the
109 individuals in the contact network that would serve as the best predictors of an outbreak, this
110 work seeks to identify the *components of the transit system* that are most vulnerable (*i.e.*,
111 most likely to have infected riders) during an outbreak (or attack). This analysis provides a
112 means to understand what role the different components of the transit system play in the
113 spreading process, and better predict how an outbreak might evolve over space (particular bus
114 routes) and time.

115 The contact network of passengers is generated using data from Capital Metropolitan
116 Transportation Authority in Austin, TX. The nodes of the network represent passengers, and
117 they are connected if they ride on the same vehicle (bus or train) during their travels. The
118 links are undirected with two attributes attached to them: the duration and the start time of the
119 contact. A range of outbreak scenarios, which vary based on the number of location of
120 initially infected individuals and properties of the disease, are then modelled atop this contact
121 network, and the resulting infection patterns (*i.e.*, set of individuals and bus routes most likely
122 to be infected) are compared.

123 We divide the analysis of the urban transportation network into two stages. The first stage
124 quantifies network-based statistics on the contact network described above, including contact
125 duration patterns, centralities and group structure. The second stage of the analysis focuses on
126 modelling epidemics. A traditional discrete compartmental model (SIR) is implemented to
127 simulate the spread of infections between individuals based on contact patterns. The
128 sensitivity analysis reveals the robustness of the system to the initial conditions of a particular
129 outbreak (*i.e.*, random versus targeted bus routes, and level of infectiousness of the disease).

130 The remainder of this manuscript includes a description of the data used in the study, the
131 methodology implemented, followed by numerical results and conclusions.

132 **3. DATA**

133 In order to model passenger movement in the transit network, we used a schedule-based
134 transit assignment model. The key point in such models is the time-dependent format of the
135 network, in order to represent service variability during the day as well as modeling transit
136 vehicle trips, as opposed to routes. This is particularly important because first, passengers’

137 route choice is more likely affected by service schedule and second, simulating their
138 movement in the network requires knowing the particular vehicle they board. The transit
139 assignment model, FAST-TripS (64) uses General Transit Feed Specifications (GTFS) data
140 for the Austin, TX region in 2015 to build the schedule-based network. Passenger movement
141 is then modeled by a previously estimated route choice model (65), in which parameters such
142 as in-vehicle time, waiting time, walking time (for access, egress and transfer), and transfer
143 penalty are incorporated. Finally, passengers' movement is simulated in the schedule-based
144 network such that passengers are loaded to the network according to their chosen path.
145 During simulation, passengers are modeled as agents, were created and loaded to a FIFO
146 queue at their first boarding stop, boarded the assigned vehicle-trip upon its arrival, alighted
147 at the alighting stop, and walked to either the next boarding stop or the destination. This
148 process is modeled by an event-based simulation in which vehicle arrivals and departures are
149 events as well as passengers' arrival at the stop (or destination), boarding, and alighting.
150 Clearly, the model captures the interaction between passengers on the vehicles and at the
151 boarding stops.

152 The transit assignment model was developed for Austin TX region. It involves a network of
153 75 routes (one commuter rail, two arterial BRT, 7 express bus routes, and 54 local bus routes,
154 and 11 university shuttle routes) and 4,859 vehicle trips serving 2,653 stops in a weekday 24-
155 hour period. On the demand side of the model, we used the transit origin-destination trip
156 matrix from the 2015 travel demand model developed by the metropolitan planning
157 organization in the region. In order to prepare the data for the assignment model, synthesized
158 passengers were generated using the OD data and an aggregate departure time profile
159 obtained from passenger count data. The assignment model was run and 88,570 passengers
160 were assigned to the network, resulting in 111,640 unlinked passenger trips.

161 The transit assignment model generates various detailed outputs including the load profile of
162 vehicles and passenger trajectories. In the load profile output, for each vehicle trip, the
163 number of boarding and alighting passengers at each stop along with the time stamp and
164 estimated vehicle dwell time is reported. Therefore, one can know how many passengers are
165 on-board a vehicle in each segment of the route. The passengers' trajectory output, for each
166 passenger, contains every activity during the course of the passenger trip along with stop ID,
167 vehicle ID, and time stamp. Therefore, one can map the passenger activities to the schedule-
168 based network and capture the interaction between passengers at each point in the space-time
169 diagram.

170 **4. NETWORK STRUCTURE**

171 The collected transit data naturally defines a network structure. Passenger trajectories from
172 the assignment model output are processed to create a contact network. Each passenger is
173 considered as a node in the contact network, and links connect any two passengers that are on
174 the same vehicle for a positive time period (i.e. in a segment of the corresponding transit
175 route). The complete network is comprised of 88,570 nodes and 2,260,373 links.

176 For the purposes of our analysis, the time dimension of the network is defined on each link by
177 two components: a *time stamp*, corresponding to the time when the second passenger gets on
178 the vehicle while the first passenger is already on-board, and the *contact duration*, which
179 indicates the temporal overlap between two passengers, i.e., the time until one of the
180 passengers gets off the vehicle. In this study we focus on the contact patterns between
181 passengers in the morning peak period between 5 AM and 10 AM, therefore we only consider

182 the subgraph induced by the set of links where the time stamp indicating the start of the
183 contact is within this period. We denote this network as the *contact network*. The contact
184 network contains 31,176 nodes (passengers) and 1,006,459 links (contacts).

185 The id of the *vehicle trip* is an additional link attribute representative of where the contact
186 takes place. The vehicle trip refers to a specific route with a specific departure time, and is
187 specific to a single vehicle. Thus, the contact network includes a set of vehicle trips assigned
188 to each passenger, and the set of passengers on each vehicle trip. We denote the passengers
189 travelling on multiple trips as *transfer passengers*, while the passengers utilizing only a single
190 vehicle trip as *bulk passengers*.

191 **5. NETWORK STATISTICS**

192 Before discussing epidemic spreading on the network, we examine the structure of the
193 generated network in detail.

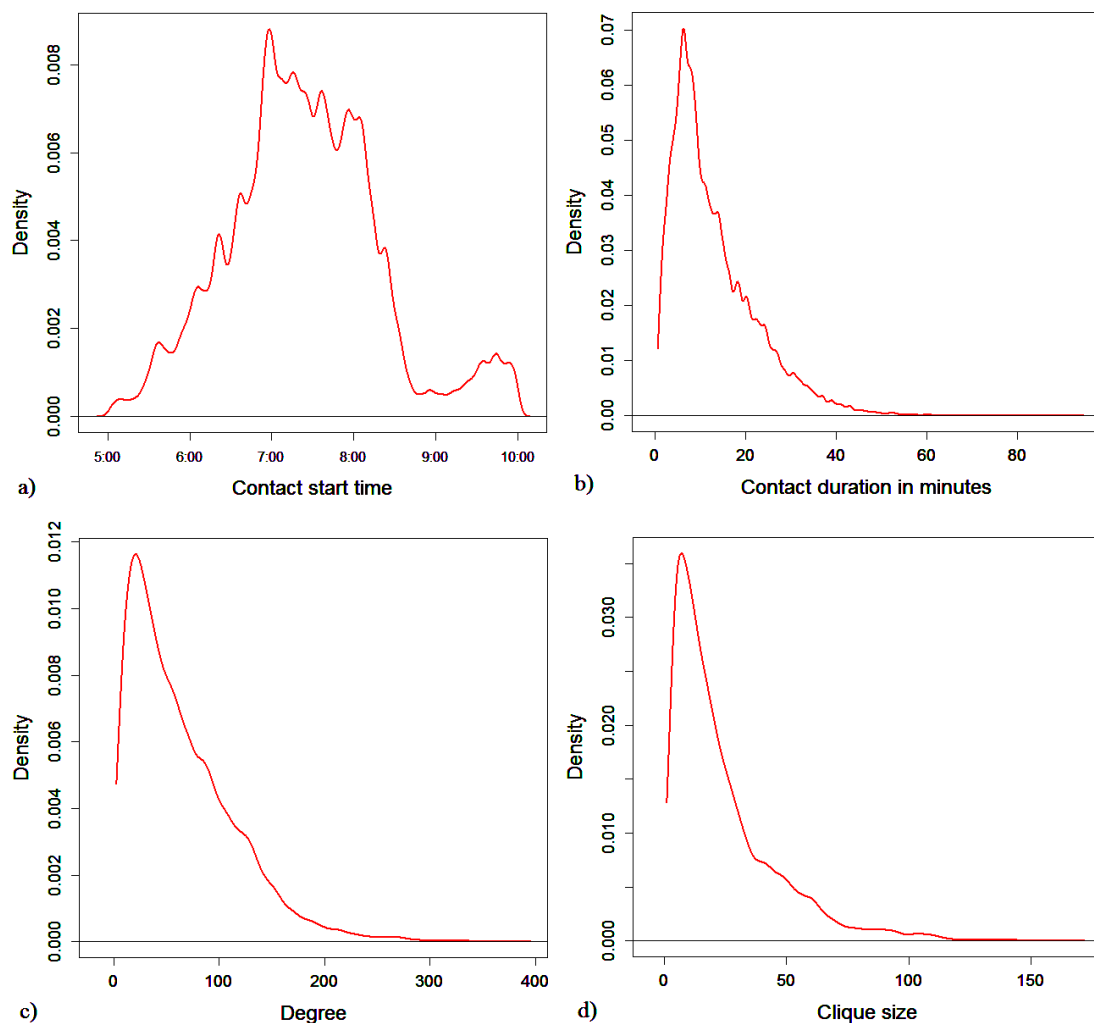
194 **5.1 Contact network**

195 We mentioned before that the contact network represents the morning peak behavior of
196 passengers on an urban transportation network on a single workday. For each contact the start
197 time and the duration is available, as well as the id of the vehicle where the contact takes
198 place.

199 The density plot of the contact start times and durations are illustrated in Figure 1/a and 1/b.
200 The start time distribution peaks at 7 AM and remains high until 9AM, which is consistent
201 with a morning weekday commute. The majority of the contacts are in the magnitude of
202 several minutes. The degree distribution of the network is shown in Figure 2/c. It follows a
203 skewed power law, where the average number of contact per person is 65 while the maximum
204 is 348. More than three quarters of the passengers do not transfer: they are bulk passengers,
205 more than 20% of the travelers' only change vehicles once, and less than 2% change vehicles
206 two or more times. While the number of transfer passengers is low, their role is critical; they
207 represent the set of individuals that connect different parts of the network together allowing
208 the transmission of diseases.

209 The clique size distribution can be seen in Figure 2/d. Since every passenger travelling on a
210 vehicle at the same time is connected and the number of transfer passengers is relatively low,
211 the network can be considered as a union of large overlapping cliques. In fact, if we examine
212 the sizes of maximal cliques we can find that there are only 137 2-cliques (single edges) and
213 around 300 triangles; the rest of the network is covered by cliques larger than this. If we
214 group the nodes together according to vehicle trips, the resulting sets will be very dense and
215 the overlaps between them (the transfer passengers) will be small, further emphasizing this
216 structure.

217



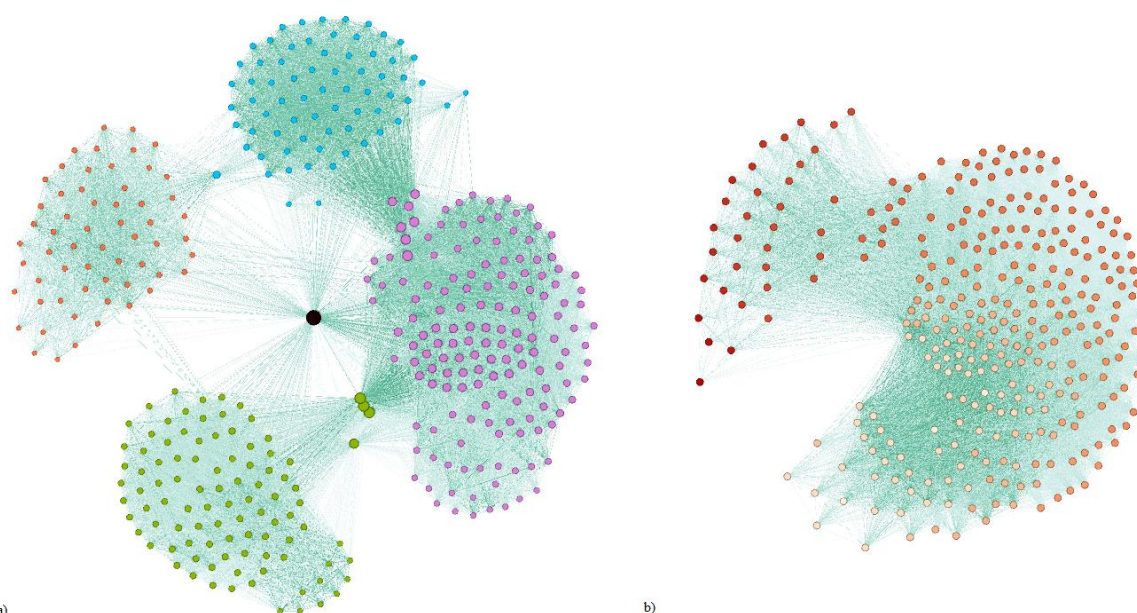
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219 **Figure 1. The distribution of (a) contact start times, (b) contact durations, (c) the degree**
 220 **distribution and (d) clique size distributions of the contact network.**

221 Since the network is too large to be appropriately visualized, we illustrate this phenomenon
 222 on the two subgraphs of Figure 2. Figure 2/a illustrates the passenger with the most contacts,
 223 which is the black node in the middle, and its direct neighborhood. The nodes are colored
 224 according to vehicle trips, and darker edges indicate longer contact durations. It is clear that
 225 nodes are easily grouped according to vehicle trips. In reality these nodes were present on
 226 multiple vehicle trips and the color only indicates the first vehicle trip with the central
 227 passenger. This graph again illustrates the low number of transfer passengers.

228 Figure 2/b illustrates the contact network of the vehicle trip with the highest ridership. Here,
 229 the nodes are colored according to the contact start time, with lighter colors corresponding to
 230 earlier connections. The links are still shaded according to contact duration. This figure
 231 depicts how contacts on a single trip change in time. It reveals that the passengers that meet
 232 earlier in the morning tend to travel together for longer durations. It should be noted that not
 233 all of the passengers are on the vehicle at the same time. This figure also again highlights that
 234 the contact network on a single vehicle trip is very dense, and a union of large cliques.

235



236

237 **Figure 2** Two subgraphs of the contact network. 2/a The passenger with the most
 238 contacts is represented by the black node in the middle, connected to all contacts. The
 239 colors are indicative of the vehicle trips the passengers first met on, with darker edges
 240 representing longer contact durations. 2/b illustrates the contact network of the most
 241 frequented vehicle trip where colors indicate contact start times.

242 6. MODELING THE SPREAD OF INFECTION

243 The focus for the remainder of this paper is the analysis of epidemic spreading scenarios
 244 within the public transit network. For this purpose, we simulate epidemic spreading on the
 245 contact network, which is assumed to represent passenger travel patterns on a typical morning
 246 workday commute. A five-day observation period is considered to represent a full work
 247 week, and the contact network is assumed to remain constant across days. We consider
 248 multiple infection scenarios differing in both the size and spatial distribution of the initially
 249 infected individuals, and the level of infectiousness of the disease. The network-based
 250 infection model used to model the spread of disease between the individuals described in the
 251 following section is.

252 6.1 Infection model

253 A discrete compartmental susceptible-infected-removed (SIR) model is used to model an
 254 epidemic on the network, and is defined as follows. The SIR infection process takes place on
 255 a network defined as $G(V, E)$, where $V(G)$ denotes the node set of G , and $E(G)$ denotes the
 256 edge set of G . The process can be defined on directed and undirected networks, but in this
 257 study we only consider the latter case. Real values denoted as edge infection probabilities are
 258 required on the edges of the network, these are defined as $w_e \in [0, 1]$ for all $e \in E(G)$. An
 259 edge infection probability w_e on an edge $e(u, v)$ denotes the probability that if node u is
 260 infected, the infection spreads to node v , or if v is infected, the infection spreads to u .

261 During the SIR infection process all nodes are required to adopt one of the three available
 262 states. A node can be susceptible (S), meaning it is not infected and can become infected,
 263 infected (I), meaning it is infected and may spread the disease to neighboring nodes, or
 264 removed (R) which means it was previously infected and has since recovered and is no longer

265 able to spread infection. A discrete time period is attached to the infected state denoted as τ_i
 266 indicating the amount of time a node spends in this state. The non-empty set of *initially*
 267 *infected* nodes will be denoted as $A_0 \subseteq V(G)$; these nodes are the *sources* of the infection.
 268 We define an *infection scenario* as the triple $S : (G, W, A_0)$, where G is a graph, $W : E(G) \mapsto$
 269 $[0,1]$ is a surjective assignment of edge infection probabilities to the edges of the graph, and
 270 A_0 denotes the set of initially infected nodes. The infection scenario defines the input of an
 271 infection process.

272 The infection process itself is iterative and takes place in discrete time steps, according to the
 273 following logic. Let $A_i \subseteq V(G)$ denote the set of infected nodes in iteration i . Each infected
 274 node $u \in A_i$ tries to infect its susceptible neighbors $v \in V(G) \setminus \bigcup_{0 \leq j \leq i} A_j$ according to w_e .
 275 If the attempt is successful v will be in an infected state starting from iteration $i + 1$. If the
 276 attempt is unsuccessful, node u it may make additional attempts to infect v depending on i
 277 and τ_i . If more than one node is trying to infect v in the same iteration, the attempts are made
 278 in an arbitrary order independent of each other. Finally, all nodes in an infected state change
 279 their state to removed if the time they became infected t_u is $t_u = i - \tau_i$.

280 The application of the SIR infection model to the contact network in this study can be
 281 interpreted as follows: Each iteration corresponds to a single day. If a person gets infected in
 282 iteration i then he is infectious for τ_i days, starting from iteration $i + 1$, indicating a latency
 283 period of a single day¹. In this study we assume an infectious period (τ_i) of five days. Results
 284 from the process provide the state of each individual (S, I or R) and each iteration. Due to the
 285 five-day observation period, any infected individual remains infectious throughout the entire
 286 simulation, i.e., there will be no recovered individuals within the timeframe considered.

287 Due to the stochastic nature of the model, we seek to estimate the likelihood of infection for
 288 each node at each iteration. In this study for each outbreak scenario considered we run the
 289 infection process k times and count the frequency of infection for each node at any given
 290 iteration to estimate the probability that a node is in an infectious state at any iteration. For all
 291 results in this paper $k = 100,000$ was used. This approach is similar to that used in (66, 67).

292 6.2 Experiment Design

293 In order to analyze the effect of an epidemic on the public transportation system we consider
 294 a diverse set of infection scenarios on the contact network. The scenarios represent a high
 295 variety of initial outbreak conditions, which can be divided into two strategies according to
 296 the definition of the infection scenario in the previous section. The edge infection probability
 297 assignment represents the level of infectiousness of a particular disease, while the set of
 298 initially infected nodes represents the source(s) of the outbreak. The set of initially infected
 299 nodes can be further divided into two factors: the size of the initial set and the spatial
 300 distribution of it.

301 Following the assumption of (57) we define the transmission probabilities as a linear function
 302 of the contact duration between individuals: $p_{u,v} = d_{u,v} * \beta$, where $p_{u,v}$ is the transmission
 303 probability between nodes u and v , $d_{u,v}$ is the contact duration between u and v in minutes,
 304 and β is a constant. We consider four different values for β which we believe represent a

¹ While in the infection model, this translates into a latency period of zero iterations.

305 wide range of outbreaks. $\beta = 0.001$ which puts the edge infection values between 0 and 0.1,
306 $\beta = 0.003$ which puts the transmission probabilities between 0 and 0.3, $\beta = 0.005$ with the
307 bounds 0 and 0.5 and $\beta = 0.007$ with 0 and 0.7.

308 The size of the initially infected node set A_0 will be 10, 50 and 100 representing a small,
309 medium and large number of initially infected individuals. These individuals are distributed
310 according to two different strategies, similar to the works of (68). The first strategy considers
311 the top 10, 50 and 100 most central² individuals corresponding to a “targeted” strategy, while
312 the second strategy selects the same amount of nodes randomly. In the random selection
313 strategy ten samples were drawn independently from $V(G)$ for each scenario³, and the test
314 results were averaged.

315 **6.3 Limitations of the study**

316 Before presenting the results certain limitations of this study should be noted. The first set of
317 limitations is due to the available data. The passenger demand used in the assignment model
318 (used to generate the contact network used in this study) was estimated using a traditional
319 *trip-based* travel forecasting model; thus individuals could not be tracked between their
320 morning and evening commute due to the lack of information about their daily tour. For this
321 reason, only the morning peak period (rather than the entire day) was used in this study.
322 Secondly, the contact network is generated using a single morning of data, but used to model
323 the spread of an outbreak over a 5-day period. This implies the assumption that morning
324 commute patterns remain constant day-to-day over a 5-day work week. While individual’s
325 travel patterns can vary day-to-day, a recent study (57) which used travel smart card data to
326 generate an in-vehicle social encounter network on public buses using a full week of travel
327 data found that physical encounters display reproducible temporal patterns. The finding that
328 repeated encounters are regular and identical, and rooted in daily behavioral regularity
329 supports the assumption used in this work. While the restriction to morning peak travel does
330 limit the applicability of our results (we underestimate the spread of disease), the proposed
331 methodology, which is the main contribution of this work, can be directly applied to a more
332 complete contact data set. Planned future research will utilize a 24 hour (or longer) travel
333 period that tracks the individual users over the entire period.

334 Implicit assumptions also result from the use of the SIR model, which restrict this study to
335 the family of infectious diseases that are transmitted from an infected to susceptible
336 individual via direct or close contact. This category includes various strands of the flu, SARS
337 and the common cold, among others and, diseases which have an infectious period greater
338 than five days. These assumptions could easily be relaxed without altering the proposed
339 methodology, and a more complex compartmental model, *e.g.*, SEIR, could be substituted.

340 Finally, this study ignores all contacts made outside of transit movements, which in reality
341 would significantly increase the size of the outbreak within a region (while also resulting in
342 more infected transit riders). However, predicting the size of the outbreak is not the intended
343 goal of this study. Instead, the contribution is the proposed methodology to identify the
344 components of the public transit system that play a critical role during the early stages of an
345 outbreak. The identified transit-trips can be prioritized for surveillance monitoring and

² Degree centrality was computed and used to identify the set of nodes.

³ For each size configuration (10, 50, 100) ten different samples were taken.

346 possible mitigation and control efforts by transit and public health authorities during an
347 outbreak.

348 7. RESULTS

349 We examine the results of the infection simulations in this section. Our key point of interest
350 is the behavior of the outbreak as a function of the initial conditions. Apart from the fraction
351 of infected individuals we are most interested in the components of the transit system that are
352 at highest risk of infection. Due to space restrictions we present all results as of day 5;
353 however analogous results for any stage of the outbreak are available.

354 7.1 Comparison in initial infection scenarios

355 Table 1 reveals the fraction of infected individuals in the contact network (on day 5) for each
356 infection scenario considered. The rows correspond to the transmission probabilities, while
357 the columns indicate the size of the initially infected set and the selection strategy. As
358 expected, the prevalence of the outbreak increases with number of initially infected
359 individuals, *i.e.*, size of A_0 , and the level of infectiousness of the disease, β . Results are also
360 provided for the targeted and random strategies, which only vary based on the spatial
361 distribution of the initial set of infected individuals.

362 **Table 1. Fraction of infected individuals on the contact network on day 5. The rows**
363 **denominate the constants used to compute the edge infection probabilities, while the**
364 **columns indicate the size of the initially infected set and the selection strategy. The**
365 **values in parenthesis for the random strategy indicate standard deviation.**

	Targeted strategy			Random strategy		
	$ A_0 = 10$	$ A_0 = 50$	$ A_0 = 100$	$ A_0 = 10$	$ A_0 = 50$	$ A_0 = 100$
$\beta = 0.001$	0.0262	0.0612	0.1015	0.0068(58%)	0.0311(22%)	0.0631(12%)
$\beta = 0.003$	0.1171	0.2177	0.2921	0.0442(47%)	0.1693(15%)	0.2826(8%)
$\beta = 0.005$	0.2075	0.3381	0.4183	0.0993(45%)	0.3131(11%)	0.4512(5%)
$\beta = 0.007$	0.2847	0.4299	0.5051	0.1623(33%)	0.4297(8%)	0.5647(4%)

366

367 The results in Table 1 generally confirm previous observations (68) that a targeted strategy
368 has a much greater impact on the network, than a random one. There is some difference in the
369 finer details however. To illustrate this, we classify the behavior of the infection process into
370 three categories. We assign the infection scenarios to the above categories as follows. The
371 pairs indicate $(\beta, |A_0|)$.

372 I. $(0.001,10), (0.001,50), (0.001,100), (0.003,10), (0.005,10), (0.007,10)$.

373 II. $(0.003,50), (0.003,100), (0.005,50), (0.007,50)$.

374 III. $(0.005,100), (0.007,100)$.

375 In the first category (I) a targeted strategy has greater impact than the random one. This is
376 true if the size of the initial outbreak or the network transitivity (edge infection values) is
377 small, and also corresponds with the observations of (68). For example, if $\beta = 0.001, |A_0| =$
378 10 the difference is almost a multiplier of 4. For scenarios with high transitivity $\beta =$
379 $0.005, |A_0| = 10$ the observation is still true albeit of a lesser magnitude. This is because
380 with a small number of sources or low levels of infectiousness, the set of the initially

381 infection individuals is critical. In other words, a small number of well-connected individuals
382 chosen as source infections have a much more substantial impact on the system compared
383 with the same number of randomly selected individuals.

384 The second category (II) corresponds to a broad range of configurations where there is little
385 difference between targeted and random strategies. In this case the network transitivity and
386 the size of the initial infections are such that the selection strategy for the source infections
387 matters less; the infection spreads around the network to a similar number of individuals. On
388 the contact network, this occurs when the prevalence of infection is moderately high.

389 In the third category the random infection strategy has greater impact than the targeted one.
390 The explanation behind this somewhat counter-intuitive, and may be due to assortativity. In
391 assortative networks nodes tend to be connected to other nodes with similar degrees. This is
392 true for the nodes with high degrees as well, grouping them together. In contrast, random
393 nodes are selected randomly so they are spread out across the network. Since the network
394 transitivity is high and there are a considerable number of initial infections, the randomly
395 selected (and more spread out) individuals may be able to infect more individuals than the
396 well-connected (but grouped) targeted individuals.

397 The categorization becomes clearer if we look at the standard deviation values for the random
398 selection strategy indicated by the values in parenthesis in Table 1. These represent how the
399 expected size of the infection varies across the ten samples for each $|A_0|$ of a random
400 scenario. High deviation values in the first category show that the selection of the initially
401 infected set is critical and has a considerable effect on the outcome. For the second and third
402 categories, this becomes less true, to the point, that it does not matter how the sources of the
403 infection are chosen, the network “takes care” of the spreading process.

404 We examined the expected number of infected passengers on each individual day and found
405 that in all infection scenarios the outbreak progresses in a near-linear fashion on subsequent
406 days, with the slope of the ascent depending on the initial conditions. There is little difference
407 between the categories.

408 **7.2 Identification of critical components of the transit system**

409 The previous section explored the evolution of the outbreak across individuals in the network.
410 However, the main objective of this study is to identify the most critical bus trips responsible
411 for furthering the spread of disease amongst passengers (and more generally within the
412 region) during an outbreak. To identify these critical components of the transit system we
413 compute and rank the vehicle trips according to their likelihood of transporting infected
414 passengers.

415 The infection model defined in Section 6 provides the likelihood of infection for the nodes of
416 the network, which represent individuals. Using the known assignment of the individuals to
417 vehicle trips and the infection probability for each individual passenger, we compute the
418 likelihood of infection on each vehicle trip by averaging the probability of infection for all
419 passengers on a given vehicle trip at each iteration, *i.e.*, day. The vehicle trips are then ranked
420 according to their corresponding probability of infection and the top 100 are considered. In
421 this study 100 is representative of the top 10% of vehicle trips in the network we evaluate, but
422 any threshold level could be chosen. The process is conducted for each infection scenario
423 simulated.

424 We first compare the robustness of the highest risk vehicle trips *within* each scenario. The
 425 maximum number of times a vehicle trip appears in the top 100 ranking across the ten
 426 samples, and the number of vehicle trips with this maximum frequency are provided in the
 427 left hand side of Table 2, with the latter in parentheses. For example, 8(3) means that 3
 428 different vehicle-trips appeared in the top 100 ranking in 8 of the ten samples (for a given
 429 random scenario). The right hand side of the table compares the overlap between each
 430 random strategy (using the top 100 most likely infected vehicle trips across all ten samples)
 431 and targeted strategy for the same initial outbreak conditions. It is clear for the more
 432 aggressive outbreaks (more contagious with a larger number of sources) the set of most likely
 433 infected vehicle trips appears in all ten samples, and is more likely to appear in the targeted
 434 strategy. Almost all outbreak scenarios share more than 50 vehicle trips across the respective
 435 random and targeted strategy, suggesting robustness in the set of riskiest vehicle trips. These
 436 results correspond to the standard deviations in Table 1, and further support the
 437 categorization introduced there.

438 **Table 2. The maximum number of times a vehicle trip appears in the top 100 ranking**
 439 **across the ten samples, and the number of vehicle trips with this maximum frequency**
 440 **are provided in the left hand side for the random strategy. The right hand side**
 441 **represents the number of vehicle trips identified in both the random and targeted**
 442 **strategies.**

	Random Strategy Frequencies			Random -Targeted Frequencies		
	$ A_0 = 10$	$ A_0 = 50$	$ A_0 = 100$	$ A_0 = 10$	$ A_0 = 50$	$ A_0 = 100$
$\beta = 0.001$	7(2)	8(3)	10(2)	42	57	55
$\beta = 0.003$	9(2)	10(4)	10(12)	55	69	69
$\beta = 0.005$	10(1)	10(10)	10(22)	56	69	73
$\beta = 0.007$	10(2)	10(14)	10(24)	60	68	74

443

444 In addition to identifying the list of highest risk routes for each scenario independently, we
 445 seek to identify a robust set of vehicle trips which are most likely to transport infected
 446 passengers across a range of outbreak scenarios (similar initial conditions). To identify this
 447 set we consider the different categories defined previously, and count the frequency of each
 448 vehicle-trip's occurrence in the top 100 vehicle trips across all scenarios in each category.
 449 The frequency list provides an aggregated ranking of the most likely infected vehicle trips
 450 across a range of scenarios. The frequency results for each category as well as the global set
 451 are illustrated in Table 3. There are a total of 132 scenarios considered⁴, with Cat I, II and III
 452 corresponding to 60, 40 and 20 scenarios respectively. The vehicle trips which overlap
 453 across all 4 categories are highlighted in Table 3.

454 The global aggregation reveals that there are 12 particular vehicle trips which occur in over
 455 100 of the 132 scenarios, implying this particular set of routes are highly likely to have
 456 infected travelers, irrespective of where the infection starts (random vs. targeted, $|A_0|$) or the
 457 level of infectiousness of the disease in question. Similarly, high occurrence of particular
 458 vehicle trips can also be seen in the other three categories. The first category contains 60

⁴ Four β values, three $|A_0|$ settings, 10 random samples of A_0 and one targeted for each $|A_0|$.

459 infection scenarios and the most common (two) vehicle trips are present in 48 of them.
 460 Category II contains 40 scenarios and three trips contained by all of them. The last category
 461 contains 20 scenarios and 21 trips are present in all of them. This again confirms our previous
 462 results that the overlap is greater between infection scenarios in the second and third
 463 categories but even in the first one we are able to identify several vehicle trips where the
 464 infection is likely to be present. if we consider infection scenarios in category I, the overlap
 465 between them is surprisingly high. For example, in the scenario with $\beta = 0.001$, $|A_0| = 10$
 466 ten passengers are selected randomly out of 31000 and still seven times out of ten, the same
 467 (two) vehicle trips carry the infection. This further confirms our observation; the outbreak has
 468 a tendency to reach the same vehicle trips in the transportation network independent of the
 469 initial conditions.

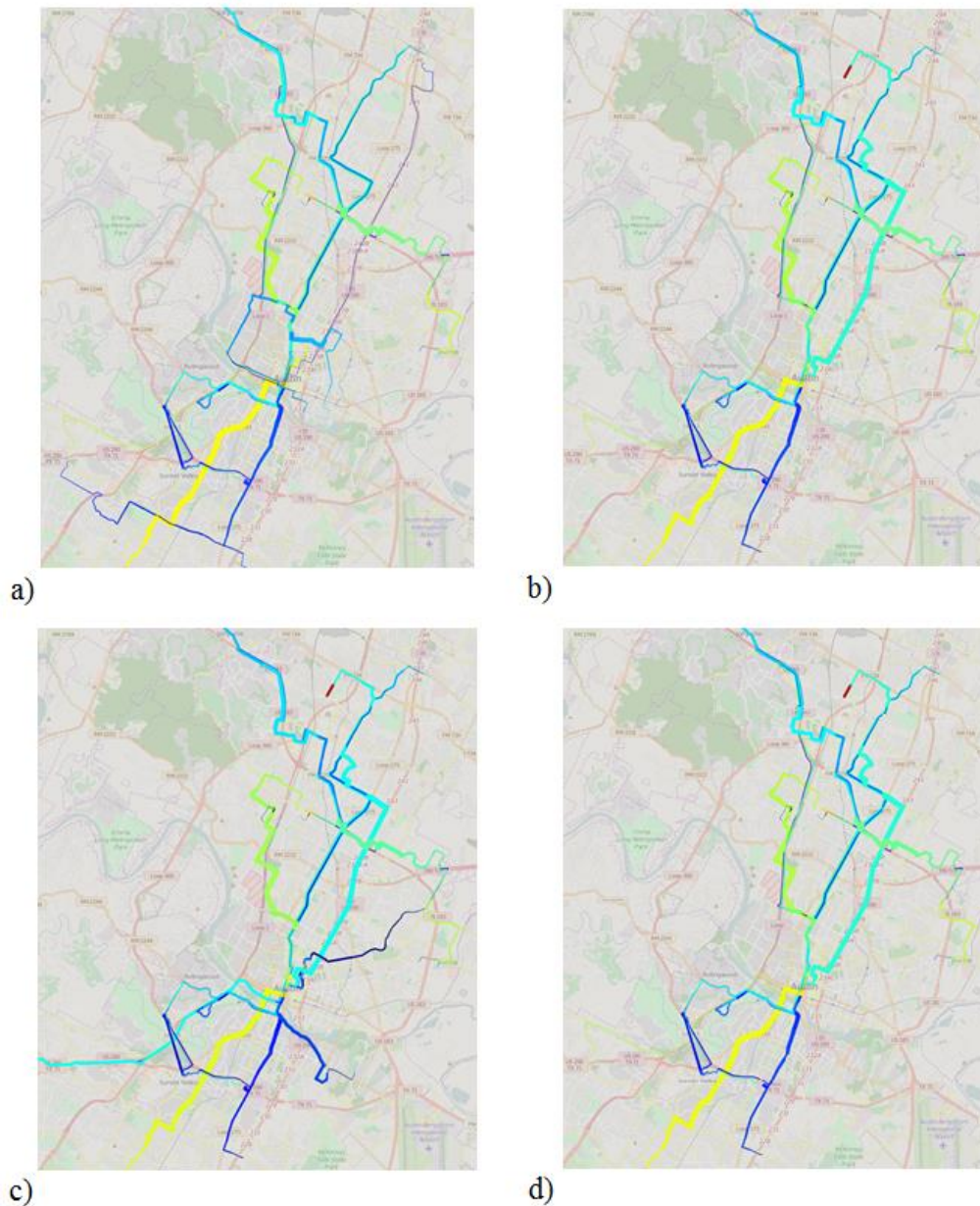
470 **Table 3. The global and categorical rankings of the vehicle trips where infection is most**
 471 **likely to be present. The numbers in the table indicate the ID of the route, the start time**
 472 **for the specific vehicle trip, and the frequency (number of scenarios) of the trip being**
 473 **infected. Trips that appear in all rankings are bolded in red.**

Rank	Global (132)	Cat. I. (60)	Cat. II. (40)	Cat. III. (20)
1	19 - 7:05 (120)	1 - 6:04 (48)	19 - 7:05 (40)	661 - 7:38 (20)
2	1 - 6:04 (119)	19 - 7:05 (48)	19 - 6:33 (40)	19 - 7:05 (20)
3	19 - 5:58 (116)	323 - 7:25 (46)	383 - 6:05 (40)	19 - 5:58 (20)
4	323 - 7:25 (114)	19 - 5:58 (45)	19 - 5:58 (39)	19 - 6:33 (20)
5	383 - 6:05 (110)	30 - 7:10 (41)	1 - 6:04 (39)	20 - 6:42 (20)
6	19 - 7:08 (109)	19 - 7:08 (40)	30 - 7:53 (39)	103 - 6:25 (20)
7	19 - 6:33 (108)	333 - 7:15 (38)	19 - 7:08 (38)	19 - 7:08 (20)
8	30 - 7:10 (107)	103 - 6:25 (38)	661 - 7:58 (38)	1 - 6:04 (20)
9	103 - 6:25 (105)	383 - 6:05 (38)	661 - 7:38 (37)	323 - 7:25 (20)
10	30 - 7:53 (104)	983 - 7:30 (37)	19 - 7:40 (37)	19 - 7:40 (20)
11	983 - 7:30 (102)	970 - 7:25 (36)	142 - 6:30 (37)	103 - 6:45 (20)
12	19 - 7:40 (101)	663 - 8:02 (36)	1 - 6:52 (36)	19 - 6:15 (20)
13	661 - 7:38 (99)	19 - 7:40 (36)	30 - 7:10 (36)	19 - 5:21 (20)
14	103 - 6:45 (99)	19 - 6:33 (36)	323 - 7:25 (36)	142 - 6:30 (20)
15	142 - 6:30 (99)	103 - 6:45 (35)	103 - 6:45 (36)	383 - 6:05 (20)
16	661 - 7:58 (98)	935 - 6:45 (34)	1 - 6:26 (36)	171 - 6:43 (20)
17	1 - 6:26 (97)	30 - 7:53 (34)	323 - 6:50 (36)	1 - 6:26 (20)
18	970 - 7:25 (97)	661 - 7:58 (33)	103 - 6:25 (35)	323 - 6:50 (20)
19	1 - 6:52 (95)	1 - 6:52 (32)	983 - 7:30 (34)	383 - 5:35 (20)
20	19 - 6:15 (93)	21 - 7:20 (32)	970 - 7:25 (34)	30 - 7:53 (20)

474

475 These transit routes with the highest likelihood of infection are visualized in Figure 3. Each
 476 subfigure illustrates a category of the routes listed in Table 2 (i.e., the categories). The width
 477 of the lines in the figure shows the ridership of the route in each segment, and the color
 478 indicates the frequency of the vehicle trip (with a spectrum from blue to yellow to red, with
 479 red representing the highest number). It can be observed that a subset of routes is common
 480 among categories with slight differences. Notably, most of the routes serve the central area
 481 where downtown and the university campus is located. More specifically, routes 19 and 103
 482 are present in all the categories with high frequency of being infected. Route 19 is a local

483 route serving residential areas where university students live and the university campus and
 484 downtown. Route 13 connects high density corridors to the downtown.



485

486 **Figure 3. The transit routes with the highest frequency of being infected. Categories I –**
 487 **III can be seen on a – c), respectively, and d) depicts the top routes of the global**
 488 **ranking.**

489 8. CONCLUSION AND FUTURE WORKS

490 We presented a novel method to identify the components of a public transit system which are
 491 most likely to transport infected passengers, and therefore play a vital role in the continued
 492 spread of infection during an outbreak. We ran a variety of epidemic scenarios atop the
 493 generated network which varied in their selection strategy of infection sources, number of the
 494 initially infected individuals, and levels of infectiousness. Based on the results from a case

495 study using the public transit network from Austin, TX, the infection scenarios can vary
496 substantially in the pattern of spread at the individual level, but the set of transit vehicle trips
497 at highest risk of infection is robust to the initial conditions of the outbreak. These vehicle-
498 trips would be optimal locations to implement vehicle surveillance, for example in the forms
499 of temperature gauges such as those recently installed on buses in Hong Kong. Identifying
500 the critical spreading components of the transit system can aid public health authorities in the
501 optimal allocation of surveillance resources, and is essential in mitigating future outbreaks.
502 The application in this study has recognized limitations due to the available data, but the
503 proposed method serves as a proof of concept, which can be directly applied to more
504 complete travel data sets, and also extended to include additional sources of human contact
505 outside of travel if a more inclusive contact data were available.

506

507

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