Modeling Bus Capacity for Isolated Bus Stops

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ABSTRACT:
This study proposed a bus capacity estimation method for bus stops, which were isolated from the influences of traffic signals and other bus stops. Data collected from the seven most common types of bus stops in China were used to validate the proposed model. The results indicated the arrival process obeyed Poisson distribution and the service time fit a lognormal distribution. In light of this, queuing models for both single-berth and multi-berth stops were developed to estimate bus stop capacity. As a comparison, the HCM model was also used for bus stop capacity estimation. The results showed that the proposed method was more accurate and reliable, with an 8.44% MAPE (Mean Absolute Percentage Error) compared to a MAPE of 17.42% from the HCM method. Sensitivity analyses were also conducted to investigate the effects of bus arrival rate, service time, and the number of bus berths on the capacity of bus stops.

Keywords: bus capacity; bus stop; arrival process; service time; bus berth
1. INTRODUCTION

With the advance of modern technologies and the requirements to improve Level-of-Service (LOS) for public transportation, there has been a transition from ‘vehicle-oriented’ (private transportation) to ‘people-oriented’ (public transportation) in the development of an urban transportation network (1-3). Urban public transportation systems perform an essential function in the mobility of citizens in metropolitan areas around the world (4-6). Recently, more and more researchers have focused on studying the role of bus capacity (7), which is relevant to optimizing resources on the basis of ridership. Many studies have found that the bus stop, which serves as a crucial part in the bus system, has a great effect on traffic flow (8,9). When serving passengers at a busy stop, buses can interact in ways that limit their discharge flows (10). Thus, a dwelling bus may become a bottleneck that constrains traffic flow near the bus stop, and may degrade the bus system’s overall service quality (11-13).

In general, any bus stop consists of a ‘stop area’ where vehicles stop and an adjacent ‘platform’ where passengers wait for vehicles and boarding/alighting operations take place. Under light ridership conditions, bus stops seem to be a rather simple mechanism to be modeled. Passenger interarrival times and bus headways are large so that the number of bus berths is enough for dwelling buses. In high demand situations, however, this is not the case. For example, at peak periods at a bus stop, if there is no unoccupied berth for an arriving bus, the bus must wait on the road blocking traffic and impeding the progress of boarding and alighting (11). Thus, transit system capacity is controlled by stop capacity in public transportation systems (14).

Two categories of methods for modeling stop capacity have been reported in the literature. The first category is the use of analytical models to estimate capacity with a steady state formula (15). The most well-known one in this category is the Highway Capacity Manual (HCM) formula (16). The parameters in this model include green time ratio, clearance time, average dwell time, standard normal variable corresponding to a desired failure rate, and coefficient of variation of dwell time. Fernández et al. (17) introduced a concept called capacity of divided bus stops. A divided bus stop contained berths that were separated to reduce bus interference and increase bus capacity. Jaiswal et al. (18) introduced bus lost time as an additional component of the minimum headway to calculate bus platform capacity. Accordingly, a busway loading bus capacity model was developed with lost time variables. The second category is the use of simulation models to calculate capacity as well as other performance measures of stop operations (11). Two specially-designed simulation models of stop operations could be found in the literature: IRENE (13) and PASSION (12,19,20). PASSION is developed as part of broader studies on the interactions between buses, passengers and traffic at bus stops.

Although a variety of methods have been developed to estimate bus stop capacity, the performance of these models varies. For instance, Fernandez and Planzer (12) reported that the HCM formulas tended to under-predict the stop capacity. The stochastic processes at bus stops may limit the usefulness of HCM formulas. In addition, existing bus stop capacity estimation methods merely considered the
perspective of modeling or simulation, and rarely involved the combination of
analytical models and statistical analysis. In light of these, an estimation method that
takes into account the statistical test for bus arrival process and service time and an
analytical model for single-berth and multi-berth stops is developed. In this study, the
bus stops are isolated from the influences of traffic signals and other bus stops, where
sufficient space exists for accommodating the bus queues.

The remaining sections of the paper are organized as follows. Section 2 and 3
describe the data collection and proposed methodology to model bus stop capacity.
Section 4 validates the proposed method and evaluates the performance by analysis
and comparison. The findings and conclusions are provided in the last section.

2. DATA COLLECTION
Based on the right-of-way, bus lanes at bus stops can be divided into three categories:
grade-separated bus lanes, at-grade bus lanes, and non-exclusive bus lanes (21).
According to TCRP Report 19 (22), the bus dwell time will be affected by the layout
of the bus stop. In general, the more exclusive the design, that is, the less interaction
that a transit vehicle has with other traffic, the fewer impacts on bus dwell time. In
terms of the form, bus stops can be classified into two categories: on-line and off-line
(21). Compared to an on-line bus stop, there is additional time required for buses at an
off-line stop to find an acceptable time gap between consecutive vehicles. It can be
thus concluded that the form of bus stop has impact on the bus dwell time. Moreover,
based on the location of the cross-section, bus stops can be divided into the two
categories of median and curbside. According to the above classifications, seven types
of bus stop designs are most commonly observed in China, as illustrated in Figure 1.

(a) Type 1: The at-grade bus lanes are separated from motor vehicle lanes by
traffic markings. Bus stops are on-line and on the curbside.
(b) Type 2: There is no exclusive bus lane. Bus stops are on-line and on the
curbside.
(c) Type 3: The at-grade bus lanes are separated from motor vehicle lanes by
traffic markings. Bus stops are off-line (bay-style) and on the curbside.
(d) Type 4: There is no exclusive bus lane. Bus stops are off-line (bay-style)
and on the curbside.
(e) Type 5: The grade-separated bus lanes are separated from motor vehicle
lanes by separation strips. Bus stops are on-line and in the median of the
cross-section.
(f) Type 6: The at-grade bus lanes are separated from motor vehicle lanes by
traffic markings. Bus stops are on-line and in the median of the cross-section.
(g) Type 7: There is no exclusive bus lane. Bus stops are on-line and on the
curbside. Buses pull over to the curbside and occupy bicycle lanes to dwell.
In this study, data were collected for these seven different types of bus stops in the cities of Nanjing, Changzhou, and Guangzhou, China. The data were collected under fine weather conditions between May 19, 2014 and June 15, 2014 to exclude potential influences of adverse weather. In addition, there was no curb parking around the stops.

Three video cameras were used at each stop to record traffic data. One camera was set up at a high location and two other video cameras were in front of and behind the bus stop, respectively. The recorded videos were reviewed by several trained graduate students to obtain bus average service time, variances in bus service time and bus stop failure rate. The site and traffic characteristics of the bus stops and service time are shown in Table 1.
TABLE 1 Site and Traffic Characteristics of the Bus Stops and Service Time

<table>
<thead>
<tr>
<th>No.</th>
<th>Bus stop</th>
<th>Type</th>
<th>BN⁶</th>
<th>TC²</th>
<th>SS⁵</th>
<th>AST⁴</th>
<th>STV⁶</th>
<th>SVC⁷</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gulou North</td>
<td>Type 1</td>
<td>4</td>
<td>Peak</td>
<td>67</td>
<td>30.13</td>
<td>83.69</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>Non-peak</td>
<td>47</td>
<td>32.38</td>
<td>79.37</td>
<td>0.27</td>
</tr>
<tr>
<td>3</td>
<td>Beiji Huitang</td>
<td>Type 2</td>
<td>2</td>
<td>Peak</td>
<td>113</td>
<td>23.90</td>
<td>82.98</td>
<td>0.38</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>Non-peak</td>
<td>108</td>
<td>19.01</td>
<td>44.53</td>
<td>0.35</td>
</tr>
<tr>
<td>5</td>
<td>Public Transport Corporation</td>
<td>Type 3</td>
<td>5</td>
<td>Peak</td>
<td>96</td>
<td>33.64</td>
<td>97.10</td>
<td>0.29</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>Non-peak</td>
<td>101</td>
<td>30.80</td>
<td>71.32</td>
<td>0.27</td>
</tr>
<tr>
<td>7</td>
<td>Xuanwu Park</td>
<td>Type 4</td>
<td>5</td>
<td>Peak</td>
<td>40</td>
<td>35.83</td>
<td>145.43</td>
<td>0.33</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td>Non-peak</td>
<td>56</td>
<td>40.96</td>
<td>156.25</td>
<td>0.30</td>
</tr>
<tr>
<td>9</td>
<td>Renmin Park</td>
<td>Type 5</td>
<td>3</td>
<td>Peak</td>
<td>34</td>
<td>34.37</td>
<td>71.35</td>
<td>0.25</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td>Non-peak</td>
<td>40</td>
<td>32.86</td>
<td>40.27</td>
<td>0.20</td>
</tr>
<tr>
<td>11</td>
<td>Gangding</td>
<td>Type 6</td>
<td>4</td>
<td>Peak</td>
<td>51</td>
<td>31.06</td>
<td>56.26</td>
<td>0.24</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td>Non-peak</td>
<td>52</td>
<td>26.77</td>
<td>61.36</td>
<td>0.29</td>
</tr>
<tr>
<td>13</td>
<td>Danfeng Street</td>
<td>Type 7</td>
<td>1</td>
<td>Peak</td>
<td>39</td>
<td>26.33</td>
<td>63.49</td>
<td>0.30</td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td>Non-peak</td>
<td>41</td>
<td>24.95</td>
<td>42.65</td>
<td>0.26</td>
</tr>
</tbody>
</table>

a: Number of bus berths at the bus stop
b: Traffic condition (peak or non-peak period)
c: Sample size
d: Average service time of buses (s), denoted by \( \mu \)
e: Variances in bus service time (s²), denoted by \( \sigma^2 \)
f: Coefficient of variation in bus service time, defined as \( \sigma \) divided by \( \mu \)

3. BUS STOP CAPACITY MODEL

In operational terms, a bus stop can be considered as a concurrent queuing system between buses and passengers (11). It is assumed that bus stops operate in the steady-state, such that the long-run average bus arrival rate never exceeds the stop’s capacity (10). Bus stop capacity is closely related to the bus average service time at bus stops (16). In addition, a bus halts at the stop area for certain duration, which depends on the number of waiting passengers. If there is no unoccupied berth for an arriving bus, the bus may have to wait on the road blocking the traffic and impeding the progress of boarding and alighting (11). Consequently, the arrival rate of buses can affect the bus service time, and thus affect the bus stop capacity.

It has been assumed that bus arrival at stops follows a Poisson process (23,24). The interarrival time is an independent, exponentially distributed random variable with arrival rate of buses \( \lambda \). Some studies also show that bus service time at stops follows some general distribution, and the distribution of an individual bus service time is independent of the number of berths (10,25,26). To further examine and explore the distributions of bus arrivals and service time, we collected 885 samples from the 14 sites, as shown in Table 1.

3.1 Bus Arrival Process

In order to validate the Poisson process for bus arrivals, the Kolmogorov-Smirnov test
(K-S test) is introduced as a criterion. The K-S test can be used to compare a sample with a reference probability distribution, or to compare two samples \(^{(27)}\). The K-S statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples. The null distribution of this statistic is calculated under the null hypothesis that the samples are drawn from the same distribution (in the two-sample case) or that the sample is drawn from the reference distribution (in the one-sample case). In this section, the Poisson process for bus arrivals is a one-sample K-S test. The results of K-S tests show that all of the \(P\) values for the 14 sites are much greater than 0.05 (varying between 0.58 and 1.00). Hence, the tests show that the results are consistent with previous studies that bus arrivals follow Poisson distribution for isolated bus stops.

### 3.2 Bus Service Time

In this section, three common distributions including normal, lognormal, and weibull distribution are selected to fit the bus service time. In addition, the exponential distribution is also introduced as a comparison. Similar to the way of validating bus arrival process, distribution for bus service time is examined using the K-S test. In general, the smaller K-S value and larger P value, the better the fitting performance. As shown in Table 2, among the four distributions, the lognormal distribution (boldface in Table 2) has the best performance for all bus stops during peak and non-peak periods.

In order to provide visual comparison of the fitting results for bus service time, the probability density function of normal distribution, lognormal distribution, weibull distribution, exponential distribution, and sample data for Beiji Huitang bus stop (Type 2) and Public Transport Corporation bus stop (Type 3) in peak and non-peak periods are chosen to be presented in Figure 2. The results show that the sample data follow the probability density function of lognormal distribution better. The distribution parameters of the lognormal distribution for each bus stop in peak and non-peak periods are also displayed in the figure.
<table>
<thead>
<tr>
<th>Type</th>
<th>Bus stop</th>
<th>State</th>
<th>Sample size</th>
<th>K-S value (the smaller, the better)</th>
<th>P value (P&gt;0.05)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Normal distribution</td>
<td>Lognormal distribution</td>
</tr>
<tr>
<td>Type 1</td>
<td>Gulou North</td>
<td>Peak</td>
<td>67</td>
<td>0.148</td>
<td><strong>0.113</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-peak</td>
<td>47</td>
<td>0.113</td>
<td><strong>0.061</strong></td>
</tr>
<tr>
<td>Type 2</td>
<td>Beiji Huitang</td>
<td>Peak</td>
<td>113</td>
<td>0.149</td>
<td><strong>0.087</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-peak</td>
<td>108</td>
<td>0.126</td>
<td><strong>0.082</strong></td>
</tr>
<tr>
<td>Type 3</td>
<td>Public Transport</td>
<td>Peak</td>
<td>96</td>
<td>0.131</td>
<td><strong>0.078</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-peak</td>
<td>101</td>
<td>0.119</td>
<td><strong>0.097</strong></td>
</tr>
<tr>
<td>Type 4</td>
<td>Xuanwuhu Park</td>
<td>Peak</td>
<td>40</td>
<td>0.127</td>
<td><strong>0.081</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-peak</td>
<td>56</td>
<td>0.112</td>
<td><strong>0.061</strong></td>
</tr>
<tr>
<td>Type 5</td>
<td>Renmin Park</td>
<td>Peak</td>
<td>34</td>
<td>0.102</td>
<td><strong>0.082</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-peak</td>
<td>40</td>
<td>0.093</td>
<td><strong>0.080</strong></td>
</tr>
<tr>
<td>Type 6</td>
<td>Gangding</td>
<td>Peak</td>
<td>51</td>
<td>0.148</td>
<td><strong>0.117</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-peak</td>
<td>52</td>
<td>0.109</td>
<td><strong>0.070</strong></td>
</tr>
<tr>
<td>Type 7</td>
<td>Danfeng Street</td>
<td>Peak</td>
<td>39</td>
<td>0.161</td>
<td><strong>0.139</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-peak</td>
<td>41</td>
<td>0.125</td>
<td><strong>0.094</strong></td>
</tr>
</tbody>
</table>
Based on the above testing results, an $M/G-L/1$ queuing model for single-berth stops and an $M/G-L/S$ queuing model for multi-berth stops are developed to estimate bus stop capacity, where $M$ represents Poisson arrivals, $G-L$ represents lognormal bus service time, and $1(S)$ represents the number of bus berths.

### 3.3 Model for Single-berth Stops

The Highway Capacity Manual (16) reports that the capacity of a bus stop is inversely proportional to the bus average service time and a second term that accounts for the variation in this service time. In this study, the expected time spent in the queuing system $W_s$ is used to represent the sum of bus average service time and time spent in the queue. Thus, the bus capacity for single-berth stops can be expressed as follows:

$$C_s = 3600/W_s$$  \hspace{1cm} (1)\

where $C_s$ is the bus capacity for single-berth stops (veh/h); $W_s$ is the expected time spent in the system (s).

The input into the system is formed by the buses approaching from upstream, and there are Poisson arrivals with a mean headway of $1/\lambda$ s. The service time at a bus stop is expressed by generally distributed variables with mean $1/\mu$ s. The service counter is the bus berths. According to Little’s Law (28,29), $W_s$ in $M/G-L/1$ system is given by:
\[ W_s = L_s / \lambda \]  
where \( L_s \) is the expected number of buses in the system (veh); \( \lambda \) is the arrival rate of buses (veh/s). The expected number of buses in the system \( L_s \) can be represented as:

\[ L_s = \left( \rho_s^2 + \lambda^2 \times D(T_n) \right) \left( 2 \times (1 - \rho_s) \right) + \rho_s \]  
(3)

\[ \rho_s = \lambda / u = \lambda \times E(T_n) \]  
(4)

where \( \rho_s \) is the traffic intensity for single-berth stops; \( D(T_n) \) represents the variance of service time for lognormal distribution (s^2); \( u \) is the service rate (veh/s); \( E(T_n) \) represents the mean service time for lognormal distribution (s). Substituting (2)-(4) into Equation (1), the bus capacity for single-berth stops can be expressed by:

\[ C_s = \left( 3600 \times 2 \times (1 - \lambda \times E(T_n)) \right) \left( 2 \times E(T_n) - \lambda \times E^2(T_n) + \lambda \times D(T_n) \right) \]  
(5)

3.4 Model for Multi-berth Stops

As mentioned above, the bus capacity for multi-berth stops can be expressed by:

\[ C_m = \left( 3600 \times N_{el} \right) / W_s \]  
(6)

where \( C_m \) is the bus capacity for multi-berth stops (veh/h); \( N_{el} \) represents the number of effective berths. In real-world bus stops, a bus's entering and exiting maneuvers will be affected by other buses dwelling at neighboring berths. Thus, the concept of the number of effective berths is introduced for multi-berth stops. According to HCM (16), the first berth is 100% efficient in terms of stop capacity. If a second berth is added, it increases the capacity by 75% \((N_{el} = 1.75)\) and a third berth may add 70% extra capacity \((N_{el} = 2.45)\). However, the fourth and fifth berths only increase capacity by 17% \((N_{el} = 2.62)\) and 13% \((N_{el} = 2.75)\), respectively.

As for the multi-berth stops, similar to single-berth stops, there are Poisson arrival headways with mean \(1/\lambda\) s and generally distributed service time with mean \(1/\mu\) s. Explicit exact solutions for the M/G-L/S system have not been obtained except for some special cases, e.g. the M/M/S system. It is, however, possible to obtain approximate solutions for a relatively wide class of service time distributions \((30-31)\).

Thus, the expected time \( W_s \) in the M/G-L/S system can be approximately given by:

\[ W_s = W_q + E(T_n) \]  
(7)

\[ W_q = \frac{\rho_s^s \times \left( D(T_n) + E^2(T_n) \right)}{(S - 1)! \times 2 \times E(T_n) \times (S - \rho_m) \times \left( \sum_{j=1}^{S-1} \frac{\rho_m^j}{j!} + \frac{\rho_m^S}{(S - 1)!} \times (S - S) \right)} \]  
(8)

\[ \rho_m = \lambda / (S \times u) = \frac{(\lambda \times E(T_n))}{S} \]  
(9)

where \( W_q \) is the expected time spent in the queue (s); \( \rho_m \) is the traffic intensity for multi-berth stops; \( S \) denotes the number of bus berths. Substituting (7)-(9) into Equation (6), the bus capacity for multi-berth stops can be expressed as follows:

\[ C_m = \frac{3600 \times N_{el} \times (S - 1)! \times 2 \times E(T_n) \times S \times E(T_n)}{\left( \frac{\lambda \times E(T_n)}{S} \right)^3 \times (D(T_n) + E^2(T_n)) \times \Pi_{(S - 1)! \times 2 \times E(T_n)} \times (S \times \lambda \times E(T_n)) \times E(T_n)} \]  
(10)

where
\[
\Pi = \left( \sum_{j=1}^{s-1} \left( \frac{\lambda \times E(T_n)}{S} \right)^j \right) \left( \frac{1}{j!} + \left( \frac{\lambda \times E(T_n)}{S} \right)^s \right) \left( S-1 \right) \times \left( S - \frac{\lambda \times E(T_n)}{S} \right) \right)^{s-1} \tag{11}
\]

In addition, as for the mean service time \( E(T_n) \) and the variance of service time \( D(T_n) \) for lognormal distribution, they can be deduced by elementary probability theory methods:

\[
E(T_n) = \int_0^{\infty} T_n f(T_n) dT_n
\]

\[
= \int_0^{\infty} \frac{T_n}{\sqrt{2\pi} \sigma T_n} \exp\left( -\frac{(\ln T_n - \mu)^2}{2\sigma^2} \right) dT_n
\]

\[
= \int_0^{\infty} \frac{\exp(t \sigma + \mu)}{\sqrt{2\pi} \sigma \exp(t \sigma + \mu)} \exp\left( -\frac{t^2}{2} \right) \exp(t \sigma + \mu) \sigma dt
\]

\[
= \exp(\mu) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left( -\frac{t^2 - 2t \sigma + \sigma^2}{2} + \frac{\sigma^2}{2} \right) dt
\]

\[
= \exp(\mu + \frac{\sigma^2}{2}) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left( -\frac{(t - \sigma)^2}{2} \right) dt
\]

\[
= \exp(\mu + \frac{\sigma^2}{2})
\]

\[
D(T_n) = \int_0^{\infty} (T_n - E(T_n))^2 f(T_n) dT_n
\]

\[
= \int_0^{\infty} \frac{(T_n - \exp(\mu + \frac{\sigma^2}{2}))^2}{\sqrt{2\pi} \sigma T_n} \exp\left( -\frac{(\ln T_n - \mu)^2}{2\sigma^2} \right) dT_n
\]

\[
= \int_0^{\infty} \frac{(\exp(t \sigma + \mu) - \exp(\mu + \frac{\sigma^2}{2}))^2}{\sqrt{2\pi} \sigma \exp(t \sigma + \mu)} \exp\left( -\frac{t^2}{2} \right) \exp(t \sigma + \mu) \sigma dt
\]

\[
= \exp(2\mu) \int_{-\infty}^{\infty} \frac{\exp\left( \frac{-t^2 - 4t \sigma + 4\sigma^2}{2} + 2\sigma^2 \right)}{\sqrt{2\pi}} dt
\]

\[-2 \exp(2\mu + \frac{\sigma^2}{2}) \int_{-\infty}^{\infty} \exp\left( \frac{-t^2 - 2t \sigma + \sigma^2}{2} + \frac{\sigma^2}{2} \right) \frac{\exp\left( -\frac{t^2}{2} \right)}{\sqrt{2\pi}} dt + \exp(2\mu + \sigma^2) \int_{-\infty}^{\infty} \exp\left( \frac{-t^2}{2} \right) dt
\]

\[= \exp(2\mu + 2\sigma^2) \int_{-\infty}^{\infty} \frac{\exp\left( -\frac{(t - 2\sigma)^2}{2} \right)}{\sqrt{2\pi}} dt - 2 \exp(2\mu + \sigma^2) \int_{-\infty}^{\infty} \exp\left( -\frac{(t - \sigma)^2}{2} \right) \frac{\exp\left( -\frac{t^2}{2} \right)}{\sqrt{2\pi}} dt
\]

\[+ \exp(2\mu + \sigma^3) \int_{-\infty}^{\infty} \frac{\exp\left( -\frac{t^2}{2} \right)}{\sqrt{2\pi}} dt
\]

\[= \exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)
\]
4. MODEL VALIDATION

4.1 Comparison of Results

In addition to the proposed method, the model based on HCM formula (16), which is one of the most well-known analytical models, is used for bus stop capacity estimation and comparison. According to the HCM, the capacity is computed as follows:

\[
C_s = \frac{3600 \times (g/c)}{t_c + (g/c) \times t_d + Z \times C_v \times t_d}
\]

(14)

\[
C_m = N_{cl} \times \frac{3600 \times (g/c)}{t_c + (g/c) \times t_d + Z \times C_v \times t_d}
\]

(15)

where \(g/c\) is the effective green time of a downstream traffic signal (as for the isolated bus stops in this study, \(g/c = 1\)); \(t_c\) is the clearance time between successive buses (s); \(t_d\) is the average dwell time (s). In this study, the bus service time includes \(t_c\) and \(t_d\); \(C_v\) is the coefficient of variation in bus service time; \(Z\) is the standard normal variable corresponding to a desired failure rate. The above values are provided in Table 1.

The proposed method and HCM method are compared using field data at the seven bus stops. The number of buses that could be served by a stop under saturated conditions is surveyed. Instrument and equipment used for the data collection include digital video cameras, tripods, stopwatch and computer. All the investigated bus stops have locations or spots to install video cameras. Between May and June 2014, 3 to 4 hours were spent to observe the number of buses at each bus stop. Taking 5 minutes, 10 minutes and 15 minutes as analysis intervals respectively, the maximum number of buses for these periods from all the three types of intervals was selected. Then the number of buses is derived from an hourly volume, which is the actual capacity of a bus stop (11,32).

The Mean Absolute Percentage Error (MAPE) is chosen to evaluate the performance of the proposed and HCM methods. MAPE has no requirement for sample size and shows an obvious advantage in evaluating discrete data (33,34). The value of MAPE in this study is calculated using the following equation:

\[
MAPE = \left| \frac{\text{Estimated capacity} - \text{Actual capacity}}{\text{Actual capacity}} \right|
\]

(16)

To evaluate the performance of two methods, the comparison data of MAPE for the two methods at seven bus stops in peak and non-peak periods are presented in Table 3. According to the results, it is the proposed method that has more accurate and reliable estimation (with 8.44% of MAPE on average) than the HCM method (with 17.42% of MAPE on average). In addition, the MAPE values of the proposed method at all bus stops are below 15%, with the lowest value of 0.53% at Gangding bus stop in non-peak period. The performance of the HCM method, however, is inferior to the proposed method, with almost half of the MAPE values over 20%.
TABLE 3 Comparison of MAPE at Bus Stops in Peak and Non-peak Periods

<table>
<thead>
<tr>
<th>No.</th>
<th>Bus stop/ number of berths</th>
<th>State</th>
<th>Field data (veh/h)</th>
<th>Proposed method</th>
<th>Competing method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Capacity (veh/h)</td>
<td>MAPE (%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Gulou North/ 4</td>
<td>Peak</td>
<td>316</td>
<td>314</td>
<td>0.63</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Non-peak</td>
<td>270</td>
<td>290</td>
<td>7.41</td>
</tr>
<tr>
<td>3</td>
<td>Beiji Huitang/ 2</td>
<td>Peak</td>
<td>282</td>
<td>245</td>
<td>13.12</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>Non-peak</td>
<td>274</td>
<td>315</td>
<td>14.96</td>
</tr>
<tr>
<td>5</td>
<td>Public Transport Corporation/ 5</td>
<td>Non-peak</td>
<td>362</td>
<td>319</td>
<td>11.88</td>
</tr>
<tr>
<td>6</td>
<td>Xuanwu Park/ 5</td>
<td>Peak</td>
<td>264</td>
<td>275</td>
<td>4.17</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>Non-peak</td>
<td>223</td>
<td>241</td>
<td>8.07</td>
</tr>
<tr>
<td>8</td>
<td>Renmin Park/ 3</td>
<td>Peak</td>
<td>231</td>
<td>256</td>
<td>10.82</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>Non-peak</td>
<td>234</td>
<td>267</td>
<td>14.10</td>
</tr>
<tr>
<td>10</td>
<td>Gangding/ 4</td>
<td>Peak</td>
<td>401</td>
<td>356</td>
<td>11.22</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>Non-peak</td>
<td>379</td>
<td>377</td>
<td>0.53</td>
</tr>
<tr>
<td>12</td>
<td>Danfeng Street/ 1</td>
<td>Peak</td>
<td>103</td>
<td>111</td>
<td>7.77</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>Non-peak</td>
<td>122</td>
<td>114</td>
<td>6.56</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>8.44</td>
</tr>
</tbody>
</table>

To show the difference between the proposed method and the HCM model, Figure 3 displays the estimated capacity at 3-berth stops with varying bus service time (15s to 45s). Overall, the estimated bus capacity for stops decreases as bus service time increases. The HCM method has significant under-estimations, with differences between 123veh/h and 41veh/h.

4.2 Sensitivity Analyses
In order to fully analyze the proposed method, we use sensitivity analyses to
investigate the effects of bus arrival rate, service time, and the number of bus berths on the capacity of bus stops. Differences in the arrival rate of the bus stream affect bus capacity at single-berth stops, as shown in Figure 4. At the same bus service time, the probability of no stopped bus at the stop decreases with the increasing arrival rates of the bus stream, and further reduces the bus stop capacity. In addition, the service time of buses at the stop affects bus capacity. At the same arrival rate, the increasing bus service time increases the probability of the conflict between buses. The stopped bus blocks bus streams near the stop and impedes the progress of boarding and alighting, and further reduces bus capacity from 217veh/h to 67veh/h at the arrival rate 0.01veh/s, from 204veh/h to 51veh/h at the arrival rate 0.015veh/s, and from 190veh/h to 29veh/h at the arrival rate 0.02veh/s, as bus service time increases from 15s to 40s. In this study, when the arrival rate is 0.02veh/s and bus service time is more than 35s, the bus stop capacity is reduced to fewer than 50veh/h.

![Figure 4. Bus capacity with different arrival rates of bus stream and bus service time at single-berth stops](image)

The results of bus capacity estimation for multi-berth stops are shown in Figure 5. Similar to single-berth stops, at a same arrival rate, the bus stop capacity reduces as the bus service time increases. For example, as for the 3-berth stops, bus capacity is reduced from 439veh/h to 170veh/h at the arrival rate 0.06veh/s, as bus service time increases from 20s to 50s. In addition, as shown in figures 5(a) and 5(b), at a same bus service time, the increase in arrival rates of the bus stream significantly reduce the bus stop capacity at 2-berth and 3-berth stops. However, the trend does not occur at the stops whose number of bus berths is more than 3. It is reasonable because the probability of unoccupied berth(s) at the stop increases with the increase of the number of bus berths. Hence, as for the 4-berth (Figure 5(c)) and 5-berth (Figure 5(d)) stops, the bus stop capacity is not sensitive to arrival rates. For instance, when the number of bus berths is equal to 4, the maximum reduction of bus stop capacity is merely 7veh/h when the arrival rate of buses changes from 0.01veh/s to 0.15veh/s.
Figure 5. Bus capacity with different arrival rates of bus stream and bus service time at multi-berth stops.
To discover the degree to which bus berths and bus service time affect bus capacity at bus stops, the proposed method is further used to explore the change of bus stop capacity when arrival rate of buses is 0.01 veh/s and bus service time is between 15s and 45s. The curves in Figure 6(a) display the differences in how the number of bus berths affects bus stop capacity. When the bus service time is short, additional berths could produce increasing returns in capacity. For example, the figure shows that when bus service time is equal to 20s, 2-berth stops increase bus capacity by 141 veh/h over 1-berth stops. This favorable trend does not continue, however, as bus service time increases. In addition, at the same bus service time, the incremental changes in bus stop capacity are diminished dramatically, as the number of bus berths increases (Figure 6(b)). When bus service time is 25s, the increasing returns in capacity are reduced from 134 veh/h to 19 veh/h for the first berth through the fifth berth, especially for the fourth and fifth berths, with increases of 25 veh/h and 19 veh/h, respectively. This finding is in accordance with the actual bus operations at bus stops; current practice states that it is not efficient to build bus stops with more than three adjacent berths (10). If more than 3 berths are necessary, the stop should be split in two multi-berth stops (e.g., a 4-berth stop can be split in two 2-berth stops). The results further indicate the method is well validated and could be practically used for the analysis and estimation of bus stop capacity in China.

Figure 6. Bus stop capacity and incremental changes in capacity versus bus service time for different berths
5. CONCLUSIONS
This study investigated bus arrival process and bus service time distribution, and applied a stochastic queuing method to estimate the bus capacity for isolated bus stops. The method not only takes into account the statistical test for bus arrival process and service time, but also considers the analytical model for single-berth and multi-berth stops.

In this study, data collected from the seven bus stops in China were used to validate the proposed model. K-S test was performed to explore bus arrival process and bus service time distribution. According to the results of P value and K-S value, it was found that the bus service time followed the lognormal distribution, and the bus arrival process obeyed Poisson distribution. An M/G-L/1 queuing model for single-berth stops and an M/G-L/S model for multi-berth stops were developed to estimate bus stop capacity. To evaluate the proposed model’s performance, another model based on HCM formula was selected for comparison. According to the results, it was the proposed method that had more accurate and reliable estimation, with an 8.44% MAPE on average compared to a 17.42% average MAPE from the HCM model. In addition, sensitivity analyses were conducted to investigate the effects of bus arrival rate, number of bus berths, and bus service time on the capacity of bus stops.

This study explored the capacity of isolated bus stops. Further research can be conducted on the capacity of those stops that are not isolated (i.e., stops under the influence of traffic signals and other adjacent stops, or those stops with limited space for accommodating bus queues).

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REFERENCES


29 Sztrik, J. Basic queueing theory. University of Debrecen: Faculty of Informatics, 2011.


