A Comprehensive Approach to Allocate Reliability and Cost in Passenger Rail System Design

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ABSTRACT

Rail system design and procurement comprise the process to identify, acquire, select, and purchase the right products to form a rail system. To acquire a new system, a variety of products with specific costs and reliability for each subsystem and corresponding components could be chosen from a number of equipment suppliers. Planners have to carefully examine the trade-off among life cycle cost (LCC), system reliability, and service reliability to allocate resources optimally. This study develops a comprehensive allocation process with four types of optimization models for passenger rail system design: (1) maximization of system reliability, (2) maximization of service reliability, (3) minimization of LCC, and (4) minimization of a combination of service unreliability (delay cost) and LCC. Based on the characteristics of a passenger rail system and possible alternatives, the proposed process can allocate LCC and service reliability optimally to determine the optimal investment plan for rail system design. Empirical case studies demonstrate that the proposed optimization process and models can evaluate efficiently and successfully all possible alternatives and determine the optimal allocation among all subsystems and corresponding components. This comprehensive approach can help users identify the ideal balance between cost and reliability so as to achieve an optimal rail system design.

Keywords

Rail Transportation, System Design, Reliability Allocation, Life Cycle Cost

INTRODUCTION

Rail system design and procurement is the process of identifying, acquiring, selecting, and purchasing the right products to form a rail system (*1, 2*). This system usually consists of a number of subsystems and corresponding components, and each has its own reliability, life cycle cost (LCC), and impact on service reliability. Determining the best investment plan from the design stage to ensure reliable service is important because rail systems are often long-term projects that include design, construction, operation, and disposal phases. Implementing significant changes becomes difficult when the rail system has been completed and starts to operate.

Reliability allocation is a well-known method of dealing with the design of a passenger rail system (*3-5*). This type of system consists of several subsystems, each one characterized by reliability and life cycle cost (LCC). To design or maintain a rail system with a particular level of system reliability, planners should carefully balance the trade-off between reliability and budget. Determining an optimal investment plan can facilitate acceptable performance of rail systems during their entire life cycle in a cost-effective manner. Various reliability allocation methods have been developed to apply total system reliability to subsystem reliability under cost-effective conditions. These methods are generally categorized into two types, namely, weighting method (*6-8*) and optimization approach (*4, 9-16*). Although the weighting method considers several criteria simultaneously, it remains subjective and case-specific. Building a common model for similar systems with the same parameters has proven difficult. On the other hand, the optimization approach has to incorporate the relationship of the cost reliability for each product, which is usually captured by exponential functions, thereby resulting in nonlinear models that cannot guarantee a global optimum.

Another deficiency in past studies is to consider the reliability of the rail systems purely on the basis of failure rate, mean time between failure (MTBF), or mean distance between failure (MDBF). With this type of attributes, the number of failures can be measured within a specific exposure rate, such as time (e.g., train-hour) or distance (e.g., train-km). However, this kind of reliability, which can be regarded as "system reliability," does not consider the effect of failure on trains or passengers, such as delay (*17*). Train delay can severely affect transit service reliability and customer satisfaction. Lai et al. (*4*) proposed to use "service reliability" in the reliability allocation process and to develop corresponding optimization models for passenger rail system design. Although service reliability, such as delay and on-time percentage, were incorporated in the process, their models could not deal with systems that have multiple level structures, and the objectives of the models are limited to cost or reliability minimization without the capability to incorporate all important attributes simultaneously. Consequently, in this research, we developed a comprehensive allocation process with four types of optimization models for passenger rail system design: (1) maximization of system reliability, (2) maximization of service reliability, (3) minimization of LCC, and (4) minimization of a combination of service unreliability (delay cost) and LCC. Based on the characteristics of a passenger rail system and possible alternatives, the proposed process can optimally allocate LCC and reliability to determine the optimal investment plan for rail system design. This comprehensive approach can help users identify the ideal balance between cost and reliability so as to achieve an optimal rail system design.

METHODOLOGY

When designing a new rail system or upgrading an existing one, a number of products for each subsystem can be selected from several equipment suppliers. Figure 1 illustrates a rail system that consists of five subsystems and seven corresponding components. This relationship is based on the structure of the design and procurement process. The product to be acquired can be the subsystem itself (e.g., rolling stock and station) or the corresponding components within the subsystems, such as primary and backup substations in electricity subsystem. The conceptual illustration in Fig. 1 can be applied to a system with more layers of subsystems (e.g., subsubsystems) and more components.

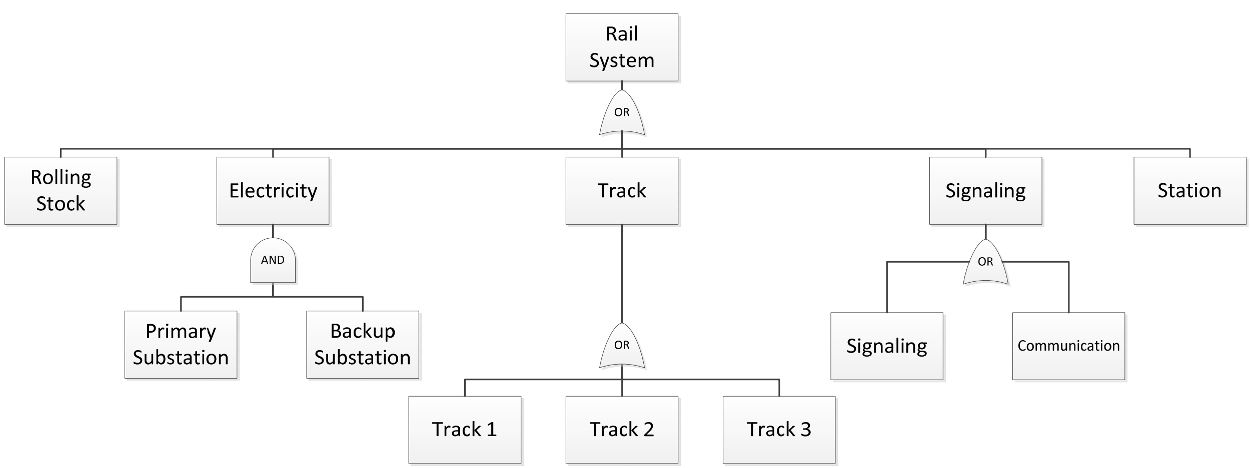


FIGURE 1 Exemplar Structure of a Rail System

Each product has its own system reliability, service reliability, and LCC, which is calculated or obtained from the data provided by suppliers and operators. The system reliability of each product can be obtained from suppliers. However, LCC and service reliability have to be derived from product specifications and an operation plan. The following items define the three important elements for decision making:

- System reliability: This element refers to the probability that the rail system is in normal condition and will satisfactorily perform the task for which it was designed or intended for a specified time and in a specified environment. This probability can be determined by failure rate, usually denoted by λ, which is the reciprocal of the MTBF or MDBF for a product (λ=1/MTBF or λ=1/MDBF). A lower failure rate results in improved system reliability.
- LCC: This element is the total cost throughout the life of the product, which includes capital investment (planning and design), operating cost, and maintenance cost. Some products have low initial costs but high operating and maintenance costs; others with high initial costs may have low operating and maintenance costs. LCC is therefore an appropriate indicator for decision making than evaluating capital investment only.
- Service reliability: This element identifies the effect on passengers, which is an important concept adopted in this study. The effect on passengers usually can be characterized as delay. Train delay is used to indicate the effect of failures. Longer delay results in a larger effect on passengers and implies lower service reliability. Service reliability is defined as on-time arrival percentage (without any buffer), which is the proportion of on-time operations in terms of system operating time in train-hour, as shown in Equation (1):

$$r_{sys} = \left(\frac{P - \sum_{i \in I} \sum_{k \in K} D_j f_j}{P} \right) \times 100\% , \tag{1}$$

where

r_{sys}	=	on-time arrival percentage,
P	=	total system operational time (in train-hour) in a defined period,
D_j	=	delay (in train-hour) of subsystem j within operational time or distance, and
f_j	=	failure rate.

Equation (2) computes the delay for subsystem j within the total operational time or distance, which is the product of the operational time or distance (T_j), average number of online trains (N), and average delay time (Q_j). T_j and N are computed by demand, cycle time, train capacity, and design (or average) loading factor. For example, N = (*Demand in Maximum Loading Section* x *Cycle Time) / (Train Capacity* x *Loading Factor).* The average delay from failure of subsystem j denoted by Q_j is the key to evaluating service reliability of an alternative. This parameter is estimated by using historical data from similar systems.

Alternatively, this parameter is determined by simulating the service effect from possible types of failures.

$$D_j = T_j N Q_j \qquad \forall j \in J , \qquad (2)$$

where

T_j = operational time or distance of subsystem *j*,

N = average number of online trains, and

Q_j = average delay (in hours) from a failure of subsystem *j*.

Conventional on-time percentage is computed by the "number of trains" that arrive at the terminal station within a buffer time (e.g., 5 min). If the scheduled and actual arrival times of a train service are within the buffer, then delay is not computed against on-time percentage. When this kind of indicator is used, frequent failures within buffer time or rare failures with long delay time may not significantly affect the conventional on-time percentage. However, these two situations can significantly affect customer satisfaction. Therefore, for the purpose of this study, "delay time" is used instead of the "number of trains" to show an accurate failure effect on customers. We consider all delays during the decision-making process without adopting any buffer to reveal actual service reliability.

These three elements are then incorporated into the optimization models to determine optimal allocation of reliability and LCC. In addition, further consideration should be given to the relationship among system, subsystems, and components. According to Figure 1, the railway system consists of five subsystems with an "OR" relationship. This relationship indicates that the system will fail if one of the five subsystems fails. Similarly, the signaling subsystem consists of two corresponding components, signaling and communication, with an "OR" relationship, which means that the subsystem will fail if one of the two components fails. Moreover, the electricity subsystem consists of two components, namely, primary and backup substations, with an "AND" relationship, which means that the subsystem will fail only if both the primary and backup substations fail at the same time. Rolling stock subsystem does not consist of any subsequent components, and therefore can be regarded as a component itself. The relationship among the system, subsystems, and components should be considered in the decision-making process to accurately capture the effect of potential failures, especially in the computation of failure rate to determine the optimal allocation of resources.

Figure 2 demonstrates the input and output of the optimization process for a rail system design. The optimization process aims to determine the optimal investment plan for rail systems based on available alternatives. The optimal investment plan identifies the best alternative for every subsystem according to acceptable LCC, system reliability, or service reliability. Four optimization models are developed based on different points of view, namely: (1) Maximize System Reliability Model (MSysRM), which only considers LCC and system reliability, and maximizes system reliability according to available LCC; (2) Maximize Service Reliability Model (MSerRM), which considers all three elements, and maximizes service reliability according to available LCC and system reliability; (3) Minimize LCC Model (MCM), which minimizes total LCC according to acceptable service reliability and system reliability; and (4) Integrated Model (IM), which minimizes the summation of LCC and delay cost according to acceptable service reliability, LCC, and system reliability.

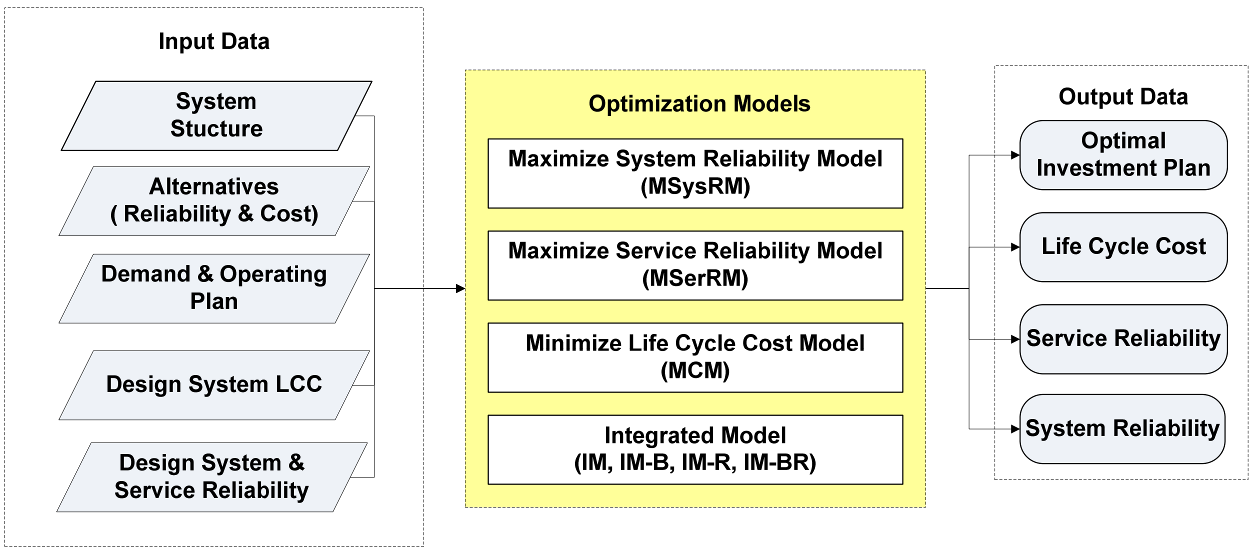


FIGURE 2 Optimization process and models for passenger rail system design

MSysRM represents the current practices with consideration of only system reliability instead of service reliability. The output of this model can be used as a benchmark to compare with outputs from other models with consideration of service reliability. Both MSerRM and MCM consider service reliability but from two different points of view, which are to minimize cost or to maximize service reliability. Finally, the integrated model combines both objectives, minimization of cost and service reliability, to determine the optimal allocation of resources. The optimization models employ the following notations: i is the index of the component (Figure 1). j is the index of subsystem; n is the index of alternatives of the component; I, J, and N are the sets of i, j, and n, respectively. k is the index of possible combinations of the alternatives within a subsystem j, and K_j denotes the set of k within subsystem j. J^T and J^r are both subsets of J, where the former represents the rail system and the latter represents all the subsystems. J^s is the subset of J, representing the subsystem with an "OR" relationship with subsequent elements (e.g., rail system, and the track subsystem in Figure 1). J^P is the subset of J, representing the subsystem with an "AND" relationship with subsequent elements (e.g., electricity subsystem in Figure 1). V_k is the set of corresponding alternatives in combination k, where $(i, n) \in V_K$. N_i is the set of alternatives of i elements; C_{in} is LCC of alternative n of component i; D_j is average delay of subsystem j; F^C_{in} is failure rate of alternative n of component i; U_j is the amount of elements in j subsystem; P is total operational time per year; R is design service reliability; B is design LCC; E is design total system reliability in terms of failure rate; G_j is design system reliability in terms of failure rate of each subsystem j; T is time value of a train; H is the planning horizon of the project, which takes net present value into account; and m represents an arbitrary small number, e.g., -1×10^8.

The following models have five types of decision variables:

- δ_{in} is a binary variable denoting the selection of the alternative, where

$$\delta_{in} = \begin{cases} 1, \text{ if alternative } n \text{ of element } i \text{ is selected} \\ 0, \text{ otherwise} \end{cases}.$$

- y_{jk} is a binary variable denoting the selection of the combination, where

$$y_{jk} = \begin{cases} 1, \text{ if combination } k \text{ of subsystem } j \text{ is selected} \\ 0, \text{ otherwise} \end{cases}.$$

- f^C_{jk} is a continuous variable denoting the failure rate of each combination k in subsystem j.
- f^B_j is a continuous variable denoting the failure rate of subsystem j.
- f is a continuous variable denoting the failure rate of the system.

The following are the formulations of the four types of optimization models.

Maximize System Reliability Model (MSysRM)

The current practices only consider system reliability and cost for reliability allocation problems. Therefore, we developed MSysRM as the benchmark model to maximize system reliability as an objective function with constraints on the design LCC. The model is expressed as follows:

$$Max \quad 1 - f \tag{3}$$

$s.t.$

$$\sum_{k \in K} f^m_{jk} y_{jk} \le G_j \qquad \forall j \in J_r \tag{4}$$

$$\sum_{n \in N_i} \delta_{in} = 1 \qquad \forall i \in I, N_i \in N \quad , \qquad (5)$$

$$\sum_{k \in K} y_{jk} = 1 \qquad \forall j \in J_r \quad , \qquad (6)$$

$$\sum_{i \in I} \sum_{n \in N} C_{in} \delta_{in} \le B \qquad (7)$$

$$\sum_{(i,n) \in V_k} \delta_{in} \le y_{jk} + (U_j - 1) \qquad \forall k \in K_j, j \in J_r \ , \qquad (8)$$

$$f_{jk}^{C} \ge F_{in}^{C} + m(1 - y_{jk}) \qquad \forall (i,n) \in V_k, k \in K_j, j \in J_s, \qquad (9)$$

$$f_{jk}^{C} \ge \prod_{(i,n) \in V_k} F_{in}^{C} + m(1 - y_{jk}) \qquad \forall k \in K_j, j \in J_p \ , \qquad (10)$$

$$f_{j}^{B} \ge f_{jk}^{C} \qquad \forall k \in K_j, j \in J_r, \qquad (11)$$

$$f \ge f_{j}^{B} \qquad \forall j \in J_r, \qquad (12)$$

and

$$\delta_{in} \in \{0,1\} \qquad \forall i \in I, n \in N_i, \qquad (13)$$

$$y_{jk} \in \{0,1\} \qquad \forall k \in K_j, j \in J \ , \qquad (14)$$

$$f_{jk}^{C} \ge 0 \qquad \forall k \in K_j, j \in J \ , \qquad (15)$$

$$f_{j}^{B} \ge 0 \qquad \forall j \in J \ , \qquad (16)$$

$$f \ge 0 \ . \qquad (17)$$

The objective function, Equation (3), maximizes system reliability, which is 1 minus the failure rate of the rail system. Equation (4) guarantees that the system reliability (in failure rate) of each subsystem should be less than the design system reliability (in failure rate). Equation (5) guarantees that only one alternative is chosen for a component. Equation (6) guarantees that only one combination (k) can be selected for a subsystem. Equation (7) is the budget constraint. Equation (8) establishes the relationship between the selection of a combination for a subsystem (such as y_{jk}=1) and the selection of corresponding components (such as $\delta_{in} = 1$). Equation (9) determines the failure rate of combination *k* of subsystem *j* (i.e., f_{jk}^{C}) consisting of components with an "OR" relationship, which returns the highest failure rate among all components *i* within the subsystem. Equation (10) determines the failure rate of combination *k* of subsystem *j* (i.e., f_{jk}^{C}) consisting of components with an "AND" relationship, which returns the failure rate as the product of the failure rate of all components *i* within the subsystem. Equation (11) returns the value of the failure rate of subsystem *j* (i.e., f_{j}^{B}) from f_{jk}^{C}. Similarly, Equation (12) returns the value of the failure rate of the system (i.e., *f*) from f_{j}^{B}. Equations (13) to (17) demonstrate the properties of the variables.

Maximize Service Reliability Model (MSerRM)

The previous model (MSysRM) focuses only on system reliability instead of service reliability, and cannot reflect the effect of failures on passengers. Thus, MSerRM is developed to maximize service reliability with constraints on the design LCC. The model is expressed as follows:

$$Max \quad \left(\frac{P - \sum_{j \in J_r} D_j f_j^B}{P} \right) \times 100\%, \tag{18}$$

s.t.

$$f \le E \tag{19}$$

$$\sum_{k \in K} f_{jk}^{m} y_{jk} \le G_j \qquad \forall j \in J_r \quad , \tag{20}$$

$$\sum_{n \in N_i} \delta_{in} = 1 \qquad \forall i \in I, N_i \in N, \tag{21}$$

$$\sum_{k \in K} y_{jk} = 1 \qquad \forall j \in J_r, \tag{22}$$

$$\sum_{i \in I} \sum_{n \in N} C_{in} \delta_{in} \le B, \tag{23}$$

$$\sum_{(i,n) \in V_k} \delta_{in} \le y_{jk} + (U_j - 1) \qquad \forall k \in K_j, j \in J_r, \tag{24}$$

$$f_{jk}^{C} \ge F_{in}^{C} + m(1 - y_{jk}) \qquad \forall (i,n) \in V_k, k \in K_j, j \in J_s, \tag{25}$$

$$f_{jk}^{C} \geq \prod_{(i,n)\in V_k} F_{in}^{C} + m(1-y_{jk}) \qquad \forall k \in K_j, j \in J_p\ , \qquad (26)$$

$$f_{j}^{B} \geq f_{jk}^{C} \qquad \forall k \in K_j, j \in J_r\ , \qquad (27)$$

$$f \geq f_{j}^{B} \qquad \forall j \in J_r\ , \qquad (28)$$

and

$$\delta_{in} \in \{0,1\} \qquad \forall i \in I, n \in N_i\ , \qquad (29)$$

$$y_{jk} \in \{0,1\} \qquad \forall k \in K_j, j \in J\ , \qquad (30)$$

$$f_{jk}^{C} \geq 0 \qquad \forall k \in K_j, j \in J\ , \qquad (31)$$

$$f_{j}^{B} \geq 0 \qquad \forall j \in J\ , \qquad (32)$$

$$f \geq 0\ . \qquad (33)$$

Equation (18) maximizes service reliability in terms of on-time arrival percentage. The on-time arrival percentage can effectively depict the feelings of passengers by considering the impact on them. Given that the objective function focuses on service reliability, Equation (19) is added to this formulation to guarantee the minimum requirement of the failure rate of the system. Equations (20) to (28) are the same as Equations (4) to (12). Equations (29) to (33) demonstrate the properties of the variables.

Minimize Cost Model (MCM)

Besides maximization of reliability, reliability allocation problems could be implemented by minimizing cost with constraints imposed on reliability. To address possible needs of planners, MCM can minimize total system LCC under the constraints of system and service reliability. The model is expressed as follows:

$$Min \quad \sum_{i\in I}\sum_{n\in N} C_{in}\delta_{in}\ , \qquad (34)$$

s.t.

$$f \leq E \qquad (35)$$

$$\sum_{k\in K} f_{jk}^{m} y_{jk} \leq G_j \qquad \forall j \in J_r\ , \qquad (36)$$

$$\left(\frac{P-\sum_{j\in J_r} D_j f_j^B}{P}\right)\times 100\% \geq R, \tag{37}$$

$$\sum_{n\in N_i} \delta_{in} = 1 \qquad \forall i\in I, N_i \in N, \tag{38}$$

$$\sum_{k\in K} y_{jk} = 1 \qquad \forall j\in J_r, \tag{39}$$

$$\sum_{(i,n)\in V_k} \delta_{in} \leq y_{jk} + (U_j - 1) \qquad \forall k\in K_j, j\in J_r, \tag{40}$$

$$f_{jk}^C \geq F_{in}^C + m(1-y_{jk}) \qquad \forall (i,n)\in V_k, k\in K_j, j\in J_s, \tag{41}$$

$$f_{jk}^C \geq \prod_{(i,n)\in V_k} F_{in}^C + m(1-y_{jk}) \qquad \forall k\in K_j, j\in J_p, \tag{42}$$

$$f_j^B \geq f_{jk}^C \qquad \forall k\in K_j, j\in J_r, \tag{43}$$

$$f \geq f_j^B \qquad \forall j\in J_r, \tag{44}$$

and

$$\delta_{in} \in \{0,1\} \qquad \forall i\in I, n\in N_i, \tag{45}$$

$$y_{jk} \in \{0,1\} \qquad \forall k\in K_j, j\in J, \tag{46}$$

$$f_{jk}^C \geq 0 \qquad \forall k\in K_j, j\in J, \tag{47}$$

$$f_j^B \geq 0 \qquad \forall j\in J, \tag{48}$$

$$f \geq 0. \tag{49}$$

The objective function, Equation (34), minimizes total LCC over the planning horizon. Total LCC refers to the sum of costs of all subsystems and components determined by the optimization process. Equation (35) indicates the constraint on system reliability for the rail system as a whole, and Equation (36) indicates the constraint on system reliability for the subsystems. Equation (37) indicates the constraint on service reliability. Equations (38) and (39) are the same as Equations (5) and (6). Equations (40) to (44) are the same as Equations (8) to (12). Equations (45) to (49) demonstrate the properties of the variables.

Integrated Model (IM)

In MSysRM, MSerRM, and MCM, the elements of service reliability and LCC are discussed separately as an objective function or constraints. To integrate those two important considerations in system design, we developed an integrated optimization model to consider cost minimization and reliability maximization simultaneously. To account for service reliability, on-time arrival percentage is replaced by using train delay, which could then be converted into delay cost by using a design time value. With this conversion, the objective function could be maintained in the same direction as that of minimization of both LCC and delay cost. The new model is depicted as follows:

$$Min \quad \sum_{i\in I}\sum_{n\in N} C_{in}\delta_{in} + HT\sum_{j\in J_r} D_j f_j^B , \qquad (50)$$

s.t.

$$f \le E \qquad (51)$$

$$\sum_{k\in K} f_{jk}^m y_{jk} \le G_j \qquad \forall j\in J_r , \qquad (52)$$

$$\sum_{n\in N_i} \delta_{in} = 1 \qquad \forall i\in I, N_i \in N , \qquad (53)$$

$$\sum_{k\in K} y_{jk} = 1 \qquad \forall j\in J_r , \qquad (54)$$

$$\sum_{(i,n)\in V_k} \delta_{in} \le y_{jk} + (U_j - 1) \qquad \forall k\in K_j, j\in J_r , \qquad (55)$$

$$f_{jk}^C \ge F_{in}^C + m(1 - y_{jk}) \qquad \forall (i,n)\in V_k, k\in K_j, j\in J_s , \qquad (56)$$

$$f_{jk}^C \ge \prod_{(i,n)\in V_k} F_{in}^C + m(1 - y_{jk}) \qquad \forall k\in K_j, j\in J_p , \qquad (57)$$

$$f_j^B \ge f_{jk}^C \qquad \forall k\in K_j, j\in J_r , \qquad (58)$$

$$f \ge f_j^B \qquad \forall j\in J_r , \qquad (59)$$

and

$$\delta_{in} \in \{0,1\} \qquad \forall i\in I, n\in N_i , \qquad (60)$$

$$y_{jk} \in \{0,1\} \qquad \forall k \in K_j, j \in J, \tag{61}$$

$$f_{jk}^{C} \geq 0 \qquad \forall k \in K_j, j \in J, \tag{62}$$

$$f_{j}^{B} \geq 0 \qquad \forall j \in J, \tag{63}$$

$$f \geq 0. \tag{64}$$

Equation (50) minimizes total LCC and delay cost, which clearly demonstrates the trade-off between cost and reliability. The first part is the sum of LCC of all subsystems and components required in the design process. The second part is the total delay cost, which is computed based on the product of expected delay time and time value within the planning horizon. Equations (51) and (52) are the same as Equations (35) and (36). Equations (53) to (59) are the same as Equations (38) to (44). Equations (60) to (64) demonstrate the properties of the variables.

The core IM model incorporates LCC and service reliability in the objective function to balance the trade-off between these two elements. Equation (65) could be added to the core model to ensure a budget constraint if needed, which guarantees that the total LCC of the optimal investment plan should be within the design LCC:

$$\sum_{i \in I} \sum_{n \in N} C_{in} \delta_{in} \leq B. \tag{65}$$

Similarly, if a minimum on-time arrival percentage is given, Equation (66) could be added to guarantee that service reliability is greater than design service reliability.

$$\left(\frac{P - \sum_{j \in J_r} D_j f_j^B}{P} \right) \times 100\% \geq R. \tag{66}$$

Three models related to IM could be derived based on the two constraints. These models, which are discussed later in the case study, are the following: (1) IM-B, where Equation (65) is used as the addiitonal constraint in IM; (2) IM-R, where Equation (66) is used as the additional constraint in IM; and (3) IM-BR, where Equations (53) and (54) are both used as additional constraints in IM.

CASE STUDY

A passenger rail system project in Taiwan is used to demonstrate how the proposed optimization process is applied to assist a rail system design. We considered a 25-km passenger rail system, which contains five subsystems, namely, rolling stock, electricity, track, signaling, and station. Figure 1 shows the structure of the system, subsystems, subsequent components, and their relationship in computing reliability (i.e., AND or OR). This relationship is based on the structure of the design and procurement process, where the rolling stock is an individual project, the electricity subsystem is separated into primary substation project and backup substation project, track is separated into three projects by sections (or components), signaling is separated into signaling and communication projects, and station is an individual project. Each project has a number of alternatives (such as suppliers, products, and so on) with specific reliability, LCC, and impact on service reliability. Figure 1 illustrates the common type of system structure for passenger rail systems in Taiwan; however, the developed process and models can be adapted to other rail systems with different subsystems and components.

Three case studies were performed by using all the developed models to demonstrate their applicability. The benefit of using service reliability is first demonstrated in Case I, where two optimization models, namely, MSysRM and MSerRM, are used to show the results with and without service reliability consideration. MCM is then applied to Case II to determine the optimal investment plan by selecting appropriate alternatives according to design service reliability. Integrated models (i.e., IM) were employed in Case III to demonstrate the advantages in considering service reliability and LCC simultaneously in the passenger rail system design, followed by a sensitivity analysis on the design time value

The estimated demand of this system is 140,000 trips/day, and the life cycle of a rail system is set at 30 years for the planning horizon. Figure 3 shows the LCC in new Taiwan dollars (NTD, $1 NTD = US$0.031 in 2016) and the failure rate of possible alternatives for every subsystem and component in the system design structure, which is obtained from the suppliers. According to this figure, LCC increases with the decrease of failure rate, which is usually with a non-linear relationship. Service reliability in terms of delay time (D_j) is calculated by Equation (2), where T_j and N are computed based on the required train service according to demand, train capacity, cycle time, and design loading factor. A result of 5,411,347 (train-km/year) is obtained as well as an average of 31 online trains. Q_j is estimated based on historical data from existing systems that have similar characteristics with the new design. P indicates 172,539 train-hours per year. After computing D_j, the LCC, system reliability (in failure rate), and service reliability of all alternatives could be inputted into optimization models to determine the optimal investment plan.

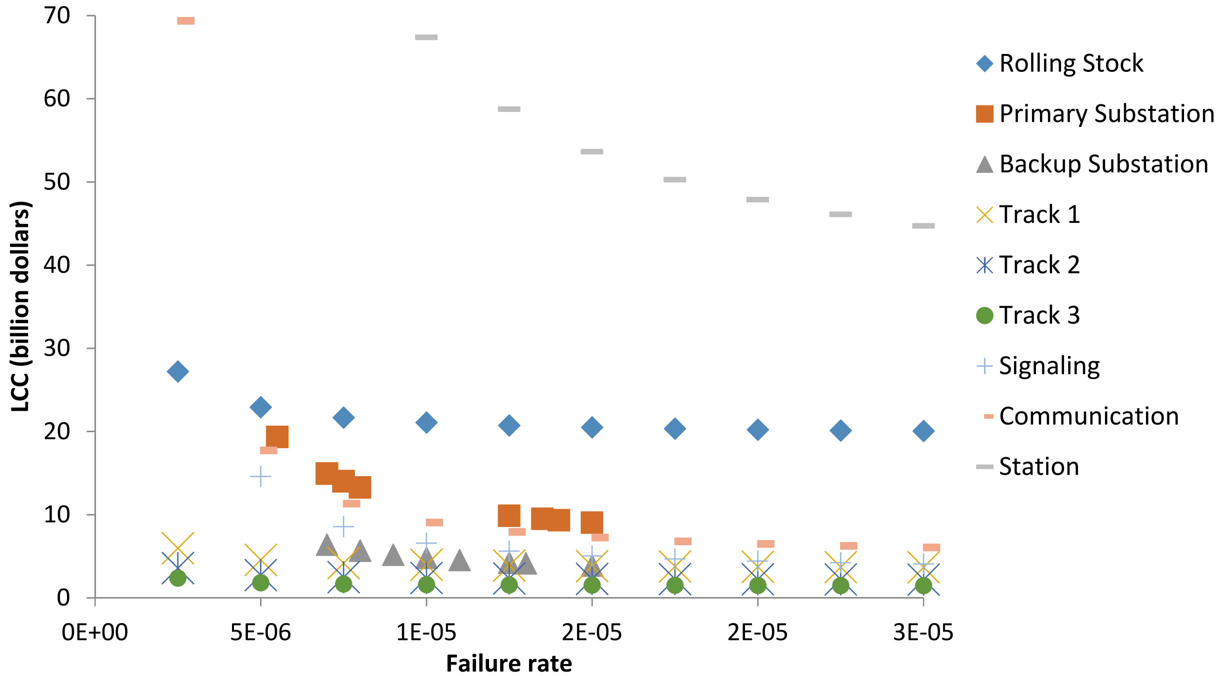


FIGURE 3 Alternatives for New System Design

Case I: Benefit of using Service Reliability - MSysRM vs. MSerRM

The system design process in the past usually focuses on "system reliability" instead of "service reliability." To demonstrate the benefit of using service reliability, two optimization models, namely, MSysRM and MSerRM, are employed with the same LCC budget to show the differences of the models.

Both optimization models are coded into General Algebraic Modeling System (GAMS), a high-level modeling system for mathematical programming and optimization. And, these models are efficiently solved by CPLEX, a popular commercial solver for linear programming problems developed by ILOG, within seconds. Table 1 shows the results for each subsystem in both models. The same budget constraint, 100 billion NTD, is used in both models. As a result, the total LCC of the optimal investment plans between these two models are similar, with less than 0.5% difference. However, the total delay in MSerRM is substantially lower than that in MSysRM by more than 34%. This result implies that the effect on passengers could be significantly reduced when service reliability is considered.

TABLE 1 Optimal Allocation by MSysRM and MSerRM

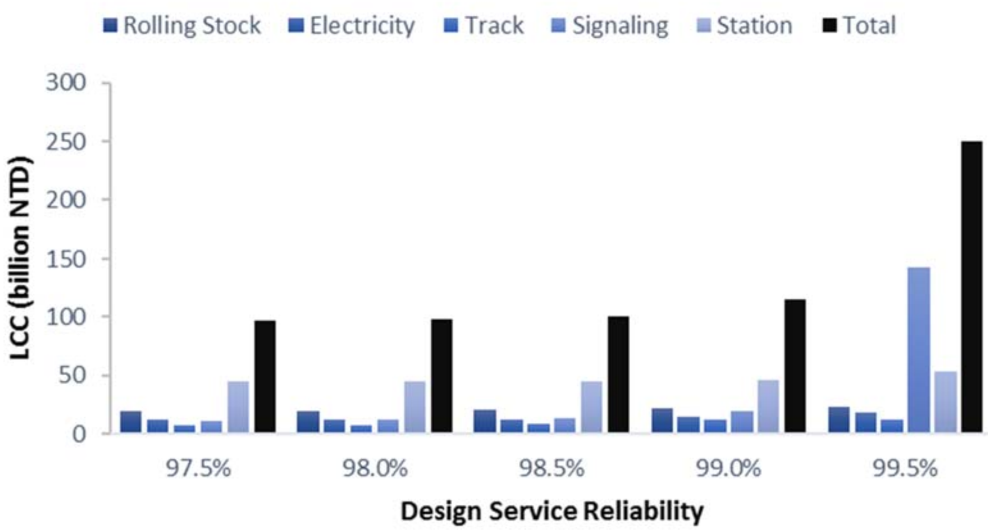
Subsystem	MSerRM			MSysRM		
	Failure rate	LCC (billion NTD)	Delay (train-hour)	Failure rate	LCC (billion NTD)	Delay (train-hour)
Rolling Stock	1.75E-05	20.35	201.48	2.00E-05	20.23	230.3
Electricity	4.50E-06	12.94	282.40	4.50E-06	12.94	282.4
Track	7.50E-06	8.36	375.23	2.00E-05	7.51	1,000.6
Signaling	1.25E-05	13.56	1,558.05	2.00E-05	10.94	2,492.9
Station	2.50E-05	44.75	471.07	2.00E-05	47.90	376.9
Total		99.95	2,888.2		99.50	4,383.0

According to the delay of each subsystem, track and signaling encounter considerable delay when only system reliability is considered. With the consideration of service reliability, the delay of track and signaling significantly decreases compared with that of other subsystems. Considering the LCC of each subsystem, we observe that MSerRM allocates more money in rolling stock, track, and signaling than MSysRM because of higher impact on service reliability. A smaller budget is allocated to the station project in MSerRM because the impact on service reliability is relatively lower than that of other systems. The results of this case show that service reliability could efficiently decrease delay and increase passenger satisfaction. MSerRM allocates the optimal service reliability of each subsystem to maximize service reliability, where the system reliability of each subsystem fulfills the design requirement.

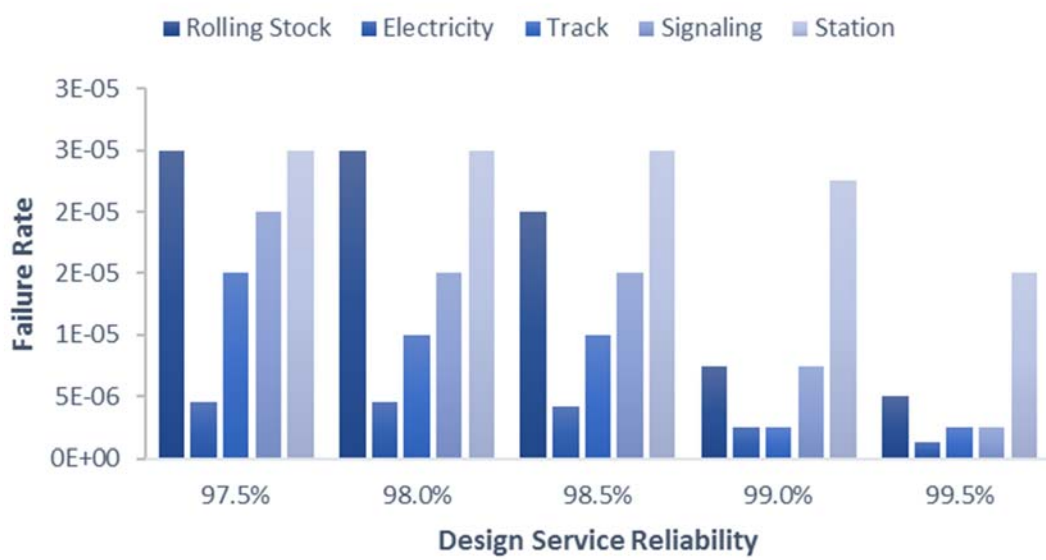
Case II: New System Design - MCM

MCM is employed in Case II to determine an optimal investment plan with minimal LCC. When designing a new rail system, cost minimization is one of the most common objectives in the process. At the same time, service and system reliability should be set to an acceptable value according to passenger satisfaction and regulations. To examine the results from MCM with different design service reliability levels, five design service reliability constraints from 97.5% to 99.5% with 0.5% increment are implemented in this case. All scenarios are again coded in GAMS and solved by CPLEX within seconds.

Figure 4a demonstrates the LCC allocation for each subsystem (denoted by the blue bars from left to right respectively: rolling stock, electricity, track, signaling, and station), and the total LCC (denoted by the black bar). The total cost range is from $97.25 billion to $259.82 billion NTD. The LCC generally increases with service reliability. The increase in total LCC from 97.5% to 98.5% for design service reliability is modest. However, total LCC increased sharply from 99.0% to 99.5% because of the nonlinear relationship between cost and reliability. The proportion of allocation is very similar when the design service reliability increased from 97.5% to 98.5%. The proportions of track and signaling became relatively higher when the design service reliability increased to 99%. The proportion allocated to the signaling system sharply increased at 99.5% design service reliability to meet high standards of service reliability.



(a)



(b)

FIGURE 4 Allocation of (a) LCC and (b) Failure rate in MCM

Figure 4b shows the corresponding failure rate of each subsystem under the desired level of service reliability. With 97.5% service reliability, electricity has the lowest failure rate among all subsystems. Significant reduction in failure rate of the other subsystems could be observed with the increase in design service reliability. High design service reliability results in low failure rate. According to Figures 4a and 4b, the reliability of several subsystems could be significantly reduced by a slight increase in LCC. Results from all these scenarios illustrate that MCM can efficiently and successfully identify the optimal investment plan according to design service reliability.

Case III: New System Design - IM

The previous cases demonstrate the importance of maximizing service reliability and minimizing LCC during the system design process. Through the use of delay and delay cost, the IM is able to incorporate service reliability and LCC simultaneously in the objective function of the optimization model. To use IM, delay time (in train-hour) has to be converted into delay cost first by using time value. When failure happens, the delay time is viewed as the out-of-vehicle time during which passengers have to wait for the next train. In this case, time value is estimated by time value per passenger and the number of average passengers on the train. The average time value per person is estimated to be 11.24 NTD/person-minute and the average number of passengers per train is 96 (*18*). The resulting time value (T) is 64,742 NTD/train-hour. This case is again coded in GAMS and solved by CPLEX within seconds. Table 2 shows the optimal allocation determined by IM. The resulting LCC is $97.25 billion NTD, the total delay cost of the planning horizon is $7.23 billion NTD, and service reliability in on-time arrival rate is 97.84%. This allocation demonstrates the optimal balance between LCC and service reliability at a given time value.

TABLE 2 Optimal Investment Plan by IM

Subsystem	Failure rate	LCC (billion NTD)	Delay cost (billion NTD)
Rolling stock	2.50E-05	20.06	0.56
Electricity	4.50E-06	12.94	0.55
Track	1.00E-05	8.00	0.97
Signaling	1.75E-05	11.51	4.24
Stations	2.50E-05	44.75	0.92
Total		**97.25**	**7.23**

IM could be regarded as a multi-objective model that considers both LCC and service reliability. By varying the design time value, a pareto front can be observed through the use of IM (Figure 5). Each point is the optimal balance between LCC and service reliability at a particular time value. A higher time value increases the relative importance of the service reliability and causes the results to move towards to the left, which indicates higher service reliability (lower delay cost).

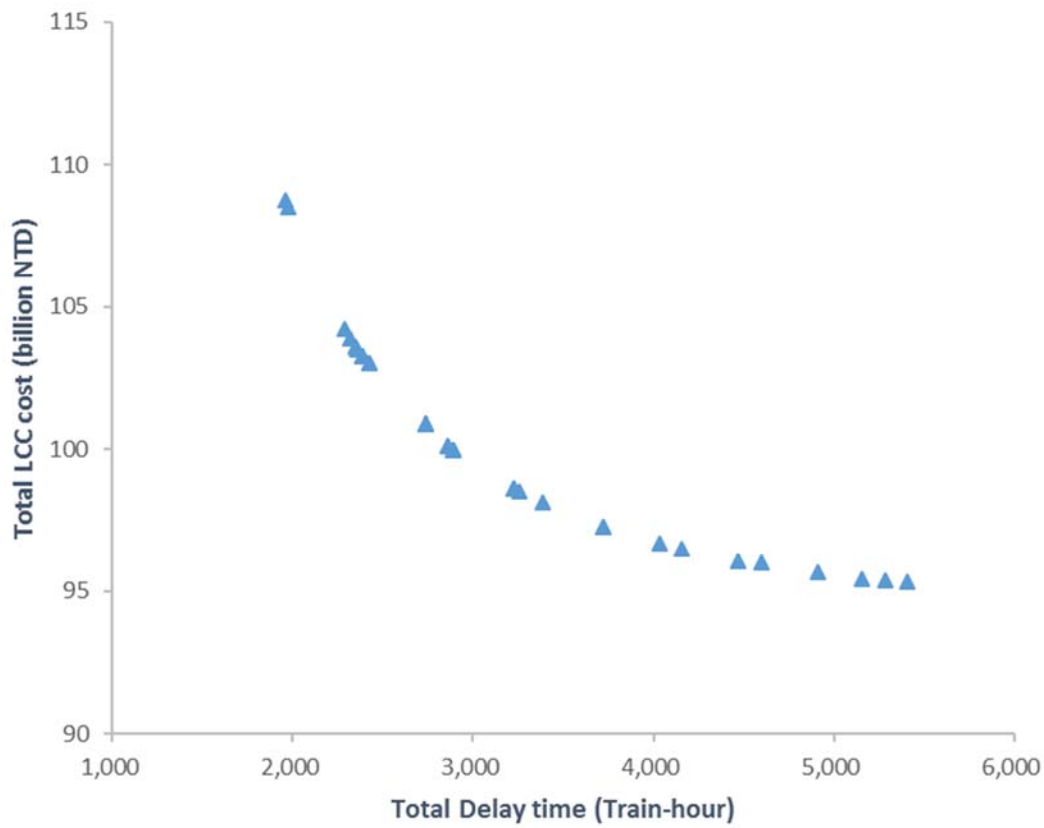


FIGURE 5 Pareto Front of Optimal Allocation

Sensitivity Analysis

A sensitivity analysis was performed by examining the impact on service reliability of different time values that varied from 10,000 to 150,000 NTD/train-hour. In addition to the core IM model, IM-B with budget constraint (Equation (53)), IM-R with service reliability constraint (Equation (54)), and IM-BR with both additional constraints were implemented to illustrate their applicability and relationship. The maximum LCC is set to be 100 billion NTD in both IM-B and IM-BR, and the minimum service reliability is set to be 97.7% in both IM-R and IM-BR. Figure 6 shows the results of service reliability from different types of models with different time values. The result from IM shows that service reliability increased with design time value, as expected.

IM-B has a similar trend with IM moving up until approximately 110,000 NTD/train-hour, whereas IM-R has a similar trend with IM after the design time value exceeded 60,000 NTD/train-hour. IM-BR is constrained by budget and service reliability; thus, it has the same trend as IM in terms of time value between 60,000 and 110,000 NTD/train-hour.

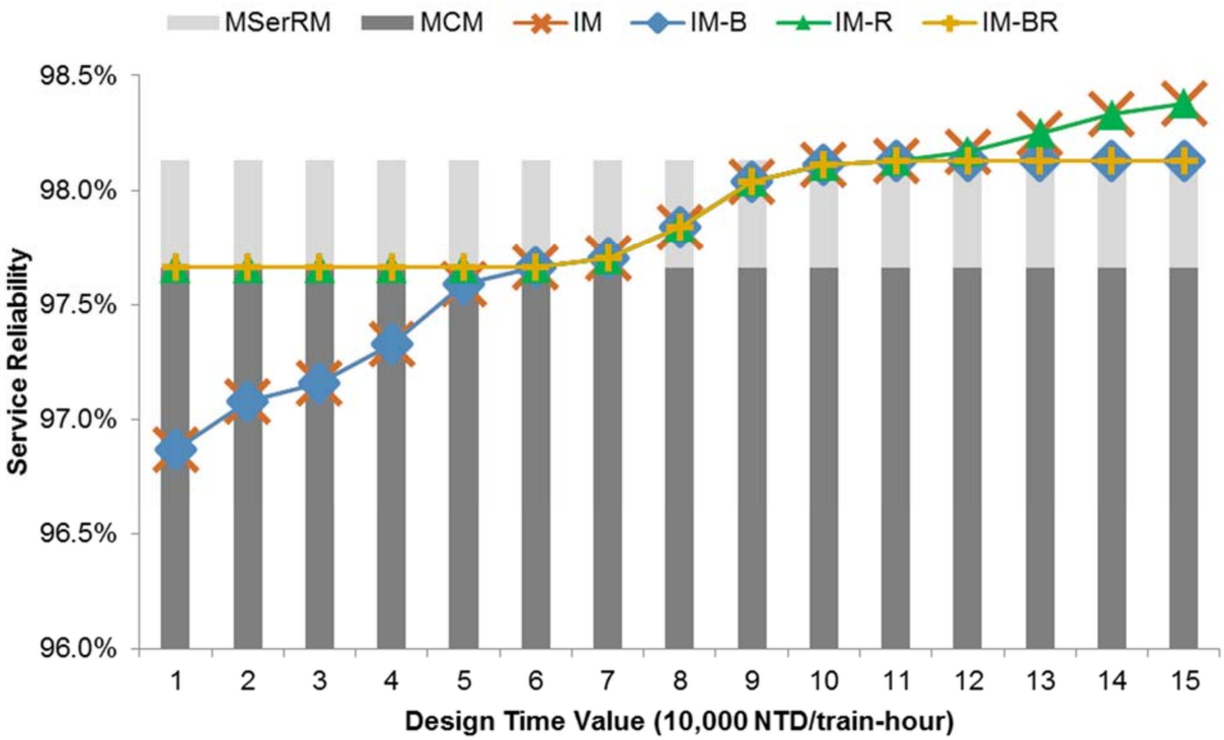


FIGURE 6 Sensitivity Analysis for IMs, MSerRM, and MCM.

In Figure 6, for sections less than 60,000 NTD/train-hour, results of IM-R and IM-BR reached the constraint of service reliability (i.e., 97.7%), which is the lower bound of the models. The models have the same results no matter what time value is used. However, IM and IM-B have no service reliability constraint; thus, results decrease with decreasing time value. For sections between 60,000 and 110,000 NTD/train-hour, all types of IM models obtained the same results because neither the budget constraint nor the service reliability constraint is binding. For the section greater than 110,000 NTD/train-hour, both IM-B and IM-BR reached the constraint of LCC (i.e., 100 billion NTD), which is the upper bound of these models. As a result, these models have the same results no matter what time value is used. However, IM and IM-R have no LCC constraint; thus, the results increase with an increasing time value.

The four types of IM can be discussed together with MSerRM and MCM. Figure 6 shows service reliability from different types of IM, MSerRM, and MCM. The results from different types of IM have been discussed. However, these results are related to MSerRM and MCM. When the time value is less than 60,000 NTD/train-hour, the results from IM-R and IM-BR are the same as that of MCM in Case II because of the same service reliability constraint. When the time value is greater than 110,000 NTD/train-hour, the results from IM-B and IM-BR are the same as that of MSerRM in Case I because of the same LCC constraint. These results demonstrated that the integrated model, IM-BR, considers all objectives and constraints in the formulation, which makes the model powerful and flexible for application in railway system design.

CONCLUSIONS

Carefully designed systems with good resource allocation can prevent costly reworking and improvement. This study develops a comprehensive allocation process with four types of optimization models for passenger rail system design: (1) maximization of system reliability, (2) maximization of service reliability, (3) minimization of LCC, and (4) minimization of a combination of service unreliability (delay cost) and LCC. Based on the characteristics of a passenger rail system and possible alternatives, the proposed process can allocate LCC and service reliability optimally to determine the optimal investment plan for rail system design. Empirical case studies demonstrate that the proposed optimization process and models can evaluate efficiently and successfully all possible alternatives and determine the optimal allocation among all subsystems and corresponding components. This comprehensive approach can help users identify the ideal balance between cost and reliability so as to achieve an optimal rail system design.

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